Trigonometry – 1

Exercise – 3

1. **(A)**

$A + B + C = \pi$ &

$$\sin\left(A + \frac{C}{2}\right) = k\sin\left(\frac{C}{2}\right)$$

$$\frac{\sin\left(A + \frac{C}{2}\right)}{\sin\frac{C}{2}} = k$$

By componendo and dividendo

$$\frac{k+1}{k-1} = \frac{\sin\left(A + \frac{C}{2}\right) + \sin\frac{C}{2}}{\sin\left(A + \frac{C}{2}\right) - \sin\frac{C}{2}}$$

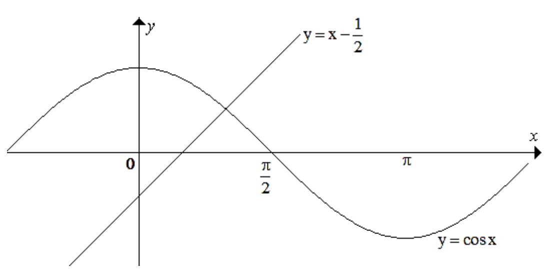
$$= \frac{2\sin\left(\frac{A+C}{2}\right)\cos\left(\frac{A}{2}\right)}{2\sin\left(\frac{A}{2}\right)\cos\left(\frac{A+C}{2}\right)}$$

$$= \frac{\tan\left(\frac{A+C}{2}\right)}{\tan\left(\frac{A}{2}\right)} = \frac{\cot\left(\frac{B}{2}\right)}{\tan\left(\frac{A}{2}\right)} = \frac{1}{\tan\frac{A}{2}\tan\frac{B}{2}}$$

$$\Rightarrow \tan\frac{A}{2}\tan\frac{B}{2} = \frac{k-1}{k+1}$$

2. **(A)**

$$\cos x = x - \frac{1}{2}$$



3. **(A)**

$$N^{r}: 2(\sin 1^\circ + \sin 2^\circ + \dots\dots + \sin 89^\circ) = \frac{2\sin\left(\frac{89^\circ}{2}\right)\sin\left(45^\circ\right)}{\sin\left(\frac{1^\circ}{2}\right)}$$

$$D^{r}: 2(\cos 1^\circ + \cos 2^\circ + \dots\dots + \cos 44^\circ) + 1 = \frac{2\sin\left(22^\circ\right)\cos\left(\frac{45^\circ}{2}\right)}{\sin\left(\frac{1^\circ}{2}\right)} + 1$$

$$= \frac{-\sin\left(\frac{1^\circ}{2}\right) + \sin\left(\frac{89^\circ}{2}\right)}{\sin\left(\frac{1^\circ}{2}\right)} + 1$$

$$= \frac{\sin\left(\frac{89^\circ}{2}\right)}{\sin\left(\frac{1^\circ}{2}\right)} - 1 + 1 = \frac{\sin\left(\frac{89^\circ}{2}\right)}{\sin\left(\frac{1^\circ}{2}\right)}$$

$$\frac{N^{r}}{D^{r}} = \frac{\dfrac{2\sin\left(\frac{89^\circ}{2}\right)\sin\left(45^\circ\right)}{\sin\left(\frac{1^\circ}{2}\right)}}{\dfrac{\sin\left(\frac{89^\circ}{2}\right)}{\sin\left(\frac{1^\circ}{2}\right)}}$$

$$= 2\sin 45^\circ = \sqrt{2}$$

4. **(A)**

$$\sin 2x = \frac{2024}{2025}\ ,\ \frac{5\pi}{4} < x < \frac{9\pi}{4}$$

$$(\sin x - \cos x)^2 = 1 - \sin 2x = 1 - \frac{2024}{2025}$$

$\therefore \left|(\sin x - \cos x)\right| = \frac{1}{45}$

i.e. $\sin x - \cos x = \pm \frac{1}{45}$

But range of x is given for $\frac{5\pi}{4} < x < \frac{9\pi}{4}$

sin x < cox x < 50 hence sin x + cos x should be negative

$\sin x - \cos x = \frac{-1}{45}$

5. **(D)**

$\sin\phi = \frac{1}{\sqrt{10}} \Rightarrow \tan\phi = \frac{1}{3}$

Now $\tan 2\phi = \frac{2\tan\phi}{1-\tan^2\phi} \Rightarrow \tan 2\phi = \frac{2\times\frac{1}{3}}{1-\frac{1}{9}}$

$\Rightarrow \tan 2\phi = \frac{3}{4}$

Now $\tan(\theta+2\phi) = \frac{\tan\theta + \tan 2\phi}{1-\tan\theta\tan 2\phi} \Rightarrow \tan(\theta+2\phi) = \frac{\frac{1}{7}+\frac{3}{4}}{1-\frac{1}{7}\times\frac{3}{4}}$

$\Rightarrow \tan(\theta+2\phi) = 1 \Rightarrow \theta+2\phi = 45^o$

6. **(D)**

$\frac{\cot A}{1+\cot A}.\frac{\cot B}{1+\cot B} = \frac{1}{(1+\tan A)(1+\tan B)}$

Now $A+B = 225^o \Rightarrow \frac{\tan A + \tan B}{1-\tan A\tan B} = 1$

$\Rightarrow \tan A + \tan B + \tan A\tan B = 1$

$\Rightarrow (1+\tan A)(1+\tan B) = 2$

$\Rightarrow \frac{\cot A}{1+\cot A}.\frac{\cot B}{1+\cot B} = \frac{1}{2}$

7. **(D)**

$$\cos^2 76^\circ + \cos^2 16^\circ - \cos 76^\circ \cos 16^\circ$$
$$= \frac{1+\cos 152^\circ}{2} + \frac{1+\cos 32^\circ}{2} - \frac{\cos 92^\circ + \cos 60^\circ}{2}$$
$$= \frac{3}{4} + \frac{\cos 152^\circ + \cos 32^\circ - \cos 92^\circ}{2}$$
$$= \frac{3}{4} + \frac{2\cos 92^\circ \cos 60^\circ - \cos 92^\circ}{2}$$
$$= \frac{3}{4}$$

8. (A)

$$\cos^2 \frac{\pi}{12} + \cos^2 \frac{\pi}{4} + \cos^2 \frac{5\pi}{12}$$
$$= \cos^2 \frac{\pi}{12} + \frac{1}{2} + \cos^2\left(\frac{\pi}{2} - \frac{\pi}{12}\right)$$
$$= \cos^2 \frac{\pi}{12} + \sin^2 \frac{\pi}{12} + \frac{1}{2}$$
$$= \frac{3}{2}$$

9. (A)

$$\frac{A}{2} + \frac{B}{2} = 45^\circ$$

Take tan on both sides

$$\frac{\tan\left(\frac{A}{2}\right) + \tan\left(\frac{B}{2}\right)}{1 - \tan\frac{A}{2}\tan\frac{B}{2}} = 1$$

Cross multiply and solve we will get

$$\left(1 + \tan\frac{A}{2}\right)\left(1 + \tan\frac{B}{2}\right) = 2$$

10. (A)

$\sum \tan A = 6$ $\qquad$ $A + B + C = \pi$

$\tan A, \tan B = 2$

$\sum \tan A = \prod \tan A = 6$

$\therefore \dfrac{\prod \tan A}{\tan A \ \tan B} = \tan C = \dfrac{6}{2} = 3$

tan C = 3

$\therefore \ \tan A + \tan B + \tan C = 6$

$\tan A + \tan B = 3$

$\tan A + \tan B = 2$

$\tan A, \ \tan B$ are roots of

$x^2 - 3x + 2 = 0$

x = 1, 2

so, tan A = 1, tan B = 2, tan C = 3

11. (A)

$p = \sin 20^\circ - \cos 20^\circ$

$p^2 = 1 - \sin 40^\circ$ or $\sin 40^\circ = 1 - p^2$

$\therefore \cos 40^\circ = \sqrt{1 - \sin^2 40^\circ} = \sqrt{1 - \left(1 - p^2\right)^2}$

$= \sqrt{\left(1 + 1 - p^2\right)\left(1 + p^2 - 1\right)}$

$= \sqrt{\left(2 - p^2\right)\left(p^2\right)}$

$\cos 40^\circ = |\,p\,| \sqrt{2 - p^2}$

$\cos 40^\circ = -\,p\sqrt{2 - p^2}$

12. (C)

$\sin \alpha = P$

$$\frac{2\tan\frac{\alpha}{2}}{1+\tan^2\frac{\alpha}{2}} = P$$

$$P\tan^2\frac{\alpha}{2} + P - 2\tan\frac{\alpha}{2} = 0$$

$$P\tan^2\frac{\alpha}{2} - 2\tan\frac{\alpha}{2} + P = 0$$

Now product of the roots $= \frac{C}{A} = \frac{P}{P} = 1$

So if one root is $\tan\frac{\alpha}{2}$ other is $\cot\frac{\alpha}{2}$

$\therefore$ equation with roots $\tan\frac{\alpha}{2}$ and $\cot\frac{\alpha}{2}$ is $px^2 - 2x + p = 0$

13. (B)

$\cos 290^\circ = \sin 20^\circ$

$\sin 250^\circ = -\cos 20^\circ$

$$\frac{1}{\sin 20^\circ} - \frac{1}{\sqrt{3}\cos 20^\circ} = \frac{\sqrt{3}\cos 20^\circ - \sin 20^\circ}{\sqrt{3}\sin 20^\circ \cos 20^\circ}$$

$$= \frac{2\left(\frac{\sqrt{3}}{2}\cos 20^\circ - \frac{1}{2}\sin 20^\circ\right)}{\frac{\sqrt{3}}{2}\sin 40^\circ}$$

$$= \frac{4}{\sqrt{3}}\,\frac{\sin\left(60^\circ - 20^\circ\right)}{\sin 40^\circ} = \frac{4}{\sqrt{3}} = \frac{4\sqrt{3}}{3}$$

14. (B)

$$\sin x + \sin y = a = 2\sin\left(\frac{x+y}{2}\right)\cos\left(\frac{x-y}{2}\right)$$

$$\cos x + \cos y = b = 2\cos\left(\frac{x+y}{2}\right)\cos\left(\frac{x-y}{2}\right)$$

$$\tan\left(\frac{x+y}{2}\right) = \frac{a}{b}$$

$$\sin(x+y) = \frac{2\left(\frac{a}{b}\right)}{1+\left(\frac{a}{b}\right)^2} = \frac{2ab}{a^2+b^2}$$

15. (B)

$$\cos 2A = \frac{1-\tan^2 A}{1+\tan^2 A} \Rightarrow \cos 2A = \frac{1-\frac{1}{4}}{1+\frac{1}{4}} = \frac{3}{5}$$

$$\sin 2B = \frac{2\tan B}{1+\tan^2 B} \Rightarrow \sin 2B = \frac{2\times\frac{1}{3}}{1+\frac{1}{9}} = \frac{3}{5}$$

$$\therefore \cos 2A = \sin 2B$$

16. (C)

$$\frac{3\cos\theta+\cos 3\theta}{3\sin\theta-\sin 3\theta} = \frac{3\cos\theta+\left(4\cos^3\theta-3\cos\theta\right)}{3\sin\theta-\left(3\sin\theta-4\sin^3\theta\right)} = \cot^3\theta$$

17. (A)

$$\frac{\left(1+\tan 11^\circ\right)\left(1+\tan 34^\circ\right)}{\left(1+\tan 17^\circ\right)\left(1+\tan 28^\circ\right)}$$

If $A+B=45^\circ$

(1 + tan A) (1 + tan B) = 2

Easy to prove just write $A+B=45^\circ$

Take tan on both sides

tan (A + B) = 1

tan A + tan B = 1   and you will get the result

1 – tan A tan B

$\therefore \left(1+\tan 11^\circ\right)\left(1+\tan 34^\circ\right) = 2$

$\left(1+\tan 17^\circ\right)\left(1+\tan 28^\circ\right) = 2$

Ans $= \dfrac{2}{2} = 1$

18. **(A)**

$\left(4\cos^2 9^\circ - 3\right)\left(4\cos 27^\circ - 3\right)$

= (Multiply by $\cos 9^\circ$ and divide)

= (Multiply & divide by $\cos 27^\circ$)

$$\frac{\left(4\cos^3 9^\circ - 3\cos 9^\circ\right)}{\cos 9^\circ}\ \frac{\left(4\cos^3 27^\circ - 3\cos 27^\circ\right)}{\cos 27^\circ}$$

$$= \frac{\cos 27^\circ}{\cos 9^\circ}\ \frac{\cos 81^\circ}{\cos 27^\circ} = \frac{\sin 9^\circ}{\cos 9^\circ} = \tan 9^\circ$$

19. **(C)**

$$32\sin\frac{A}{2}\sin\frac{5A}{2} = 16\left(\cos 2A - \cos 3A\right)$$

$$= 16\left(2\cos^2 A - 1 - 4\cos^3 A + 3\cos A\right)$$

$$= 16\left(2\left(\frac{3}{4}\right)^2 - 1 - 4\left(\frac{3}{4}\right)^3 + 3\left(\frac{3}{4}\right)\right)$$

$$= 11$$

20. **(B)**

$$25\cos^2\theta + 5\cos\theta - 12 = 0 \Rightarrow \cos\theta = -\frac{4}{5}, \frac{3}{5}$$

But $\dfrac{\pi}{2} < \alpha < \pi$, so $\cos\alpha = -\dfrac{4}{5}$ & $\sin\alpha = \dfrac{3}{5}$

Now $\sin 2\alpha = 2\sin\alpha\cos\alpha = -\dfrac{24}{25}$

21. (B)

$\sin(1765^\circ) = \sin(1800^\circ - 35^\circ)$

$= \sin(20 \times 90^\circ - 35^\circ)$

$= -\sin 35^\circ$

$\sin(325^\circ) = -\sin 35^\circ$

$\operatorname{cosec}(-1465^\circ) = -\operatorname{cosec}(1465^\circ)$

$= -\operatorname{cosec}(1440^\circ + 25^\circ)$

$= -\operatorname{cosec}(16 \times 90^\circ + 25^\circ)$

$= -\operatorname{cosec} 25^\circ$

$\sec 295^\circ = \operatorname{cosec} 25^\circ$

$\boxed{\text{Ans} = 0}$

22. (A)

$2\sec 2\alpha = \tan\beta + \cot\beta \Rightarrow 2\sec 2\alpha = 2\operatorname{cosec} 2\beta$

$\Rightarrow \sin 2\beta = \cos 2\alpha = \sin\left(\frac{\pi}{2} - 2\alpha\right)$

$\Rightarrow 2\beta = \frac{\pi}{2} - 2\alpha$

$\Rightarrow \alpha + \beta = \frac{\pi}{4}$

23. (C)

$\cos x + \cos y + \cos\alpha = 0 \Rightarrow 2\cos\frac{x+y}{2}\cos\frac{x-y}{2} = -\cos\alpha$

$\sin x + \sin y + \sin\alpha = 0 \Rightarrow 2\sin\frac{x+y}{2}\cos\frac{x-y}{2} = -\sin\alpha$

$\Rightarrow \cot\frac{x+y}{2} = \cot\alpha$

24. **(B)**

$\sin 2\theta + \sin 2\phi = \frac{1}{2} \Rightarrow \sin^2 2\theta + \sin^2 2\phi + 2\sin 2\theta \sin 2\phi = \frac{1}{4}$ ...(i)

& $\cos 2\theta + \cos 2\phi = \frac{3}{2} \Rightarrow \cos^2 2\theta + \cos^2 2\phi + 2\cos 2\theta \cos 2\phi = \frac{9}{4}$ ...(ii)

Adding (i) & (ii) gives

$2 + 2\sin 2\theta \sin 2\phi + 2\cos 2\theta \cos 2\phi = \frac{5}{2}$

$\Rightarrow \cos(2\theta - 2\phi) = \frac{1}{4}$

$\Rightarrow 2\cos^2(\theta - \phi) - 1 = \frac{1}{4}$

$\Rightarrow \cos^2(\theta - \phi) = \frac{5}{8}$

25. **(B)**

$\sin\theta = \frac{p-6}{8-p} \Rightarrow 0 \le \frac{p-6}{8-p} < 1$

$\frac{p-6}{8-p} < 1 \Rightarrow \frac{p-6}{8-p} - 1 < 0$

$\Rightarrow \frac{p-7}{p-8} > 0$

$\Rightarrow p < 7$ or $p > 8$ ...(i)

$\frac{p-6}{8-p} \ge 0 \Rightarrow \frac{p-6}{p-8} \le 0$

$\Rightarrow 6 \le p < 8$ ...(ii)

From (i) & (ii) $6 \le p < 7$

26. **(A)**

$\sin^2\theta = \frac{x^2 + y^2}{2xy}$

$= \frac{1}{2}\left(\frac{x}{y} + \frac{y}{x}\right)$

$0 \le \sin^2\theta \le 1$

For positive $\frac{x}{y}$, $\frac{1}{2}\left(\frac{x}{y}+\frac{y}{x}\right) \geq I$ [A.M. ≥ G.M.]

L.H.S. ≤ 1 and R.H.S. ≥ 1, so they can be equal only when both

L.H.S. = R.H.S. = 1 so $\frac{x}{y}=1 \Rightarrow x=y$

27. (C)

$5\tan\theta = 4$ implies $\tan\theta = \frac{4}{5}$

$$\frac{\sin\theta}{4} = \frac{\cos\theta}{5} = k$$

$\sin\theta = 4k$ & $\cos\theta = 5k$

$$\frac{5\sin\theta - 3\cos\theta}{5\sin\theta + 2\cos\theta} = \frac{5(4k)-3(5k)}{5(4k)+2(5k)}$$

$$= \frac{k(20-15)}{k(20+10)} = \frac{1}{6}$$

28. (A)

$\sin x + \operatorname{cosec} x = 2$

$$\sin x + \frac{1}{\sin x} = 2$$

$\sin^2 x - 2\sin x + 1 = 0$

$(\sin x - 1)^2 = 0$

$\sin x = 1 \therefore \sin^n x + \operatorname{cosec}^n x = 1^n + 1^n = 2$

29. (B)

$$\cos\frac{6\pi}{7} = -\cos\frac{\pi}{7},\ \cos\frac{5\pi}{7} = -\cos\frac{2\pi}{7},\ \cos\frac{4\pi}{7} = -\cos\frac{3\pi}{7}$$

$$\text{L.H.S.} = \cos\frac{\pi}{7} + \cos\frac{2\pi}{7} + \cos\frac{3\pi}{7} + \cos\frac{4\pi}{7} + \cos\frac{5\pi}{7} + \cos\frac{6\pi}{7} + \cos\frac{7\pi}{7}$$

$$= \cos\frac{\pi}{7} + \cos\frac{2\pi}{7} + \cos\frac{3\pi}{7} - \cos\frac{3\pi}{7} - \cos\frac{2\pi}{7} - \cos\frac{\pi}{7} + \cos\pi = 0 + (-1) = -1$$

30. (B)

$$\sin^4\theta + \cos^4\theta = 1 - 2\sin^2\theta\cos^2\theta$$

$$= 1 - \frac{\sin^2 2\theta}{2}$$

$$0 \le \frac{\sin^2 2\theta}{2} \le \frac{1}{2}$$

$$\frac{-1}{2} \le \frac{-\sin^2 2\theta}{2} \le 0$$

$$\frac{1}{2} \le 1 - \frac{\sin^2 2\theta}{2} \le 1$$

31. (D)

$\cot(\alpha+\beta) = 0$, $\sin(\alpha+2\beta) = ?$

$$\sin(\alpha+2\beta) = \sin(\alpha+\beta+\beta)$$

$$= \sin(\alpha+\beta)\cos\beta + \cos(\alpha+\beta)\sin\beta$$

$$= \sin(\alpha+\beta)\cos\beta + 0$$

$$\cot(\alpha+\beta) = 0 \Rightarrow \cos(\alpha+\beta) = 0$$

$$\therefore \sin^2(\alpha+\beta) = 1$$

$$\sin(\alpha+\beta) = \pm 1$$

$$\sin(\alpha+2\beta) = \pm\cos\beta$$

32. (D)

$$1 \le \sin\theta_1, \sin\theta_2, \sin\theta_3 \le 1$$

so, $\sin\theta_1 + \sin\theta_2 + \sin\theta_3 = 3 \Rightarrow \theta_1 = \theta_2 = \theta_3 = 90^\circ$

and

$\cos\theta_1 + \cos\theta_2 + \cos\theta_3 = 0$

33.

$\operatorname{cosec}\theta - \cot\theta = 9$

$\operatorname{cosec}\theta - \cot\theta = \dfrac{1}{\operatorname{cosec}\theta + \cot\theta}$

$\therefore \dfrac{1}{\operatorname{cosec}\theta + \cot\theta} = q$

$\dfrac{1}{q} = \operatorname{cosec}\theta + \cot\theta$

$q = \operatorname{cosec}\theta - \cot\theta$

$q + \dfrac{1}{q} = 2\operatorname{cosec}\theta$

$\operatorname{cosec}\theta = \dfrac{1}{2}\left(q + \dfrac{1}{q}\right)$

34. (B)

$A + B + C = \pi \Rightarrow \cos A = -\cos(B + C)$

$\cos B\cos C - \sin B\sin C = -\cos A$

But $\cos A = \cos B\cos C$

$\therefore \cos B\cos C - \sin B\sin C = -\cos B\cos C$

$\Rightarrow \sin B\sin C = 2\cos B\cos C$

$\Rightarrow \tan B\tan C = 2$

35. (C)

$\sin x + \sin^2 x = 1$ ……….. (1)

$A = \cos^{12} + 3\cos^{10} x + 3\cos^8 x + \cos^6 x - 2$

$= \cos^6 x\left(\cos^6 x + 3\cos^4 x + 3\cos^2 x + 1\right) - 2$

$= \cos^6 x\left(\cos^2 x + 1\right)^3 - 2$

From (1)

$\sin x = 1 - \sin^2 x$

$\sin x = \cos^2 x$

$A = \cos^6 x\left(\sin x + 1\right)^3 - 2$ ……… $\cos^6 x\left(\sin x + 1\right)^3 = \left(\cos^2 x(\sin x + 1)\right)^3$

$= \left(\sin^2 x + \sin x\right)^3 - 2$

$= (1)^3 - 2 = -1$

36. (A)

$\cos(A - B) = \dfrac{3}{5}$

tan A tan B = 2

sin A sin B = 2 cos A cos B ………..(1)

$\cos A \cos B + \sin A \sin B = \dfrac{3}{5}$

$\therefore 3\cos A \cos B = \dfrac{3}{5}$ …….from (1)

$\cos A \cos B = \dfrac{1}{5}$ & $\sin A \sin B = \dfrac{2}{5}$

37. (D)

$\dfrac{1}{4}\left[\sqrt{3}\cos 23^\circ - \sin 23^\circ\right] = \dfrac{2}{4}\left[\dfrac{\sqrt{3}}{2}\cos 23^\circ - \dfrac{1}{2}\sin 23^\circ\right]$

$= \dfrac{1}{2}\left[\sin(60^\circ - 23^\circ)\right]$

$= \dfrac{1}{2}\left[\sin 37^\circ\right]$

$= \dfrac{1}{2}\cos 53^\circ$

38. (D)

$100 + 125^\circ = 225^\circ$

taking tan on both sides

$\tan\left(100^\circ + 125^\circ\right) = \tan\left(225^\circ\right)$

$\frac{\tan\left(100^\circ\right) + \tan 125^\circ}{1 - \tan 100^\circ \tan 125^\circ} = 1$

$\tan 100^\circ + \tan 125^\circ = 1 - \tan 100^\circ \tan 125^\circ$

$\tan 100^\circ + \tan 125^\circ + \tan 100^\circ \tan 125^\circ = 1$

39. (B)

$\tan\left(20^\circ + 40^\circ\right) = \tan 60^\circ = \sqrt{3}$

$\frac{\tan 20^\circ + \tan 40^\circ}{1 - \tan 20^\circ \tan 40^\circ} = \sqrt{3}$

$\therefore \tan 20^\circ + \tan 40^\circ + \sqrt{3}\tan 20^\circ \tan 40^\circ = \sqrt{3}$

40. (D)

$4x^2 - 16x + 15 < 0 \Rightarrow (2x-3)(2x-5) < 0$

$\Rightarrow \frac{3}{2} < x < \frac{5}{2}$

$\Rightarrow \tan A = 2$

Also $\cos B = 1$

$\Rightarrow \sin A = \pm\frac{2}{\sqrt{5}}$ & $\sin B = 0$

Now $\sin(A+B)\sin(A-B) = \sin^2 A - \sin^2 B = \frac{4}{5}$