TRIGONOMETRY I

Exercise – 1

Q. 1 (b)

Given the diameter of circular wire = 14 *cm*.

Therefore length of wire = $14\pi\, cm$

Hence, required angle $= \frac{\text{Arc}}{\text{Radius}} = \frac{14\pi}{12} = \frac{7\pi}{6}$ radian

$\Rightarrow \frac{7\pi}{6} \times \frac{180^o}{\pi} = 210^o$.

Q. 2 (d)

We have, π radians $= 180^o$

$\therefore \quad 1^c = \left[\frac{180}{\pi}\right]^o$;

$\therefore \left[\frac{2\pi}{15}\right]^c = \left[\frac{2\pi}{15} \times \frac{180}{\pi}\right]^o = 24^o$.

Q. 3 (a)

Let the angles in degrees be $\alpha - 3\delta, \alpha - \delta, \alpha + \delta, \alpha + 3\delta$

Sum of the angles $= 4\alpha = 360^o \qquad \therefore \quad \alpha = 90^o$

Also greatest angle $= \alpha + 3\delta = 120^o$,

Hence, $3\delta = 120^o - \alpha = 120^o - 90^o = 30^o \quad \therefore \ \delta = 10^o$

Hence the angles are $90^o - 30^o, 90^o - 10^o, 90^o + 10^o$ and $90^o + 30^o$

That is, the angles in degrees are $60^o, 80^o, 100^o$ and 120^o

$\therefore$ In terms of radians the angles are $\frac{\pi}{3}, \frac{4\pi}{9}, \frac{5\pi}{9}$ and $\frac{2\pi}{3}$.

Q. 4 (b)

We know that the tip of the minute hand makes one complete round in one hour *i.e.* 60 *minutes* since the length of the hand is 10 *cm*.

the distance moved by its tip in 60 *minutes* $= 2\pi \times 10\, cm = 20\pi\ cm$

Hence the distance in 20 *minutes* $= \frac{20\pi}{60} \times 20\, cm = \frac{20\pi}{3} cm$.

Q. 5 (c)

Required angle = $\frac{\text{Arc}}{\text{radius}} = \frac{1}{3}$ radian .

Q. 6 (b)

Area of sector subtending an angle of radian at centre of a circle of radius r is $\frac{1}{2} r^2 \theta$

Also area of triangle having sides a & b and included angle θ is $\frac{1}{2} ab \sin\theta$.

Hence required area = $\left(\frac{1}{2} \times 4^2 \times \frac{\pi}{4} - \frac{1}{2} \times 4 \times 4 \times \sin\frac{\pi}{4}\right) + \left(\frac{1}{2} \times 4^2 \times \frac{\pi}{3} - \frac{1}{2} \times 4 \times 4 \times \sin\frac{\pi}{3}\right) = \frac{14\pi}{3} - 4\left(\sqrt{2} + \sqrt{3}\right)$

Q. 7 (c)

$n(m^2 - 1) = (\sec\theta + \text{cosec}\theta).2\sin\theta.\cos\theta \qquad [\because m^2 = 1 + 2\sin\theta.\cos\theta]$

$$= \frac{\sin\theta + \cos\theta}{\sin\theta.\cos\theta}.2\sin\theta.\cos\theta \quad = 2m \ .$$

Q. 8 (b)

$$x\sin\phi = \tan\theta - x\cos\phi.\tan\theta$$

$$x = \frac{\tan\theta}{\sin\phi + \cos\phi\tan\theta} = \frac{\sin\theta}{\sin\theta\cos\phi + \cos\theta\sin\phi}$$

Similarly, $y = \dfrac{\sin\phi}{\sin\theta\cos\phi + \cos\theta\sin\phi}$;

$$\therefore \ \frac{x}{y} = \frac{\sin\theta}{\sin\phi}.$$

Q. 9 (a)

$$\sec\theta + \tan\theta = 3 \Rightarrow \sec\theta - \tan\theta = \frac{1}{3} \ \{\because \sec^2\theta - \tan^2\theta = 1\}$$

Hence $\sec\theta = \dfrac{5}{3}$ & $\tan\theta = \dfrac{4}{3}$

$\Rightarrow \sin\theta = \dfrac{4}{5}$ & $\cos\theta = \dfrac{3}{5}$

$$\Rightarrow \sin\theta + \cos\theta = \frac{8}{5}$$

Q. 10 (c)

$$\sqrt{\frac{(1-\sin\theta)^2}{(1-\sin^2\theta)}} = \frac{1-\sin\theta}{\cos\theta} = \sec\theta - \tan\theta \ .$$

Q. 11 (c)

$$\sin^6 x + \cos^6 x = \frac{1}{2} \Rightarrow \left(\sin^2 x\right)^3 + \left(\cos^2 x\right)^3 = \frac{1}{2}$$

$$\Rightarrow \left(\sin^2 x + \cos^2 x\right)^3 - 3\sin^2 x\cos^2 x\left(\sin^2 x + \cos^2 x\right) = \frac{1}{2}$$

$$\Rightarrow 1 - 3\sin^2 x\cos^2 x = \frac{1}{2} \Rightarrow \sin^2 2x = \frac{1}{3}$$

$$\Rightarrow 1 - \cos 4x = \frac{2}{3} \Rightarrow \cos 4x = \frac{1}{3}$$

Q. 12 (d)

$$\sin x + \cos x = \frac{1}{5} \ \Rightarrow \ \sin^2 x + \cos^2 x + 2\sin x\cos x = \frac{1}{25}$$

$$\sin 2x = -\frac{24}{25} \ \Rightarrow \ \cos 2x = -\frac{7}{25} \ \Rightarrow \ \tan 2x = \frac{24}{7}.$$

Q. 13 (b)

$$\cos x = \sqrt{1-\sin^2 x} = \sqrt{1-\left(\frac{-24}{25}\right)^2} = \frac{7}{25} \ \Rightarrow \ \tan x = \frac{\sin x}{\cos x} = \frac{-24}{7}.$$

Q. 14 (b)

$\tan\theta + \sec\theta = e^x$ ....(i)

$\therefore \ \sec\theta - \tan\theta = e^{-x}$ ....(ii)

From (i) and (ii), $\Rightarrow \ 2\sec\theta = e^x + e^{-x} \ \Rightarrow \ \cos\theta = \dfrac{2}{e^x + e^{-x}}$.

Q. 15 (b)

$$xy = \frac{1-\sin\theta}{\cos\theta} \times \frac{1+\sin\theta}{\cos\theta} = \frac{1-\sin^2\theta}{\cos^2\theta} = 1$$

Q. 16 (d)

$P+Q=\frac{2\sin\theta}{1+\sin\theta+\cos\theta}+\frac{\cos\theta}{1+\sin\theta}$

After solving, $P+Q=1$.

Q. 17 (c)

$= 6(\sin^6\theta+\cos^6\theta)-9(\sin^4\theta+\cos^4\theta)+4$

$= 6[(\sin^2\theta+\cos^2\theta)^3-3\sin^2\theta\cos^2\theta(\sin^2\theta+\cos^2\theta)]-9[(\sin^2\theta+\cos^2\theta)^2-2\sin^2\theta.\cos^2\theta]+4$

$= 6[1-3\sin^2\theta\cos^2\theta]-9[1-2\sin^2\theta\cos^2\theta]+4 = 6-9+4=1$.

Q. 18 (d)

$\frac{\sin\theta.\sin\theta}{\sin\theta(1-\cot\theta)}+\frac{\cos\theta.\cos\theta}{(1-\tan\theta)\cos\theta} = \frac{\sin^2\theta}{(\sin\theta-\cos\theta)}+\frac{\cos^2\theta}{(\cos\theta-\sin\theta)} = \frac{\cos^2\theta-\sin^2\theta}{(\cos\theta-\sin\theta)}=\cos\theta+\sin\theta$.

Q. 19 (b)

$\sin 75^o=\sin(45^o+30^o)=\sin 45^o\cos 30^o+\sin 30^o\cos 45^o$

$=\frac{1}{\sqrt{2}}\times\frac{\sqrt{3}}{2}+\frac{1}{2}\times\frac{1}{\sqrt{2}}=\frac{\sqrt{3}+1}{2\sqrt{2}}$.

Q. 20 (c)

$\cos\frac{5\pi}{4}-\sin\frac{5\pi}{4}\Rightarrow-\cos\frac{\pi}{4}+\sin\frac{\pi}{4}$.

$\Rightarrow-\frac{1}{\sqrt{2}}+\frac{1}{\sqrt{2}}=0$

Q. 21 (a)

$\tan A+\cot A+(-\tan A)+(-\cot A)=0$.

Q. 22 (a)

$\frac{\sqrt{3}+1}{2\sqrt{2}}-\frac{\sqrt{3}-1}{2\sqrt{2}} = \frac{1}{\sqrt{2}}$.

Q. 23 (a)

$\tan\theta+\cot\theta=2\operatorname{cosec}2\theta\Rightarrow\tan\frac{\pi}{8}+\cot\frac{\pi}{8}=2\operatorname{cosec}\frac{\pi}{4}=2\sqrt{2}$

Q. 24 (b)

$\tan\theta-\cot\theta=-2\cot 2\theta\Rightarrow\tan\frac{\pi}{12}-\cot\frac{\pi}{12}=-2\cot\frac{\pi}{6}=-2\sqrt{3}$

Q. 25 (b)

Since $\sin x+\sin^2 x=1 \Rightarrow \sin x=1-\sin^2 x=\cos^2 x$ ....(i)

From given expression, $\cos^8 x(\cos^8 x+4\cos^6 x+6\cos^4 x+4\cos^2 x+1) = \cos^8 x(\cos^2 x+1)^4$

From (i) $\sin x=\cos^2 x$

$\therefore \sin^4 x(\sin x+1)^4 = (\sin^2 x+\sin x)^4=1$.

Q. 26 (b)

Given $4\sin\theta=3\cos\theta \Rightarrow \tan\theta=\frac{3}{4}$

The given expression is $\frac{\sec^2\theta}{4[1-\tan^2\theta]}=\frac{1+\tan^2\theta}{4(1-\tan^2\theta)} = \frac{1+\frac{9}{16}}{4\left(1-\frac{9}{16}\right)}=\frac{25}{28}$.

Q. 27 (d)

$= \dfrac{1-\tan 12^o}{1+\tan 12^o} + \tan 147^o = \tan(45^o - 12) + \tan(180^o - 33^o) = \tan 33^o + (-\tan 33^o) = 0$.

Q. 28 (c)

We know $|\sin\theta| \le 1 \,\&\, |\cos\theta| \le 1$; So, each $\sin\theta_1, \cos\theta_2$ and $\sin\theta_3$ must be equal to 1

$\therefore \quad |\cos\theta_1| + |\sin\theta_2| + |\cos\theta_3| = 1$.

Q. 29 (b)

$\cos A + 2\cos 600^o \cos A = \cos A + 2\cos(540^o + 600^o)\cos A = \cos A - 2\cos 60^o \cos A = 0$

Q. 30 (b)

$\dfrac{2(\sin^2 A - \sin^2 B)}{2\sin A\cos A - 2\sin B\cos B} = \dfrac{2\sin(A+B).\sin(A-B)}{\sin 2A - \sin 2B} = \dfrac{2\sin(A+B)\sin(A-B)}{2\sin(A-B)\cos(A+B)} = \tan(A+B)$.

Q. 31 (c)

$\cos^2(A-B) + \sin^2 B + \cos(A-B)[\cos(A-B) + \cos(A+B)]$

$\Rightarrow \quad \sin^2 B + \cos(A-B)\cos(A+B) \Rightarrow \sin^2 B + (\cos^2 A - \sin^2 B) = \cos^2 A = 1 - \sin^2 A$

Q. 32 (b)

We have $\tan\alpha = \dfrac{m}{m+1}$ and $\tan\beta = \dfrac{1}{2m+1}$

$\tan(\alpha+\beta) = \dfrac{\dfrac{m}{m+1} + \dfrac{1}{2m+1}}{1 - \dfrac{m}{(m+1)}.\dfrac{1}{(2m+1)}} = \dfrac{2m^2 + m + m + 1}{2m^2 + m + 2m + 1 - m} \qquad \left[\because \tan(\alpha+\beta) = \dfrac{\tan\alpha + \tan\beta}{1 - \tan\alpha\tan\beta}\right]$

$= \dfrac{2m^2 + 2m + 1}{2m^2 + 2m + 1} = 1 \Rightarrow \tan(\alpha+\beta) = \tan\dfrac{\pi}{4}$

Hence $\alpha + \beta = \dfrac{\pi}{4}$

Trick : As $\alpha + \beta$ is independent of *m*, therefore put $m = 1$, then $\tan\alpha = \dfrac{1}{2}$ and $\tan\beta = \dfrac{1}{3}$.

Therefore, $\tan(\alpha+\beta) = \dfrac{\frac{1}{2} + \frac{1}{3}}{1 - \frac{1}{6}} = 1$, Hence $\alpha + \beta = \dfrac{\pi}{4}$ (Also check for other values of *m*)

Q. 33 (d)

Given that $\tan\theta - \cot\theta = a$ …….(i) and $\sin\theta + \cos\theta = b$ …….(ii)

Now, $(b^2 - 1)^2(a^2 + 4) = \{(\sin\theta + \cos\theta)^2 - 1\}^2\{\tan\theta - \cot\theta)^2 + 4\}$

$= [1 + \sin 2\theta - 1]^2[\tan^2\theta + \cot^2\theta - 2 + 4] = \sin^2 2\theta(\text{cosec}^2\theta + \sec^2\theta) = 4\sin^2\theta\cos^2\theta\left[\dfrac{1}{\sin^2\theta} + \dfrac{1}{\cos^2\theta}\right] = 4$

Trick : Obviously the value of expression $(b^2 - 1)^2(a^2 + 4)$ is independent of θ, therefore put any suitable value of θ. Let $\theta = 45^o$, we get $a = 0$, $b = \sqrt{2}$ so that $[(\sqrt{2})^2 - 1]^2(0^2 + 4) = 4$.

Q. 34 (c)

$\dfrac{\sin(2A+B)}{\sin B} = \dfrac{5}{1}$

by componendo and Dividendo,

$$\frac{\sin(2A+B)+\sin B}{\sin(2A+B)-\sin B}=\frac{5+1}{5-1}$$

$$\frac{2\sin(A+B).\cos A}{2\cos(A+B).\sin A}=\frac{6}{4} \Rightarrow \frac{\tan(A+B)}{\tan A}=\frac{3}{2}.$$

Q. 35 (c)

$$\frac{\sin 80^o-\sin 170^o}{\cos 10^o-\cos 100^o}=\frac{\sin 70^o+\cos 40^o}{\cos 70^o+\sin 40^o}$$

$$=\frac{\sin 70^o+\sin 50^o}{\sin 20^o+\sin 40^o}=\frac{2\sin 60^o\cos 10^o}{2\sin 30^o\cos(-10^o)}=\frac{\sin 60^o}{\sin 30^o}=\frac{\sqrt{3}}{2}.\frac{2}{1}=\sqrt{3}.$$

Q. 36 (d)

$$\sin 133^o-\sin 241^o+\sin 191^o-\sin 155^o=\sin 47^o+\sin 61^o-(\sin 11^o+\sin 25^o)$$

$$=2\sin 54^o.\cos 7^o-2\sin 18^o\cos 7^o$$

$$=2\cos 7^o(\sin 54^o-\sin 18^o)=2\cos 7^o.2\cos 36^o.\sin 18^o=4.\cos 7^o.\frac{\sqrt{5}+1}{4}.\frac{\sqrt{5}-1}{4}=\cos 7^o.$$

Q. 37 (b)

$$\frac{\cos 10^o-\sin 10^o}{\cos 10^o+\sin 10^o}=\frac{1-\tan 10^o}{1+\tan 10^o}=\tan 35^o=\tan(90^o-35^o)=\cot 55^o.$$

Q. 38 (d)

$$2A=\{(A+B)+(A-B)\} \Rightarrow \tan 2A=\frac{\tan(A+B)+\tan(A-B)}{1-\tan(A+B).\tan(A-B)} \Rightarrow \tan 2\text{A}=-1 \text{ or } 2\text{A}=-\frac{\pi}{4}$$

Q. 39 (b)

$$\sin(720^o+90^o+73^o).\cos(720^o-13^o)+\sin\left(720+73\right)^o.\sin(720^o+180^o-13^o)$$

$$=\cos 73^o.\cos 13^o+\sin 73^o.\sin 13^o=\cos(73^o-13^o)=\cos 60^o=\frac{1}{2}.$$

Q. 40 (b)

$$\cot 70^o+4\cos 70^o=\frac{\cos 70^o+4\sin 70^o.\cos 70^o}{\sin 70^o}=\frac{\cos 70^o+2\sin 140^o}{\sin 70^o}$$

$$=\frac{\cos 70^o+2\sin(180-40^o)}{\sin 70^o}=\frac{\sin 20^o+\sin 40^o+\sin 40^o}{\sin 70^o}$$

$$=\frac{2\sin 30^o\cos 10^o+\sin 40^o}{\sin 70^o}=\frac{\sin 80^o+\sin 40^o}{\sin 70^o}=\frac{2\sin 60^o\cos 20^o}{\sin 70^o}=\sqrt{3}.$$

Q. 41 (b)

$$\tan(\alpha+\beta)=\frac{\tan\alpha+\tan\beta}{1-\tan\alpha\tan\beta} \Rightarrow \tan(\alpha+\beta)=\frac{\frac{1}{1+\frac{1}{2^x}}+\frac{1}{1+2^{x+1}}}{1-\frac{1}{1+\frac{1}{2^x}}.\frac{1}{1+2^{x+1}}}$$

$$\Rightarrow \tan(\alpha+\beta)=\frac{2^x+2.2^{x+x}+2^x+1}{1+2^x+2.2^x+2.2^{x+x}-2^x}$$

$$\Rightarrow \tan(\alpha+\beta)=1=\tan\frac{\pi}{4} \Rightarrow \alpha+\beta=\frac{\pi}{4}.$$

Q. 42 (b)

$$\frac{\frac{\sin 7\theta}{\cos 7\theta}-\frac{\sin 2\theta}{\cos 2\theta}}{\frac{\sin 5\theta}{\cos 5\theta}}=\frac{\frac{\sin 7\theta\cos 2\theta-\cos 7\theta\sin 2\theta}{\cos 7\theta.\cos 2\theta}}{\frac{\sin 5\theta}{\cos 5\theta}}$$

$$=\frac{2}{2}\times\frac{\sin(7\theta-2\theta)\cos 5\theta}{\cos 7\theta.\cos 2\theta.\sin 5\theta}=\frac{2\sin 5\theta.\cos 5\theta}{2\cos 7\theta.\cos 2\theta.\sin 5\theta}$$

$$=\frac{2\cos 5\theta}{\cos 9\theta+\cos 5\theta}=2.$$

Q. 43 (d)

$$\pi<\alpha<\frac{3\pi}{2}\Rightarrow\frac{\pi}{2}<\frac{\alpha}{2}<\frac{3\pi}{4}\Rightarrow\cos\frac{\alpha}{2}=-ve$$

$$\Rightarrow\ \therefore\cos\alpha=\frac{4}{5}\Rightarrow\cos\frac{\alpha}{2}=\sqrt{\frac{1+\cos\alpha}{2}}=-\sqrt{\frac{1+\frac{4}{5}}{2}}=-\sqrt{\frac{9}{10}}=\frac{-3}{\sqrt{10}}.$$

Q. 44 (c)

Since $2\cos(\alpha+\beta)=2\cos^2(\alpha+\beta)-1$,

$2\sin^2\beta=1-\cos 2\beta=-\cos 2\beta+2\cos(\alpha+\beta)[2\sin\alpha\sin\beta+\cos(\alpha+\beta)]$

$=-\cos 2\beta+2\cos(\alpha+\beta).\cos(\alpha-\beta)=-\cos 2\beta+\cos 2\alpha+\cos 2\beta=\cos 2\alpha$.

Q. 45 (b)

$$\frac{1-\tan^2 15}{1+\tan^2 15}=\frac{1-[\tan(45^o-30^o)]^2}{1+[\tan(45^o-30^o)]^2}=\frac{1-\left[\frac{1-\frac{1}{\sqrt{3}}}{1+\frac{1}{\sqrt{3}}}\right]^2}{1+\left[\frac{1-\frac{1}{\sqrt{3}}}{1+\frac{1}{\sqrt{3}}}\right]^2}$$

$$=\frac{1-\left[\frac{\sqrt{3}-1}{\sqrt{3}+1}\right]^2}{1+\left[\frac{\sqrt{3}-1}{\sqrt{3}+1}\right]^2}=\frac{[\sqrt{3}+1]^2-[\sqrt{3}-1]^2}{[\sqrt{3}+1]^2+[\sqrt{3}-1]^2}=\frac{4\sqrt{3}}{8}=\frac{\sqrt{3}}{2}$$

Trick : $\cos 2\theta=\frac{1-\tan^2\theta}{1+\tan^2\theta}\Rightarrow\frac{1-\tan^2 15^o}{1+\tan^2 15^o}=\cos 30^o=\frac{\sqrt{3}}{2}$.

Q. 46 (d)

$\sin 6\theta=2\sin 3\theta.\cos 3\theta = 2[3\sin\theta-4\sin^3\theta][4\cos^3\theta-3\cos\theta]$

$= 24\sin\theta.\cos\theta(\sin^2\theta+\cos^2\theta)-18\sin\theta\cos\theta-32\sin^3\theta\cos^3\theta$

$= 32\cos^5\theta.\sin\theta-32\cos^3\theta.\sin\theta+3\sin 2\theta$

On comparing, $x=\sin 2\theta$

Trick : Put $\theta=0^o$, then $x=0$. So, option (c) and (d) are correct.

Now put $\theta=30^o$, then $x=\frac{\sqrt{3}}{2}$. Therefore, Only option (d) is correct.

Q. 47 (b)

Given, $\sqrt{x}+\frac{1}{\sqrt{x}}=2\cos\theta$ ....(i)

On squaring both sides we get, $x+\frac{1}{x}+2=4\cos^2\theta \Rightarrow x+\frac{1}{x}=4\cos^2\theta-2$

$\Rightarrow x+\frac{1}{x}=2(2\cos^2\theta-1)$

$=2\cos 2\theta$ ....(ii)

Again squaring both sides,

$x^2+\frac{1}{x^2}+2=4\cos^2 2\theta \Rightarrow x^2+\frac{1}{x^2}=4\cos^2 2\theta-2=2(2\cos^2 2\theta-1)$

$\Rightarrow x^2+\frac{1}{x^2}=2\cos 4\theta$ ......(iii)

Now taking cube of both sides;

$\left(x^2+\frac{1}{x^2}\right)^3=(2\cos 4\theta)^3 \Rightarrow x^6+\frac{1}{x^6}+3x^2.\frac{1}{x^2}\left(x^2+\frac{1}{x^2}\right)=8\cos^3 4\theta$

$\Rightarrow x^6+\frac{1}{x^6}+3(2\cos 4\theta) = 8\cos^3 4\theta \Rightarrow x^6+\frac{1}{x^6}=8\cos^3 4\theta-6\cos 4\theta$

$\Rightarrow x^6+\frac{1}{x^6}= 2(4\cos^3 4\theta-3\cos 4\theta)=2\cos 3(4\theta)=2\cos 12\theta$.

Q. 48 (c)

For $A=133^o, \frac{A}{2}=66.5^o \Rightarrow$ $\sin\frac{A}{2}>\cos\frac{A}{2}>0$

Hence, $\sqrt{1+\sin A}=\sin\frac{A}{2}+\cos\frac{A}{2}$ ......(i)

and $\sqrt{1-\sin A}=\sin\frac{A}{2}-\cos\frac{A}{2}$ ......(ii)

Subtract (ii) from (i) we get,

$2\cos\frac{A}{2}=\sqrt{1+\sin A}-\sqrt{1-\sin A}$.

Q. 49 (b)

$2\tan A=3\tan B \Rightarrow \tan A=\frac{3}{2}\tan B=\frac{3}{2}t$

(Let $\tan B=t$)

$\Rightarrow \sin 2B=\frac{2t}{1+t^2}, \cos 2B=\frac{1-t^2}{1+t^2}$

$\therefore \frac{\sin 2B}{5-\cos 2B}=\frac{\left(\frac{2t}{1+t^2}\right)}{5-\left(\frac{1-t^2}{1+t^2}\right)}=\frac{2t}{4+6t^2}=\frac{t}{2+3t^2}=\tan(A-B)$.

Q. 50 (d)

$\sin A=\frac{4}{5} \Rightarrow \tan A=-\frac{4}{3}$, $(90^o<A<180^o)$

$\tan A = \dfrac{2\tan\frac{A}{2}}{1-\tan^2\frac{A}{2}}$,

Let $\tan\frac{A}{2} = P$

$\Rightarrow \quad \dfrac{-4}{3} = \dfrac{2P}{1-P^2}$

$\Rightarrow 4P^2 - 6P - 4 = 0 \Rightarrow P = -\frac{1}{2}, 2 \Rightarrow P = -\frac{1}{2}$ (impossible)

So, $P = 2$ *i.e.*, $\tan\frac{A}{2} = 2.$

Q. 51 (a)

$\tan\alpha = \frac{1}{7}, \sin\beta = \frac{1}{\sqrt{10}} \Rightarrow \tan\beta = \frac{1}{3}$

$\Rightarrow \tan 2\beta = \dfrac{\frac{2}{3}}{1-\frac{1}{9}} = \frac{3}{4}$

$\therefore \tan(\alpha + 2\beta) = \dfrac{\frac{1}{7}+\frac{3}{4}}{1-\frac{3}{28}} = \dfrac{4+21}{25} = 1$

Q. 52 (a)

$\dfrac{1-t^2}{1+t^2} = \dfrac{1-\tan^2\frac{\theta}{2}}{1+\tan^2\frac{\theta}{2}} \quad (\because \tan\frac{\theta}{2} = t) = \cos(2.\frac{\theta}{2}) = \cos\theta$.

Q. 53 (d)

$\cos 3\theta = 4\cos^3\theta - 3\cos\theta$ & $\sin 3\theta = 3\sin\theta - 4\sin^3\theta \Rightarrow \cos 3\theta - \sin 3\theta = 4(\cos^3\theta + \sin^3\theta) - 3(\cos\theta + \sin\theta)$

$\Rightarrow \cos 3\theta - \sin 3\theta = 4(\cos\theta + \sin\theta)^3 - 12\sin\theta\cos\theta(\cos\theta + \sin\theta) - 3(\cos\theta + \sin\theta)$

$\Rightarrow \cos 3\theta - \sin 3\theta = \left(4(\cos\theta + \sin\theta)^2 - 12\sin\theta\cos\theta - 3\right)(\cos\theta + \sin\theta)$

$\Rightarrow \cos 3\theta - \sin 3\theta = (1 - 4\sin\theta\cos\theta)(\cos\theta + \sin\theta)$

Hence $\left(\cos\frac{\pi}{18} + \sin\frac{\pi}{18}\right)\left(1 - 4\sin\frac{\pi}{18}\cos\frac{\pi}{18}\right) = \cos\frac{\pi}{6} - \sin\frac{\pi}{6} = \frac{\sqrt{3}-1}{2}$

Q. 54 (a)

$\tan 2\theta + \sec 2\theta = \dfrac{2\tan\theta}{1-\tan^2\theta} + \dfrac{1+\tan^2\theta}{1-\tan^2\theta}$

Given $\tan\theta = t \Rightarrow \therefore \tan 2\theta + \sec 2\theta = \dfrac{2t}{1-t^2} + \dfrac{1+t^2}{1-t^2} = \dfrac{2t+1+t^2}{1-t^2} = \dfrac{(t+1)^2}{1-t^2} = \dfrac{1+t}{1-t}$.

Q. 55 (b)

Given, $\sin 2\theta + \sin 2\phi = \frac{1}{2}$ ....(i)

and $\cos 2\theta + \cos 2\phi = \frac{3}{2}$ ....(ii)

Squaring and adding,

$$\therefore (\sin^2 2\theta + \cos^2 2\theta) + (\sin^2 2\phi + \cos^2 2\phi) + 2[\sin 2\theta . \sin 2\phi + \cos 2\theta . \cos 2\phi] = \frac{1}{4} + \frac{9}{4}$$

$$\Rightarrow \cos 2\theta . \cos 2\phi + \sin 2\theta . \sin 2\phi = \frac{1}{4} \Rightarrow \cos(2\theta - 2\phi) = \frac{1}{4} \Rightarrow \cos^2(\theta - \phi) = \frac{5}{8}.$$

Q. 56 (b)

Given $\tan x = \frac{b}{a} \Rightarrow \sqrt{\frac{a+b}{a-b}} + \sqrt{\frac{a-b}{a+b}} = \sqrt{\frac{1+b/a}{1-b/a}} + \sqrt{\frac{1-b/a}{1+b/a}}$

$= \sqrt{\frac{1+\tan x}{1-\tan x}} + \sqrt{\frac{1-\tan x}{1+\tan x}} = \frac{2}{\sqrt{1-\tan^2 x}}$

Now, multiplying by $\sqrt{1+\tan^2 x}$ in $N'r$ and $D'r$

$$= \frac{2}{\frac{\sqrt{1-\tan^2 x}}{\sqrt{1+\tan^2 x}} . \sqrt{1+\tan^2 x}} = \frac{2}{\sqrt{\cos 2x} . \sqrt{\sec^2 x}} = \frac{2\cos x}{\sqrt{\cos 2x}}.$$

Q. 57 (b)

We have $\frac{x}{1} = \frac{y}{-2} = \frac{z}{-2} = \lambda$ (say)

$\therefore \quad x = \lambda,\ y = -2\lambda, z = -2\lambda; \quad \therefore\ xy + yz + zx = -2\lambda^2 + 4\lambda^2 - 2\lambda^2 = 0$

Q. 58 (a)

If secθ = $\sqrt{2}$

or, cosθ = $\frac{1}{\sqrt{2}}$, sinθ = $\pm \sqrt{1-\cos^2\theta}$

$= \pm\sqrt{1-\frac{1}{2}} = \pm\frac{1}{\sqrt{2}}$

But θ lies in the fourth quadrant in which sinθ is negative.

sinθ = $-\frac{1}{\sqrt{2}}$, cosec θ = $-\sqrt{2}$

tanθ = $\frac{\sin\theta}{\cos\theta}$ $\Rightarrow$ tanθ = $-\frac{1}{\sqrt{2}} \times \sqrt{2}$

$\Rightarrow$ tanθ = –1 $\Rightarrow$ cotθ = –1

then, $\frac{1+\tan\theta+\text{cosec}\theta}{1+\cot\theta-\text{cosec}\theta} = \frac{1-1-\sqrt{2}}{1-1+\sqrt{2}} = -1.$

Q. 59 (d)

L.H.S., = $\sqrt{2+\sqrt{2+\sqrt{2+2\cos 8\theta}}}$

$\therefore \left[1+\cos 8\theta = 2\cos^2\left(\frac{8\theta}{2}\right)\right]$

= $\sqrt{2+\sqrt{2+\sqrt{2(2\cos^2 4\theta)}}}$

= $\sqrt{2+\sqrt{2+2\cos 4\theta}}$ = $\sqrt{2+\sqrt{2(1+\cos 4\theta)}}$

= $\sqrt{2+\sqrt{2(2\cos^2 2\theta)}}$ = $\sqrt{2+2\cos 2\theta}$

= $\sqrt{2(1+\cos 2\theta)}$ = $\sqrt{2(2\cos^2\theta)}$ = $2\cos\theta$.

Q. 60 (a)

As $\sin\theta+\sin^2\theta+\sin^3\theta=1 \Rightarrow \sin\theta\left(1+\sin^2\theta\right)=\cos^2\theta$

$\Rightarrow \sin^2\theta\left(1+\sin^2\theta\right)^2=\cos^4\theta \Rightarrow \left(1-\cos^2\theta\right)\left(2-\cos^2\theta\right)^2=\cos^4\theta$

$\Rightarrow \cos^6\theta-4\cos^4\theta+8\cos^2\theta=4$