Q. 1) The directed angles of different measures having same positions of initial ray and terminal ray are called----
   a) Opposite angles  
   b) Straight Angles  
   c) Coterminal angles  
   d) Quadrantal angles

Q. 2) A directed angle in standard position whose terminal ray lies along ........ is a quadrantal angle.
   a) x – axis only  
   b) y – axis only  
   c) x axis or y axis  
   d) neither on x axis nor on y axis.

Q. 3) 60° = ............  
   a) \( \frac{\pi}{3} \)  
   b) \( \frac{\pi}{4} \)  
   c) \( \frac{\pi}{3} \)  
   d) \( \pi \)

Q. 4) –270° = ............  
   a) \( -\frac{2\pi}{3} \)  
   b) \( -\frac{3\pi}{2} \)  
   c) \( -\frac{\pi}{4} \)  
   d) \( -\frac{4\pi}{5} \)

Q. 5) –\( \frac{5\pi}{4} \) = ............  
   a) –200°  
   b) –180°  
   c) –225°  
   d) –335°

Q. 6) A wheel makes 180 revolutions in one minute then the angle made by the wheel in one second is ............
   a) 2 \( \pi \)  
   b) 4 \( \pi \)  
   c) 6 \( \pi \)  
   d) 8 \( \pi \)

Q. 7) The angles of a triangle are in A.P. and the least angle is 30° then the radian measure of greatest angle is ............
   a) \( \frac{\pi}{2} \)  
   b) \( \frac{2\pi}{3} \)  
   c) \( \frac{5\pi}{6} \)  
   d) \( \frac{3\pi}{2} \)

Q. 8) A railroad curve is to be laid out on a circle. What radius should be used if the track is to change direction by 25° in a distance of 40 meters ?
   a) \( \frac{\pi}{288} \) m  
   b) \( \frac{288}{\pi} \) m  
   c) \( \frac{\pi}{144} \) m  
   d) \( \frac{144}{\pi} \) m
1. Angle and its Measurements

Marks - 2

Q. 1) Express the angles in degree, minute and second
   a) 11.0133°
   b) 94.3366°

Q. 2) Find the length of the arc of a circle of diameter 20 cm., if the arc subtends an angle 36° at the centre.

Q. 3) Convert the angles
   a) 4.4°
   b) $-\frac{9\pi}{2}$ into degree.

Q. 4) Convert the angles
   a) $-\frac{10}{5}$
   b) 405° into radian.

Q. 5) If $x^0 = 405^0$; $y^0 = -\frac{\pi}{12}$ find $x$ and $y$.

Q. 6) If $\theta^0 = \frac{5\pi}{9}$, $\phi^0 = 900^0$ find $\theta$ and $\phi$.

Q. 7) In $\triangle ABC$, $m\angle A = \frac{2\pi}{3}$, $m\angle B = 45^0$. Find $m\angle C$ in degree.

Q. 8) Find the angle between the minute hand and hour hand of a circle at 4.30 p.m.

Q. 9) Find the angle in radian through which a pendulum swings if its length is 75 cm and the tip described an arc of length 21 cm.
1. Angle and its Measurements

Q. 1) Find the number of sides of a regular polygon if each of its interior angle is \( \frac{3\pi}{4} \).

Q. 2) The measures of angles of a quadrilateral are in the ratio 2:3:6:7. Find their measures in degree and radian.

Q. 3) If the arcs of the same length in two circles are subtending angles 65° and 130° then find the ratio of their radii.

Q. 4) The difference between two acute angles of a right angled triangle is \( \frac{3\pi}{10} \). Find angles in degree.

Q. 5) The perimeter of a sector of a circle of area \( 64\pi \) sq.cm. is 56 cm. Find the area of the sector.

Q. 6) Find the degree and radian measure of the angle between the hour hand and the minute hand of a clock at
   i) twenty minutes past two.
   ii) quarter past six.

Q. 7) A wire of length 10 cm. is bend so as to form an arc of radius 4 cm. What is the angle subtending at the centre in degree?

Q. 8) The sum of two angles is \( 5\pi \) and their difference is 60°, find the angles in degree.

Q. 9) One angle of a quadrilateral has measure \( \frac{2\pi}{5} \) and the measure of other three angles are in the ratio 2:3:4. Find their measures in radian.

Q. 10) Find the interior angle of a regular octagon in radian and degree.
1. Angle and its Measurements

Q. 1) In a circle of radius 12 cm, arc PQ subtends the angle of $30^0$ at the centre. Find the area between the arc PQ and chord PQ.

Q. 2) The angles of triangle are in A.P. and the greatest angle is $84^0$. Find all angles in degree and radian.

Q. 3) ΔPQR is an equilateral triangle with side 18 cm. A circle is drawn on segment QR as diameter. Find the length of the arc of this circle intercepted within the triangle.

Q. 4) Show that the minute hand of a clock gains $5^0 \ 30'$ on the hour hand in one minute.

Q. 5) A horse is tied a post by a rope. If the horse moved along a circular path, always keeping the rope tight and describes 88 meter when it traces the angle of $72^0$ at the centre. Find the length of the the rope. ($\pi = \frac{\pi}{7}$)

Q. 6) In a circle of diameter 40 cm, the length of a chord is 20 cm. Find the length of minor arc of the chord.

Q. 7) The angles of triangle are in A.P. and the ratio of the number of degree in the least to the number of radians in the greatest is $60 : \pi$. Find the angles of the triangle in degree and radian.
2. Trigonometric Functions

Q. 1) If $0^0 \leq \theta \leq 90^0$; $\cos \theta$ is .................
   a) Decreasing function
   b) Increasing function
   c) Constant function
   d) identity function

Q. 2) Value of $\frac{\tan 30^0 + \tan 60^0}{-\tan 30^0 + \tan 60^0} = ................$
   a) 0
   b) 1
   c) $\sqrt{3}$
   d) not defined

Q. 3) $\sec^2 30^0 - \tan^2 30^0 = ................$
   a) 0
   b) 1
   c) $\frac{1}{\sqrt{3}}$
   d) not defined

Q. 4) The value of $\sin 30^0 \sin 45^0 \cosec 45^0 \cos 30^0 = .............$
   a) $\frac{\sqrt{3}}{2}$
   b) $\frac{\sqrt{3}}{4}$
   c) $\frac{4}{\sqrt{3}}$
   d) $\sqrt{3}$

Q. 5) $\log [(\sec \theta + \tan \theta) (\sec \theta - \tan \theta)] = ................$
   a) 1
   b) 0
   c) $\sec^2 \theta - \tan^2 \theta$
   d) not defined

Q. 6) If $\sin \theta + \cos \theta = \frac{1}{2(\cos \theta - \sin \theta)}$; $0^0 < \theta < 90^0$ then $\sin \theta = .............$
   a) 0
   b) 1
   c) $\frac{1}{2}$
   d) $\frac{1}{4}$

Q. 7) The value of $(\sin A - \cos A)^2 + (\sin A + \cos A)^2 = .............$
   a) 0
   b) 1
   c) 2
   d) $\frac{1}{2}$

Q.8) Eliminating $\theta$ from the equation $P = a \cosec \theta$, $Q = a \cot \theta$ we get
   a) $P - Q = a$
   b) $P^2 - Q^2 = a^2$
   c) $P^2 Q^2 = a^2$
   d) $\frac{P^2}{Q^2} = a^2$

Q.9) If $\sin \beta + \cos \beta = \frac{5}{4}$ then $\sin \beta \cdot \cos \beta = .............$
   a) $\frac{1}{4}$
   b) $\frac{9}{32}$
   c) $\frac{5}{16}$
   d) $\frac{11}{32}$
Q. 10) The value of \( \cos 10^0 - \sin 10^0 \) is ...............
   a) Positive   b) Negative   c) 0   d) 1

Q. 11) The value of \( \cos 1^0 \cdot \cos 2^0 \cdot \cos 3^0 \cdots \cdot \cos 90^0 \cdots \cdot \cos 179^0 \) is ...............
   a) \( \frac{1}{\sqrt{2}} \)   b) 0   c) 1   d) none of these

Q. 12) If \( \tan \theta = -\frac{4}{3} \) then \( \sin \theta \) is ...............
   a) \(-\frac{4}{5}\) but not \(\frac{4}{5}\)   b) \(-\frac{4}{5}\) or \(\frac{4}{5}\)
   c) \(\frac{4}{5}\) but not \(-\frac{4}{5}\)   d) Neither \(\frac{4}{5}\) nor \(-\frac{4}{5}\)
2. Trigonometric Functions

Q. 1) Simplify \( \sec^4 \alpha - \tan^4 \alpha \) in their least exponents.

Q. 2) Eliminate \( \theta \) from the equations \( a = x \sin \theta - y \cos \theta \) and \( b = x \cos \theta + y \sin \theta \).

Q. 3) Express \( \sin \theta \) in terms of \( \cot \theta \).

Q. 4) Evaluate \( \cos 0^0 + \sqrt{2} \sec 45^0 - \sqrt{3} \tan 30^0 \).

Q. 5) If \( \sec^2 \alpha + \cos^2 \alpha = 2 \) then find the value of \( \sec \alpha + \cos \alpha \).

Q. 6) If \( \sin A = \cos B \), where \( A \) and \( B \) are acute angles then find \( A + B \).

Q. 7) If \( \frac{1 - \tan^2 60^0}{1 + \tan^2 60^0} = \cos X \) then find the value of \( X \).

Q. 8) If \( \sin \theta \) and \( \cos \theta \) are the roots of quadratic equation \( \lambda x^2 - mx - n = 0 \) then show that \( \lambda^2 - m^2 = 2n \).

Q. 9) Find the values of \( \cosec (-1410^0) \) and \( \tan \frac{19\pi \c}{3} \).
2. Trigonometric Functions

Q. 1) If \( \cos^2 \alpha + \cos \alpha = 1 \) then find the value of \( 4 \sin^2 \alpha + 4 \sin^4 \alpha + 2 \).

Q. 2) Eliminate \( \theta \), if \( x = a^2 \cos^3 \theta \), \( y = b^2 \sin^3 \theta \).

Q. 3) If \( \sin \alpha \) and \( \cos \alpha \) are the roots of the equation \( x^2 - bx - 1 = 0 \) then show that.
\[ a^2 - b^2 = 2a. \]

Q. 4) Express the equations in terms of \( x \) and \( y \) if \( x \cos \theta + y \sin \theta = a; \)
\( x \sin \theta - y \cos \theta = b. \)

Q. 5) Eliminate \( \theta \) if \( x = 2 \sec \theta + 3 \tan \theta ; y = 3 \sec \theta - 2 \tan \theta \).

Q. 6) If \( \tan \theta + \cot \theta = 3 \) then show that \( \tan^4 \theta + \cot^4 \theta = 47. \)

Q. 7) Prove that
\[ \frac{\cosec^2 \theta + \sec^2 \theta}{\cosec^2 \theta - \sec^2 \theta} = \frac{1 + \tan^2 \theta}{1 - \tan^2 \theta} \]

Q. 8) Show that
\[ \frac{\cos^2 \theta}{1 - \tan \theta} + \frac{\sin^3 \theta}{\sin \theta - \cos \theta} = 1 + \sin \theta \cdot \cos \theta. \]
2. Trigonometric Functions

Q. 1) If \( \tan \theta + \sin \theta = m \), \( \tan \theta - \sin \theta = n \) then prove that \( m^2 - n^2 = 4 \sqrt{mn} \).

Q. 2) Show that \( \frac{\tan^3 \theta}{1 + \tan^2 \theta} + \frac{\cot^3 \theta}{1 + \cot^2 \theta} = \sec \theta \csc \theta - 2 \sin \theta \cos \theta \).

Q. 3) If \( \cos A = \sin B = -\frac{1}{3} \) and \( \frac{\pi}{2} < A < \pi \), \( \pi < B < \frac{3\pi}{2} \) then find \( \frac{\tan A + \tan B}{\tan A - \tan B} \).

Q. 4) If \( 2 \cos^2 \theta + 7 \sin \theta = 5 \) then find the possible values of \( \sin \theta \).

Q. 5) Prove that \( \frac{\sin \theta}{\cot \theta + \csc \theta} = 2 + \frac{\sin \theta}{\cot \theta - \csc \theta} \).

Q. 6) If \( \frac{\cos^4 A}{\cos^2 B} + \frac{\sin^4 A}{\sin^2 B} = 1 \) then show that \( \frac{\cos^4 B}{\cos^2 A} + \frac{\sin^4 B}{\sin^2 A} = 1 \).

Q. 7) Prove that \( \sec^6 \theta - \tan^6 \theta - 3 \sec^2 \theta \tan^2 \theta = 1 \).

Q. 8) If any \( \square ABCD \) prove that

a) \( \sin (A + B) + \sin (C + D) = 0 \)  
b) \( \cos (A + B) = \cos (C + D) \)
3. Trigonometric Functions of Compound Angles

Marks - 1

Q. 1) \( \sin 75^\circ = \) .................. 
   a) \(- \frac{\sqrt{3}}{2}\)  b) \(\frac{\sqrt{3} - 1}{\sqrt{2}}\)  c) \(\frac{\sqrt{3} - 1}{2\sqrt{2}}\)  d) \(\frac{\sqrt{3} + 1}{2\sqrt{2}}\)

Q. 2) Compound angle means .................
   a) Product of two angles  b) Sum or difference of two angles  
   c) Division of two angles  d) Expression of any angle in radian

Q. 3) \(\frac{\sin 2\theta}{1 - \cos 2\theta} = \) ............
   a) \(\tan \theta\)  b) \(\cos \theta\)  c) \(\cot \theta\)  d) \(\sin \theta\)

Q. 4) If \(A + B = 45^0\) then \((\tan A + 1) (\tan B + 1) = \) ............
   a) 1  b) 0  c) -1  d) 2

Q. 5) If \(A + B = 60^0\) then \(\sin A \cos B + \cos A \sin B = \) ............
   a) \(\frac{\sqrt{3}}{2}\)  b) \(\frac{2}{\sqrt{3}}\)  c) \(\sqrt{3}\)  d) \(\frac{1}{\sqrt{3}}\)

Q. 6) If \(\tan (A + B) = \sqrt{3} \); \(\sin A = \frac{1}{\sqrt{2}}\) then the value of B in radian is ............
   a) \(\frac{\pi}{3}\)  b) \(\frac{\pi}{12}\)  c) \(\frac{\pi}{6}\)  d) 2 \(\pi\)

Q. 7) \(\tan (\frac{3\pi}{2} + \theta) = \) ............
   a) \(-\cos \theta\)  b) \(-\tan \theta\)  c) \(-\cot \theta\)  d) \(\sin \theta\)

Q. 8) If \(\tan \alpha = 3\), \(\tan \beta = \frac{1}{2}\) then which of the following is true ?
   a) \(\alpha + \beta = \frac{\pi^c}{4}\)  b) \(\alpha \beta = \frac{\pi^c}{4}\)  c) \(\alpha - \beta = \frac{\pi^c}{4}\)  d) \(\alpha \beta = \frac{\pi^c}{2}\)

Q. 9) If \(A, B, C, D\) are the angles of a cyclic quadrilateral then
\[\cos A + \cos B + \cos C + \cos D = \] ............
   a) \(\frac{1}{2}\)  b) 1  c) 2  d) 0

Q. 10) \(\sin^2\left(\frac{\pi}{8} + \frac{A}{2}\right) - \sin^2\left(\frac{\pi}{8} - \frac{A}{2}\right) = \) ............
   a) \(\frac{1}{\sqrt{2}} \sin A\)  b) \(\sin A\)  c) \(\frac{1}{\sin A}\)  d) \(\frac{1}{2} \sin A\)

(10)
3. Trigonometric Functions of Compound Angles

Q. 1) Show that \(\frac{\sin 25^0}{\cos 35^0} - \frac{\cos 25^0}{\sin 35^0} = -\csc 70^0\)

Q. 2) Find the value of \(\csc^2 \alpha \left[1 + \frac{1}{\sec \alpha}\right] \left[1 - \frac{1}{\sec \alpha}\right]\)

Q. 3) If \(\csc \theta - \cot \theta = x\) then find the value of \(\csc \theta + \cot \theta\).

Q. 4) Find the value of \(\frac{\sin 20^0 \cos 70^0 + \cos 20^0 \sin 70^0}{\sin 23^0 \csc 23^0 + \cos 23^0 \sec 23^0}\)

Q. 5) Prove that \(\tan 10^0 + \tan 35^0 + \tan 10^0 \tan 35^0 = 1\).

Q. 6) Show that \(\sec A \cdot \sec B \cdot \sin (A + B) = \tan A + \tan B\).

Q. 7) Show that \(\cos^2 \left(\frac{\pi}{4} - x\right) + \cos^2 \left(\frac{\pi}{4} + x\right) = 1\).

Q. 8) Find the value of \(\tan (-945^0)\)

Q. 9) Show that \((\cos \alpha + \cos \beta)^2 + (\sin \alpha + \sin \beta)^2 = 4 \cos^2 \left(\frac{\alpha - \beta}{2}\right)\)

Q. 10) Find the value of \(\tan 56^0 - \tan 11^0 - \tan 56^0 \tan 11^0\).
3. Trigonometric Functions of Compound Angles

Q. 1) Show that \( \sqrt{2} \cos \left( \frac{\pi}{4} - A \right) = \cos A + \sin A \).

Q. 2) If \( \sin \alpha = -\frac{5}{13} \), \( \pi < \alpha < \frac{3\pi}{2} \); \( \cos \beta = \frac{3}{5} \), \( \frac{3\pi}{2} < \beta < 2\pi \) then find \( \sin (\alpha + \beta) \).

Q. 3) Find the value of \( (\tan 15^0 + \tan 75^0) \).

Q. 4) If \( 2 \sin \left( \theta + \frac{\pi}{3} \right) = \cos \left( \theta - \frac{\pi}{6} \right) \) then prove that \( \tan \theta + \sqrt{3} = 0 \).

Q. 5) Prove that \( \sqrt{2 + \sqrt{2 + \sqrt{2 + 2 \cos 8\theta}}} = 2 \cos \theta \).

Q. 6) If \( \tan \theta = \frac{5}{6} \) and \( \tan \phi = \frac{1}{11} \) then find \( \theta + \phi \).

Q. 7) Express \( \sin \theta - \cos \theta \) as a single trigonometric ratio.

Q. 8) Show that \( \tan 20^0 \tan 40^0 \tan 60^0 \tan 80^0 = 3 \).
3. Trigonometric Functions of Compound Angles

Marks - 4

Q. 1) Prove that \[
\frac{2 \cos 4x + 1}{2 \cos x + 1} = (2 \cos x - 1) (2 \cos 2x - 1).
\]

Q. 2) Prove that \[
\cos 70^\circ \cos 140^\circ \cos 280^\circ \cos 560^\circ = \frac{\sin 68^\circ}{16 \cos 83^\circ}.
\]

Q. 3) Show that \[
\frac{\sin (-160^\circ)}{\sin^2 70^\circ} + \frac{\sin (180^\circ - \theta)}{\sin \theta} = \sec^2 20^\circ.
\]

Q. 4) If \(\tan x = -\frac{3}{4}\), \(\frac{3}{2} \pi < x < 2 \pi\) then find i) \(\sin 2x\) ii) \(\cos 2x\) iii) \(\tan 2x\).

Q. 5) Prove that \[
\frac{\cos^3 \theta - \cos 3\theta}{\cos \theta} + \frac{\sin^3 \theta - \sin 3\theta}{\sin \theta} = 3.
\]

Q. 6) Prove that \(4 \sin A \cos^3 A - 4 \cos A \sin^3 A = \sin 4A\).

Q. 7) Show that \[
(\cos \theta + i \sin \theta)^3 = (\cos 3\theta + i \sin 3\theta) \text{ where } i^2 = -1.
\]

Q. 8) Show that \[
\frac{1}{\sin 10^\circ} - \frac{\sqrt{3}}{\cos 10^\circ} = 4.
\]
4. Factorization Formulae

Q. 1) Which of the following is correct?
\[ \cos (360^0 + 75^0) = \ldots \ldots \ldots \ldots \]
 a) \( \cos 360^0 + \cos 75^0 \)
 b) \( \cos 45^0 + \cos 30^0 \)
 c) \( \frac{\sqrt{3} + 1}{2 \sqrt{2}} \)
 d) \( \frac{\sqrt{3} - 1}{2 \sqrt{2}} \)

Q. 2) In \( \triangle ABC \), \( \tan \left( \frac{A + B}{2} \right) = \ldots \ldots \ldots \ldots \)
 a) \( \tan \frac{C}{2} \)
 b) \( \cot \frac{C}{2} \)
 c) \( -\tan \frac{C}{2} \)
 d) \( -\cot \frac{C}{2} \)

Q. 3) \( \frac{\cos \theta + \cos (-\phi)}{\sin \theta - \sin (-\phi)} = \ldots \ldots \ldots \ldots \)
 a) \( \cot \frac{\theta - \phi}{2} \)
 b) \( \cot \frac{\theta + \phi}{2} \)
 c) \( \tan \phi \)
 d) \( \cot \theta \)

Q. 4) \( \cos \left( \frac{3\pi}{4} + x \right) \cdot \cos \left( \frac{3\pi}{4} - x \right) = \ldots \ldots \ldots \ldots \)
 a) \( \sqrt{2} \sin x \)
 b) \( \frac{1}{2} \sin x \)
 c) \( -2 \sin x \)
 d) \( -\sqrt{2} \sin x \)

Q. 5) \( \cos^2 27^0 + \cos^2 93^0 + \cos^2 147^0 = \ldots \ldots \ldots \ldots \)
 a) 1
 b) 0
 c) -1
 d) 1/2

Q. 6) In \( \triangle ABC \) which of the following is NOT true?
 a) \( \tan A + \tan B + \tan C = \tan A \cdot \tan B \cdot \tan C \)
 b) \( 3 \tan 60^0 = (\tan 60^0)^3 \)
 c) \( \sqrt{3} \tan 75^0 = \sqrt{3} + 1 + \tan 75^0 \)
 d) \( \tan 30^0 + \tan 60^0 + \tan 45^0 = \tan 30^0 \cdot \tan 60^0 \cdot \tan 45^0 \)

Q. 7) \( \sin^2 18^0 = \ldots \ldots \ldots \ldots \)
 a) \( 1 - \cos 18^0 \)
 b) \( \frac{1 + \cos 18^0}{2} \)
 c) \( 1 + \cos 36^0 \)
 d) \( \frac{1 - \cos 36^0}{2} \)
Q. 8) One of the factor of $\cos 75^0 - \cos 15^0$ is .....  
   a) $- \frac{1}{\sqrt{2}}$  
   b) 0  
   c) $\frac{\sqrt{3}}{2}$  
   d) $-\frac{\sqrt{3}}{2}$  

Q. 9) $\sin (45^0 + A) \cdot \sin (45^0 - A) =$ ...............  
   a) $\sin 2A$  
   b) $\cos 2A$  
   c) $\frac{1}{2} \sin 2A$  
   d) $\frac{1}{2} \cos 2A$.  

Q. 10) $\cos 20^0 \cos 40^0 \cos 80^0 =$ ...............  
   a) $\frac{1}{8} \cosec 10^0$  
   b) $\frac{1}{16}$  
   c) $\frac{1}{8}$  
   d) $\frac{1}{8} \sin 20^0$
Q. 1) Express \(12 \sin 2\chi \sin 3\chi \cos 2\chi\) as a sum / difference of two trigonometric functions.

Q. 2) If \(\cos 10^0 - \cos 50^0 = A \sin B\). Find the values of A, B.

Q. 3) Find the Value of \(\cos 300^0 + \sin 135^0 - \tan 120^0\)

Q. 4) Express the product \(\frac{1}{3} \cos 6\chi \cos 4\chi\) as a sum / difference.
Q. 1) Simplify:
\[ \sin(x + y) + \sin(x - y), \text{ verify your answer with } x = 60^\circ, \text{ & } y = 30^\circ. \]

Q. 2) If \( \sin(A + B) = \sin C \) and \( \cos(A + B) = -\cos C \) then, what should be the condition on angles \( A, B, C \) ?

Q. 3) Show that: \( \cos 20^\circ \cos 40^\circ - 1 = \cos 60^\circ \cos 20^\circ \)

Q. 4) Simplify: \[ \cos \left( \frac{2\pi}{3} + x \right) - \cos \left( \frac{2\pi}{3} - x \right) \]

Q. 5) Show that, \( \cos^2 A + \cos^2 (A + 60^\circ) + \cos^2 (A - 60^\circ) = \frac{3}{2} \)

Q. 6) Prove that \( 2 \sin 37.5^\circ \sin 52.5^\circ = \frac{\sqrt{3} + 1}{2\sqrt{2}} \)

Q. 7) If \( A, B, C \) are angles of triangle, Simplify:
\[ \cos 2A + \cos 2B - \cos 2C. \]

Q. 8) Find 4 angles in A.P. such that
\[ \frac{\sin A \sin B \sin C \sin D}{2} = \frac{\sin B \cdot \sin C}{2} \]
4. Factorization Formulae

Q. 1) Prove that: \[ \frac{\sin 5x - 2\sin 3x + \sin x}{\cos 5x - \cos x} = \tan x \]

Q. 2) Prove that: \[ \frac{\cos 4x + \cos 3x + \cos 2x}{\sin 4x + \sin 3x + \sin 2x} = \cot 3x \]

Q. 3) Simplify:
\[ \sin (n+1)x \sin (n+2)x + \cos (n+1)x \cos (n+2)x \]

Q. 4) Prove that:
\[ \cot x \cot 2x - \cot 2x \cot 3x - \cot 3x \cot x = 1 \]

Q. 5) Show that:
\[ \cos^2 2x - \cos^2 6x = \sin 4x \sin 4x \sin 8x \]

Q. 6) Prove that: In \triangle ABC,
\[ \cos^2 \left( \frac{A}{2} \right) + \cos^2 \left( \frac{B}{2} \right) + \cos^2 \left( \frac{C}{2} \right) = 2 + 2 \sin \frac{A}{2} \sin \frac{B}{2} \sin \frac{C}{2} \]

Q. 7) Find the Value of:
\[ \cos^2 x + \cos^2 (x + 60^\circ) + \cos^2 (x - 60^\circ) \]
Q. 1) Which of the following is NOT a point of the locus: 
\[
\frac{x^2}{16} + \frac{y^2}{25} = 1 ?
\]
\[a) \ (5 \cos \theta, 4 \sin \theta) \quad b) \ (4 \cos (-\theta), 5 \sin (-\theta)) \]
\[c) \ (4 \cos \frac{3\pi}{4}, 5 \sin \frac{3\pi}{4}) \quad d) \ (4 \cos 300^0, 5 \sin 300^0) \]

Q. 2) \{ P (x', y') | d(O, P) > 0 \} always traces.
\[a) \ line \quad b) \ circle \quad c) \ triangle \quad d) \ Plane \]

Q. 3) Point P is moving in the same direction but parallel to the locus: \(ax + by + c = 0, a, b, c \in \mathbb{R} \) (not all zero at a time), then equation of locus of P is:
\[a) \ ax + by + c = a \quad a) \ ax - by + c = 0 \quad c) \ bx - ay + c = 0 \quad d) \ ax + by + c = \]

Q. 4) Where the origin is to be shifted so that the equation.
\[(x + y)^2 + (y - 1)^2 = 1 \]
becomes
\[x^2 + y^2 = 1 ?
\[a) \ (1, 0) \quad b) \ (0, 1) \quad c) \ (-1, 1) \quad d) \ (1, -1) \]

Q. 5) Geometrically the set:
\[
\{ P (x, y) | (x + 3)^2 + (y - 2)^2 = 3 \}
\]
interpretes.
\[a) \ Ellipse \quad b) \ parabola \quad c) \ Joint \ equation \ of \ lines \quad d) \ Circle \]

Q. 6) If \((2, 0), (0, b)\) are the points of the locus \(2x + 3y = k\), then the locus passes through the point
\[a) \ (-8, 4) \quad b) \ (B, -4) \quad c) \ (4, 8) \quad d) \ (-8, -4)\]
5. LOCUS

Q. 1) Find the equation of locus of a point if difference between the distance of point P from co-ordinate axes is constant.

Q. 2) If the locus : \(x^2 - 7x + ky = 0\) passes through the point \((-2, 2)\) and \((3, a)\) then find a.

Q. 3) Where the origin is to be shifted if the point \((2, 3)\) becomes \((-2, -3)\) ?

Q. 4) Given \(A (-2, 0), B (2, 0), P (x, y)\) and \(PA - 2PB = 0\) then write the equation of locus of P.

Q. 5) What will be the new equation of a line passing through the two points \((2, 0), (0, 1)\) if the origin is shifted to the point \((-1, -1)\) ?

   a) \(|x - y| = c\)    b) \(-4/3\)    c) \((4, 6)\)    d) \(3x + 2 = 0\)
5. LOCUS

Q. 1) A variable point R is such that P – R – Q forms a semi circle.

If P (−1, 2), Q (−2, −3); find the equation of the path traced by R.

Q. 2) Find the locus of a point P, if difference between its distances from A (3, 0) and B (−3, 0) is 4.

Q. 3) If A (1, 2), B (5, 6) then find the inclination of

{ P (x, y) | d (P, A) = d (P, B) }.

Q. 4) Locus of point P (x, y') is linear. It passes through the point (1/2, 1/2) and forms right angled with co-ordinate axes. Discuss which of the following equation will always represent the path of point.

a) \(x + y = 1\)  
b) \(x = y\)  
c) \(bx + ay = ab, \ ab > 0\)  
d) \(bx + y = ab, \ \text{atmost one constant is -ve.}\)

Q. 5) After shifting the origin, axes remaining parallel, if point (6, 3) lies on new x axis and point (2, 5) lies on new Y axis, then find new co-ordinates of Point (1, 1).
Q. 1) Given conic : \( y^2 = 12x \).
Now origin is shifted to the point (3, 3). Show through neat and clean diagram what will be the corresponding effect on focus and directrix of the conic.
Also using definition derive the new equation of the conic.

Q. 2) A ladder of length 8 m standing vertically, moves slowly down till it rests horizontally. If P is a point on the ladder deviding the ladder in the ratio 5 : 3, find the locus of P, also state range for the co-ordinates of point P.

Q. 3) If Q is a point on the locus \( 3x^2 + 3y^2 = 12 \). Given point A = (3 , 3). Find the equation of locus of a point P which divides the segment AQ internally in the ratio 1 : 2.

Q. 4) Obtain the new equation of the locus \((a - b)(x^2 + y^2) - 2abx = 0\), if the origin is shifted to the point \(\left(\frac{ab}{a - b}, 0\right)\), axes remaining parallel.

Q. 5) If Q is a point on the locus \( x^2 + y^2 + 6x - 4y + 5 = 0 \)
Find the equation of locus of a point P which divides segment OQ externally in the ratio 1 : 2

Q. 6) Find the co-ordinates of new origin, axes remaining parallel
\(4x^2 + y^2 + 8x - 4y + 4 = 0\) will not contain terms in \(x\), \(y\) and constant term.

Q. 7) How can we make the equation \( xy - 3x + 2y - 7 = 0\) equivalent to \( xy = 1\)? Explain.
6. STRAIGHT LINE

Marks - 1

Q. 1) The slope of a line which cuts intercepts of equal lengths on axes is
   a) ±1  b) 0  c) 2  d) \( \sqrt{3} \)

Q. 2) If the equation \( y = mx + c \) and \( x \cos \alpha + y \sin \alpha = p \) represent the same straight line then........
   a) \( p = c \sqrt{1 + m^2} \)  b) \( c = p \sqrt{1 + m^2} \)  c) \( cp = \sqrt{1 + m^2} \)  d) \( p^2 + c^2 + m^2 = 1 \)

Q. 3) If the straight line \( ax + by + c = 0 \) always passes through \((1, -2)\) then a, b, c are........
   a) in A.P.  b) in H.P.  c) in G.P.  d) None of these

Q. 4) If the coordinates of the vertices A, B, C of \( \triangle ABC \) be \((-4, 2), (12, -2) \) and \((8, 6)\) respectively then \( \angle B = \ldots \ldots \).
   a) \( \tan^{-1} \left( \frac{-6}{7} \right) \)  b) \( \tan^{-1} \left( \frac{6}{7} \right) \)  c) \( \tan^{-1} \left( \frac{7}{6} \right) \)  d) \( \tan^{-1} \left( \frac{7}{6} \right) \)

Q. 5) The distance of the point \((-2, 3)\) from the line \( x - y = 5 \) is .........
   a) \( 5 \sqrt{2} \)  b) \( 2 \sqrt{5} \)  c) \( 3 \sqrt{5} \)  d) \( 5 \sqrt{3} \)

Q. 6) The straight lines \( x + 2y - 9 = 0, 3x + 5y - 5 = 0 \) and \( ax + by - 1 = 0 \) are concurrent if the straight line \( 35x - 22y + 1 = 0 \) passes through the point........
   a) \((a, b)\)  b) \((b, a)\)  c) \((-a, -b)\)  d) \((-b, -a)\)

Q. 7) The diagonals of the parallelogram PQRS are along the line \( x + 3y = 4 \) and \( 6x - 2y = 7 \) then \( \square PQRS \) must be ......
   a) rectangle  b) square  c) cyclic quadrilateral  d) rhombus

Q. 8) If the normal form of the equation \( ax + by + c = 0 \) is \( x \cos \alpha + y \sin \alpha = p \), then \( p = \ldots \ldots \).
   a) \( \frac{1}{\sqrt{a^2 + b^2}} \)  b) \( \frac{a}{\sqrt{a^2 + b^2}} \)  c) \( \frac{b}{\sqrt{a^2 + b^2}} \)  d) \( \frac{c}{\sqrt{a^2 + b^2}} \)

Q. 9) The slope of a line passing through the points \((2, 5)\) and \((x, 3)\) is 2 then the value of \((x - 1)\) is...........
   a) 3  b) 2  c) 1  d) 0
Q. 10) The equation of the line parallel to Xaxis and at a distance of 3 units above it is....
   a) x = − 3  
   b) x = 3  
   c) y = 3  
   d) y = − 3

Q. 11) The equation of a line passing through the point (4, 3) and perpendicular to the line 3x − 2y + 5 = 0 is ..........
   a) 3x + 2y − 17 = 0  
   b) 2x + 3y − 17 = 0  
   c) 2x − 3y + 17 = 0  
   d) 3x − 2y + 17 = 0

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6. STRAIGHT LINE

Marks - 2

Q. 1) Find the slope of the line which makes an angle of 30° with the positive direction of Y axis measured anticlock wise.

Q. 2) Write down the equation of the straight line which makes an angle of tan^{-1} 2 with X axis and cut off an intercept 5 from the positive direction of Y axis.

Q. 3) Find the equation of the line passing through (−3, 5) and perpendicular to the line through the points (2, 5) and (−3, 6).

Q. 4) A straight line passes through the point (2, 3) and the portion of the line intercepted between the axes is bisected at this point. Find the equation of the line.

Q. 5) The perpendicular distance of a line from the origin is 7 cm and its slope is -1. Find the equation of the line.

Q. 6) Test whether the lines 3x + 4y = 7 and 4x − 3y 8 = 0 are parallel or perpendicular to each other.

Q. 7) Find the equation of the line passing through (−3, 2) and cutting an intercepts equal in magnitude but opposite in sign from the axes.

Q. 8) Find the equation of the perpendicular bisector of the line segment joining A (−2, 3) and B (6 − 5).

Q. 9) If the lines 4x + 3y = 1 ; y = x + 5 and 5y + bx = 3 are concurrent then find the value of b.

Q. 10) Find the value of \( \sqrt{k} \) if the slope of the line passing through the points (2, 4), (5, k) is \( \frac{5}{3} \).
6. STRAIGHT LINE

Q. 1) The slope of a line is double of the slope of another line. If tangent of the angle between them is \( \frac{1}{3} \), then find the slopes of these lines?

Q. 2) Find the equation of a line which has y intercept 2 and is perpendicular to \( y = 2x + 1 \).

Q. 3) Find the equations of straight lines which are at a distance of \( \frac{1}{2} \) from the origin and pass through the point (0,1).

Q. 4) The vertices of \( \triangle PQR \) are P (2, 1), Q (–2, 3) and R (4, 5). Find the equation of the median through the vertex R.

Q. 5) A line forms a triangle of area \( 54\sqrt{3} \) sq. units with the coordinate axes. Find the equation of the line, if the perpendicular drawn from the origin to the line makes an angle of 60° with the X axis.

Q. 6) Find the tangent of the angle between the lines whose intercepts on the axes are respectively \( p, -q \), and \( q, -p \).

Q. 7) Find the points on the Y axis whose perpendicular distance from the line \( 4x - 3y - 12 = 0 \) is 3.

Q. 8) Find the distance of the point of intersection of the lines \( 2x + 3y = 21 \) and \( 3x - 4y + 11 = 0 \) from the line \( 8x + 6y + 5 = 0 \).

Q. 9) In what ratio does the line joining the points (2, 3) and (4, 1) divides the segment joining the points (1, 2) and (4, 3)?

Q. 10) A person standing at a junction of two straight paths represented by the equation \( 2x - 3y - 4 = 0 \) and \( 3x - 4y - 5 = 0 \) wants to reach the path whose equation is \( 6x - 7y + 8 = 0 \) in the least time. Find the equation of the path that he should follow.

Q. 11) Find the equation of a line through (2, 3) which makes an angle of 45° with the line \( 2x + y - 7 = 0 \).
6. STRAIGHT LINE

Marks - 4

Q. 1) A quadrilateral has the vertices at the points (−4, 2) (2, 6), (8, 5) and (9, −7). Show that the mid points of the sides of this quadrilateral are the vertices of a parallelogram use slope method.

Q. 2) Show that the perpendicular drawn from the point (4, 1) on the line segment joining (6, 5) and (2, −1) divides it in the ratio 8 : 5.

Q. 3) Two consecutive sides of a parallelogram are 4x + 5y = 0 and 7x + 2y = 0. If the equation of one diagonal is 11x + 7y = 9, find the equation of the other diagonal.

Q. 4) If p and q are the lengths of perpendiculars from the origin to the line $x \cos \theta - y \sin \theta = k \cos 2\theta$ and $x \sec \theta + y \cosec \theta = k$ respectively then prove that $p^2 + 4q^2 = k^2$.

Q. 5) A line passing through the point A (3, 0) makes 30° angle with the positive direction at X axis. If this line is rotated through an angle of 15° about A in clockwise direction. Find its equation in new position.

Q. 6) Find equations of the medians of triangle formed by the lines $x + y - 6 = 0$, $x - 3y - 2 = 0$ and $5x - 3y + 2 = 0$.

Q. 7) If $A (4, 3), B (0, 0)$ and $C (2, 3)$ are the vertices of $\triangle ABC$. Find the equation of the bisector of $\angle A$.

Q. 8) The points (1, 2) and (3, 8) are the opposite vertices of a square. Find the equation of its two adjacent sides through (1, 2).

Q. 9) Prove that the length of perpendiculars from the points $P(m^2, 2m), Q(mn, m+n)$ and $R(n^2, 2n)$ to the line $x \cos^2 \theta + y \sin \theta \cos \theta + \sin^2 \theta = 0$ are in G.P.

Q. 10) Find the equation of line which passes through the point of intersection of the lines $x + 2y - 5 = 0$ and $3x + 7y - 17 = 0$ and is perpendicular to the line $3x + 4y - 10 = 0$. 

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7. CIRCLE AND CONICS

Marks - 1

Q. 1) If a square is inscribed in the circle \( x^2 + y^2 - 2x + 4y + 3 = 0 \) whose sides are parallel to the coordinate axes then one vertex of the square is........
   a) \( (1 + \sqrt{2}, -2) \)    b) \( (1 - \sqrt{2}, -2) \)    c) \( (2, -1) \)    d) \( (0, 1) \)

Q. 2) If the lines \( 2x + 3y + 1 = 0 \) and \( 3x - y - 4 = 0 \) lie along diameters of a circle of circumference \( 10\pi \) then the equation of the circle is ......
   a) \( x^2 + y^2 + 2x - 2y - 23 = 0 \)    b) \( x^2 + y^2 - 2x - 2y - 23 = 0 \)
   c) \( x^2 + y^2 + 2x + 2y - 23 = 0 \)    d) \( x^2 + y^2 - 2x + 2y - 23 = 0 \)

Q. 3) A parabola has the origin as its focus and the line \( x = 2 \) as the directrix then the vertex of the parabola is at
   a) \( (1,0) \)    b) \( (0,1) \)    c) \( (2,0) \)    d) \( (0,2) \)

Q. 4) The length of latus rectum of the parabola \( 4y^2 + 2x - 20y + 17 = 0 \) is........
   a) 3    b) 6    c) \( \frac{1}{3} \)    d) 9

Q. 5) If the latus rectum of an ellipse be equal to half of its minor axis then its eccentricity is........
   a) \( \frac{3}{2} \)    b) \( \frac{\sqrt{3}}{2} \)    c) \( \frac{2}{3} \)    d) \( \frac{\sqrt{2}}{3} \)

Q. 6) The foci of the ellipse \( 25 (x + 1)^2 + 9 (y + 2)^2 = 225 \) are ........
   a) \( (-1, 2) \) and \( (-1, -6) \)    b) \( (-1, 2) \) and \( (6, 1) \)
   c) \( (1, -2) \) and \( (1, -6) \)    d) \( (-1, -2) \) and \( (1, 6) \)

Q. 7) The distance between the foci of the hyperbola \( x^2 - 3y^2 - 4x - 6y - 11 = 0 \) is...
   a) 4    b) 6    c) 8    d) 10

Q. 8) The difference of the focal distances of any point on the hyperbola \( 9x^2 - 16y^2 = 144 \) is....
   a) 8    b) 7    c) 6    d) 4

Q. 9) The eccentricity of the hyperbola conjugate to \( x^2 - 3y^2 = 2x + 8 \) is....
   a) \( \frac{2}{\sqrt{3}} \)    b) \( \sqrt{3} \)    c) 2    d) \( \sqrt{2} \)
Q. 10) The radius of the circle concentric to $x^2 + y^2 - 8x - 10y + 25 = 0$ is 6 then the equation of the concentric circle is....

a) $x^2 + y^2 - 8x - 10y - 5 = 0$

b) $x^2 + y^2 - 8x - 10y + 5 = 0$

c) $x^2 + y^2 + 8x - 10y + 5 = 0$

d) $x^2 + y^2 + 8x + 10y + 5 = 0$
7. CIRCLE AND CONICS

Q. 1) Find the equation of a circle passing through the origin and making intercepts 4, 5 on the axes of co-ordinate.

Q. 2) Find the centre and radius of the circle $2x^2 + 2y^2 - x = 0$.

Q. 3) Find the equation of the parabola when its vertex at (0, 0) ; passes through (5, 2) and symmetric with respect to Y axis.

Q. 4) Find focus and vertex of the parabola $y^2 - 2y - 4x - 19 = 0$.

Q. 5) Find the equation of the parabola when its vertex at (0, 0) ; passes through (5, 2) and symmetric with respect to Y axis.

Q. 6) Find the equation of the ellipse whose foci are (± 2, 0) and $e = \frac{1}{2}$.

Q. 7) Find the equation of the parabola with focus (0, 0) and directrix $x + y = 4$.

Q. 8) If the eccentricity of the two ellipse $\frac{x^2}{169} + \frac{y^2}{25} = 1$ and $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ are equal then find the value of $\frac{a}{b}$.

Q. 9) Find the centre of the hyperbola $9x^2 - 16y^2 + 18x + 32y - 151 = 0$.

Q. 10) Find the equation of hyperbola whose foci are at (± 5, 0) and transverse axis is of length 8.
Q. 1) Find the equation of the ellipse with major axis along the X axis and passing through the points (4, 3) and (−1, 4).

Q. 2) The focus of a parabolic mirror in figure is at a distance of 5 cm form its vertex.
   If the mirror is 45 cm deep find the distance AB.

Q. 3) Find the equation of a circle of radius 5, whose centre lies on X axis and passes through the point (2, 3)

Q. 4) Find the vertex, focus, directrix, length of latus rectum of the parabola

   \[4y^2 + 12x − 20y + 67 = 0.\]

Q. 5) Find the equations of circles which touch the lines x = 0, y = 0 and x = a.

Q. 6) Find the equations of the image of the circle \(x^2 + y^2 + 8x − 16y + 64 = 0\) in the mirror \(x = 0\).

Q. 7) An equilateral triangle is inscribed in the parabola \(y^2 = 4ax\) whose vertex is at the vertex of the parabola. Find the length of its side.

Q. 8) A man running a race-course notes that the sum of the distances from the two flag posts from him is always 10 m and the distance between the flag posts is 8 m. Find the equation of the path traced by the man.

Q. 9) If \(e\) and \(e'\) be the eccentricities of the hyperbola and its conjugate then prove that

\[
\frac{1}{e^2} + \frac{1}{e'^2} = 1.
\]

11) Show that the equation \(16x^2 − 3y^2 − 32x − 12y − 4 = 0\) represents a hyperbola. Find the lengths of axes and eccentricity.

12) The foci of a hyperbola coincide with the foci of the ellipse

\[
\frac{x^2}{16} + \frac{y^2}{9} = 1.
\]

Find the equation of hyperbola if its eccentricity is \(\sqrt{2}\).
Q. 1) A rod AB of length 15 cm rests in between two coordinate axes in such a way that the end point A lies on X axis and end point B lies on Y axis. P is point on rod AB such that AP = 6 cm. Show that the locus of P is an ellipse.

Q. 2) Find the equation of the circle which is circumscribed about the triangle whose vertices are A (−2, 3), B (5, 2) and C (6, −1).

Q. 3) Derive the equation of parabola \( y^2 = 4ax \) in its standard form.

Q. 4) Derive the equation of ellipse \( \frac{x^2}{a^2} + \frac{y^2}{b^2} = 1 \) in its standard form.

Q. 5) Derive the equation of hyperbola \( \frac{x^2}{a^2} - \frac{y^2}{b^2} = 1 \) in its standard form.

Q. 6) Find the equation of parabola with vertex (2, −3) and focus (0, 5).

Q. 7) Find the equation of the circle which passes through the point (2, 0) and whose centre is the limit of the point of intersection of the lines 3x + 5y = 1 and (2+c)x + 5c^2y = 1 as C → 1.

Q. 8) A rectangle ABCD is inscribed in a circle with a diameter lying along the line 3y = x + 10. If A and B are the points (−6, 7) and (4, 7) respectively. Find the area of the rectangle and equation of the circle.

Q. 9) Find the length of axes, coordinate of vertices eccentricity, coordinates of foci, equations of directrices and length of latus rectum of the hyperbola \( y^2 - 16x^2 = 16 \).

Q. 10) Find the equation of the parabola whose focus is at (3, −4) and directrix is the line \( x + y − 2 = 0 \). Also find the length of latus rectum and the equation of its axis.
8. Vectors

Marks - 1

Q. 1) If \( |\textbf{a}| = 11, |\textbf{a} - \textbf{b}| = 30\) and \( |\textbf{a} + \textbf{b}| = 20\) then \( |\textbf{b}| = \ldots\ldots\ldots\ldots\)
   a) 11 b) 41 c) 23 d) 19

Q. 2) If \( \text{\textit{AO}} + \text{\textit{OB}} = \text{\textit{BO}} + \text{\textit{OC}}\) then \( \text{A, B, C}\) form
   a) Equilateral triangle b) Right angled triangle
c) isosceles triangle d) Line

Q. 3) If \( \theta\) is the angle between unit vectors \( \textbf{a}\) and \( \textbf{b}\) then \( \cos \frac{\theta}{2} = \ldots\ldots\ldots\ldots\)
   a) \( \frac{1}{2} |\textbf{a} - \textbf{b}|\) b) \( \frac{1}{2} |\textbf{a} + \textbf{b}|\)
c) \( \frac{|\textbf{a} - \textbf{b}|}{|\textbf{a} + \textbf{b}|}\) d) \( \frac{|\textbf{a} + \textbf{b}|}{|\textbf{a} - \textbf{b}|}\)

Q. 4) If \( \theta\) is the acute angle between the vectors \( \textbf{a}\) and \( \textbf{b}\) and \( |\textbf{a} \times \textbf{b}| = |\textbf{a} \cdot \textbf{b}|\)
   then \( \theta = \ldots\ldots\ldots\ldots\)
   a) \( \pi\) b) \( \frac{\pi}{2}\) c) \( \frac{\pi}{4}\) d) 0

Q. 5) The moment of a force \( \textbf{i} + 2\textbf{j} + 3\textbf{k}\) about the point \( 2\textbf{i} - \textbf{j} + \textbf{k}\) is \ldots\ldots\ldots\ldots
   a) 5\textbf{i} - 5\textbf{j} + 5\textbf{k}\) b) 5\textbf{i} + 5\textbf{j} - 5\textbf{k}\) c) - 5\textbf{i} + 5\textbf{j} + 5\textbf{k}\) d) - 5\textbf{i} - 5\textbf{j} + 5\textbf{k}\)

Q. 6) If \( |\textbf{a}| = 2, |\textbf{b}| = 5\) and \( |\textbf{a} \times \textbf{b}| = 8\) then \( \textbf{a} \cdot \textbf{b} = \ldots\ldots\ldots\ldots\)
   a) 0 b) 2 c) 4 d) 6

Q. 7) The position vectors of the points \( \text{A, B and C}\) are \( \hat{i} + \hat{j} + \hat{k}\) and \( \hat{i} + \hat{j} + \hat{k}\) respectively.
   The vector area of the \( \triangle ABC = \ldots\ldots\ldots\ldots\), where \( \text{\textit{a}}\) = \ldots\ldots\ldots\ldots
   a) \( \hat{i} + \hat{j} + \hat{k}\) b) \( \hat{i} + \hat{j} + \hat{k}\) c) \( \hat{i} + \hat{j} + \hat{k}\) d) \( \hat{i} + \hat{j} + \hat{k}\)

Q. 8) If \( \textbf{a} = 3\hat{i} - 5\hat{j}\) and \( \textbf{b} = 6\hat{i} + 3\hat{j}\) are two vectors and \( \textbf{c}\) is a vector such that \( \textbf{c} = \textbf{a} \times \textbf{b}\)
   then \( |\textbf{a}| : |\textbf{b}| : |\textbf{c}|\) is \ldots\ldots\ldots\ldots
   a) \( \sqrt{34} : \sqrt{45} : \sqrt{39}\) b) \( \sqrt{34} : \sqrt{45} : 39\)
c) 34 : 39 : 45 d) 39 : 35 : 34

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Q. 9) In the adjoining figure which vectors are conital vectors?

![Diagram with vectors a, b, c, d]

a) $\overrightarrow{a}$, $\overrightarrow{c}$, $\overrightarrow{d}$  

b) $\overrightarrow{a}$ and $\overrightarrow{c}$  

c) $\overrightarrow{b}$, $\overrightarrow{c}$ and $\overrightarrow{d}$  

d) $\overrightarrow{a}$, $\overrightarrow{b}$ and $\overrightarrow{c}$

Q. 10) Which of the following is a vector?

a) Area  

b) density  

c) weight  

d) calorie

Q. 11) Let the $\overrightarrow{a}$ and $\overrightarrow{b}$ be such that $|\overrightarrow{a}| = 3$ and $|\overrightarrow{b}| = \frac{\sqrt{2}}{3}$. If $\overrightarrow{a} \times \overrightarrow{b}$ is a unit vector then the angle between $\overrightarrow{a}$ and $\overrightarrow{b}$ is ............

a) $\frac{\pi}{6}$  

b) $\frac{\pi}{4}$  

c) $\frac{\pi}{3}$  

d) $\frac{\pi}{2}$
8. Vectors

Q. 1) If \( \vec{a} = 4\hat{i} + 2\hat{j} + 2\hat{k} \) and \( \vec{b} = 3\hat{i} + 6\hat{j} + 2\hat{k} \) then find a vector in the direction of \( \vec{a} \) and having magnitude of \( |\vec{b}| \).
   a) 11  
   b) 41  
   c) 23  
   d) 19

Q. 2) Find the value of \( p \) for which \( \vec{a} = 3\hat{i} + 2\hat{j} + 9\hat{k} \) and \( \vec{b} = \hat{i} + p\hat{j} + 3\hat{k} \) are parallel vectors.

Q. 3) Find the magnitude of two vectors \( \vec{a} \) and \( \vec{b} \) having the same magnitude and such that the angle between them is 60° and their scalar product is \( \frac{1}{2} \).

Q. 4) Find the projection \( \vec{a} \) on \( \vec{b} \) if \( \vec{a} \cdot \vec{b} = 8 \) and \( \vec{b} = 2\hat{i} + 6\hat{j} + 3\hat{k} \).

Q. 5) Prove that \( |\vec{a} \times \vec{b}| = (\vec{a} \cdot \vec{b}) \tan \theta \) where \( \theta \) is the angle between the vector \( \vec{a} \) and \( \vec{b} \).

Q. 6) If \( \hat{i}, \hat{j}, \hat{k} \) are the usual three perpendicular unit vectors then find the value of
   \( \hat{i} \cdot (\hat{j} \times \hat{k}) + \hat{j} \cdot (\hat{i} \times \hat{k}) + \hat{k} \cdot (\hat{i} \times \hat{j}) \)

Q. 7) Find \( (\vec{a} - \vec{b}) \times (\vec{a} + \vec{b}) \)

Q. 8) Find the area of the parallelogram whose adjacent sides are
   \( \hat{i} - 2\hat{j} + 3\hat{k} \) and \( 2\hat{i} + \hat{j} - 4\hat{k} \)

Q. 9) If \( D \) and \( E \) are the mid points of sides \( AB \) and \( AC \) of triangle \( ABC \) respectively then show that \( \vec{BE} + \vec{DC} = \frac{3}{2} \vec{BC} \).

Q. 10) Find the values of \( x \) for which the angle between the vectors \( \vec{a} = 2x^2\hat{i} + 4x\hat{j} + \hat{k} \) and \( \vec{b} = 7\hat{i} - 2\hat{j} + x\hat{k} \) is obtuse?

Q. 11) If \( |\vec{a}| = 2, |\vec{b}| = 7 \) and \( \vec{a} \times \vec{b} = 3\hat{i} + 2\hat{j} + 6\hat{k} \). Find the angle between \( \vec{a} \) and \( \vec{b} \).
8. Vectors

Marks - 3

Q. 1) If \( \vec{a} = 3\hat{i} + 2\hat{j} + 9\hat{k} \) and \( \vec{b} = \hat{i} + \lambda\hat{j} + 3\hat{k} \) find the value of \( \lambda \) so that \((\vec{a} + \vec{b})\) is perpendicular to \((\vec{a} - \vec{b})\).

Q. 2) Let \( \vec{a} = x^2\hat{i} + 2\hat{j} - 2\hat{k}, \vec{b} = \hat{i} - \hat{j} + \hat{k} \) and \( \vec{c} = x^2\hat{i} + 5\hat{j} - 2\hat{k} \) be three vectors. Find the values of \( x \) for which the angle between \( \vec{a} \) and \( \vec{b} \) is acute and the angle between \( \vec{b} \) and \( \vec{c} \) is obtuse.

Q. 3) If \( \vec{a} \) and \( \vec{b} \) are unit vectors and \( \theta \) is the angle between them then show that

\[
\sin^{\frac{2}{3}} = \frac{1}{2} \left| \vec{a} \cdot \vec{b} \right|
\]

Q. 4) If \( \vec{a}, \vec{b}, \vec{c} \) are unit vectors such that \( \vec{a} + \vec{b} + \vec{c} = 0 \), find the value of \((\vec{a} \cdot \vec{b} + \vec{b} \cdot \vec{c} + \vec{c} \cdot \vec{a})\).

Q. 5) Given \( \vec{a} = \frac{1}{7}(2\hat{i} + 3\hat{j} + 6\hat{k}), \vec{b} = \frac{1}{7}(3\hat{i} - 6\hat{j} + 2\hat{k}), \vec{c} = \frac{1}{7}(6\hat{i} + 2\hat{j} - 3\hat{k}) \), \( \hat{i}, \hat{j}, \hat{k} \) being a right handed orthogonal system of unit vectors in space then show that \( \vec{a}, \vec{b}, \vec{c} \) is also another such system.

Q. 6) If \( P_1, P_2, P_3, P_4 \) are points in space or in a plane and \( O \) the origin of vectors then show that \( P_4 \) coincides with \( O \) iff \( \vec{OP}_1 + \vec{P}_1P_2 + \vec{P}_2P_3 + \vec{P}_3P_4 = 0 \).

Q. 7) If \( \vec{a} = \hat{i} + 2\hat{j} + 3\hat{k} \) and \( \vec{b} = 2\hat{i} + 4\hat{j} - 5\hat{k} \) represent two adjacent sides of a parallelogram. Find unit vectors parallel to the diagonals of the parallelogram.

Q. 8) Let \( \vec{a} = 4\hat{i} + 5\hat{j} - \hat{k}, \vec{b} = \hat{i} - 4\hat{j} + 5\hat{k} \) and \( \vec{c} = 3\hat{i} + \hat{j} - \hat{k} \) Find a vector \( \vec{T} \) which is perpendicular to both \( \vec{a} \) and \( \vec{b} \) and satisfying \( \vec{T} \cdot \vec{c} = 21 \).

Q. 9) Let \( \vec{OA} = \vec{a}, \vec{OB} = 10\vec{a} + 2\vec{b} \) and \( \vec{OC} = \vec{b} \) where \( O \) is origin. Let \( p \) denote the area of the quadrilateral OABC and \( q \) denote the area of the parallelogram with OA and OC as adjacent sides, Prove that \( p = 6q \).

Q. 10) Let \( \vec{a} = 2\hat{i} + \hat{k}, \vec{b} = \hat{i} + \hat{j} + \hat{k} \) and \( \vec{c} = 4\hat{i} - 3\hat{j} + 7\hat{k} \), find a vector \( \vec{T} \) which satisfies \( \vec{T} \times \vec{b} = \vec{c} \times \vec{b} \) and \( \vec{T} \cdot \vec{a} = 0 \).

Q. 11) A force of magnitude 3 units in the direction \( 2\hat{i} + 3\hat{j} + 6\hat{k} \) acts at \((1,1,1)\). Find its moment about the point \((-1, 2, 3)\).
8. Vectors

Q. 1) The scalar product of the with a unit vector \( \overrightarrow{a} = \hat{i} + \hat{j} + \hat{k} \) along the sum of the vectors 
\( \overrightarrow{b} = 2\hat{i} + 4\hat{j} - 5\hat{k} \) and \( \overrightarrow{c} = \lambda\hat{i} + 2\hat{j} + 3\hat{k} \) is equal to one. Find the value of \( \lambda \).

Q. 2) If \( \overrightarrow{a} = \hat{i} + \hat{j} + \hat{k} \) and \( \overrightarrow{b} = \hat{j} - \hat{k} \) then find a vector \( \overrightarrow{c} \) such that \( \overrightarrow{a} \times \overrightarrow{c} = \overrightarrow{b} \) and \( \overrightarrow{a} \cdot \overrightarrow{c} = 3 \).

Q. 3) If \( \overrightarrow{a}, \overrightarrow{b}, \overrightarrow{c} \) are three mutually perpendicular vectors of equal magnitude then prove that \( \overrightarrow{a} + \overrightarrow{b} + \overrightarrow{c} \) is equally inclined with vectors \( \overrightarrow{a}, \overrightarrow{b} \) and \( \overrightarrow{c} \).

Q. 4) Two adjacent sides of a parallelogram are \( 2\hat{i} - 4\hat{j} + 5\hat{k} \) and \( \hat{i} - 2\hat{j} - 3\hat{k} \). Find the unit vector parallel to its diagonal. Also find its area.

Q. 5) If \( \overrightarrow{c} = 3\overrightarrow{a} + 4\overrightarrow{b} \) and \( 2\overrightarrow{c} = \overrightarrow{a} - 3\overrightarrow{b} \) then show that
   i) \( \overrightarrow{c} \) and \( \overrightarrow{a} \) have the same direction and \( |\overrightarrow{c}| > |\overrightarrow{a}| \)
   ii) \( \overrightarrow{c} \) and \( \overrightarrow{b} \) have opposite direction and \( |\overrightarrow{c}| > |\overrightarrow{b}| \)

Q. 6) If \( \overrightarrow{a}, \overrightarrow{b}, \overrightarrow{c} \) be three vectors of magnitude 3, 4 and 5 respectively. If each one is perpendicular to the sum of the other two vectors then prove that \( \overrightarrow{a} + \overrightarrow{b} + \overrightarrow{c} = 5\sqrt{2} \)

Q. 7) If \( \overrightarrow{a}, \overrightarrow{b}, \overrightarrow{c} \) be unit vectors such that \( \overrightarrow{a} \cdot \overrightarrow{b} = \overrightarrow{a} \cdot \overrightarrow{c} = 0 \) and the angle between \( \overrightarrow{b} \) and \( \overrightarrow{c} \) is \( \frac{\pi}{6} \) then prove that \( \overrightarrow{a} = \pm 2 (\overrightarrow{b} \times \overrightarrow{c}) \).

Q. 8) If \( \overrightarrow{a}, \overrightarrow{b}, \overrightarrow{c} \) are three vectors such that \( \overrightarrow{a} + \overrightarrow{b} + \overrightarrow{c} = \overrightarrow{0} \) then prove that \( \overrightarrow{a} \times \overrightarrow{b} = \overrightarrow{b} \times \overrightarrow{c} = \overrightarrow{c} \times \overrightarrow{a} \)

Q. 9) Two forces of magnitudes 5 and 6 units acts on a particle along the direction of vectors 
\( 2\hat{i} + 9\hat{j} + 6\hat{k} \) and \( -9\hat{i} + 2\hat{j} + 6\hat{k} \) respectively. If the particle is displaced from (2, 3, 4) to (1, 3, 5). Find the total work done by the two forces.

Q. 10) If with reference to a right handed system of mutually perpendicular unit vectors 
\( \hat{i}, \hat{j}, \hat{k} \) \( \overrightarrow{a} = 3\hat{i} - \hat{j}, \overrightarrow{b} = 2\hat{i} + \hat{j} - 3\hat{k} \). Express \( \overrightarrow{b} \) in the form \( \overrightarrow{b} = \overrightarrow{a} + \overrightarrow{b} \) where \( \overrightarrow{b} \) is parallel to \( \overrightarrow{a} \) and \( \overrightarrow{b} \) is perpendicular to \( \overrightarrow{a} \).
9. Linear Inequation

Marks - 1

Q. 1) The solution set of the inequations $8x - 25 < 17 + 2x$ where $x \in \mathbb{R}$ is....
   a) $(- \infty, 7)$  b) $(7, \infty)$  c) $(- \infty, - 7)$  d) $(- 7, \infty)$

Q. 2) The solution set of the inequations $|x| < 1$ and $|x - 1| < 1$ where $x \in \mathbb{R}$ is....
   a) $(0, 1)$  b) $(0, 1]$  c) $(0, 1]$  d) $[0, 1)$

Q. 3) The solution set of the inequations $x \geq 5$ and $x < 1$ where $x \in \mathbb{R}$ is
   a) $(0, 5)$  b) $(0, 5]$  c) $(- \infty, \infty)$  d) $\phi$

Q. 4) If $k$ belongs to the solution set of the inequations $x + 3 > 2(x + 1)$ and $8 + x < 2(6 + x)$ then 'k' does not belong to....
   a) $(- 5, 1)$  b) $(- 4, 2]$  c) $(1, 2)$  d) $(- 3, 0)$

Q. 5) $x + 2 < 2(x + 3)$ and $8 + x > 2(x + 1)$ where $x \in \mathbb{R}$ then $x$ belongs to
   a) $(- 4, 6)$  b) $(- 5, - 4]$  c) $\phi$  d) $(6, \infty)$

Q. 6) Which of the following values belong to the solution set of the inequations $x + 3 > 2(x + 1)$ and $8 + x < 2(x + 5)$ where $x \in \mathbb{R}$.
   a) 1  b) -1  c) 2  d) -2

Q. 7) Which of the following point does not belong to the feasible solution of the inequations $x + 2y \leq 10; x + y \leq 6; x \leq 4; x \geq 0; y \geq 0$.
   a) $(1, 2)$  b) $(3, 3)$  c) $(5, 2)$  d) $(3, 1)$

Q. 8) If real numbers $h$ and $k$ belong to the solution set of the inequation $3x < 2$ then which of the following is possible.
   a) $h + k = 3/2$  b) $h + k = 2$  c) $h + k = 1$  d) $h + k = 5/3$

Q. 9) Which of the following shaded region represent the feasible solution of the inequations $x + 2 \leq 9; y \geq x; x \geq 0; y \geq 0$.
   a)  b)  c)  d)
9. Linear Inequation

Q. 1) Find non-negative integers satisfying the inequation $-3 \leq 3 - 2x \leq 9$.

Q. 2) Find the values of $x$ which do not belong to the solution set of the inequations $2x - 3 < x + 2$ and $4x + 7 \geq 3x + 5$ where $x \in \mathbb{R}$.

Q. 3) The marks obtained by a student of class XI in mathematics in two semesters are 62 and 48 respectively. Find the minimum marks he should get in annual examination to have average of at least 65 marks.

Q. 4) Find the real values of $x$ for which $3x^2 + 5x - 2 < 0$.

Q. 5) Solve the inequation $x - 3y < 0$ graphically.

Q. 6) Solve the inequation $\frac{1}{|x - 3|} < 1$ where $x \in \mathbb{R}$.

Q. 7) Solve the inequation $\frac{3}{|x - 2|} > 2$ where $x \in \mathbb{R}$.
9. Linear Inequation

Q. 1) Find real numbers 'x' such that

\[ 2x + 3 > x \text{ and } \frac{7 + x}{3} < 8 \text{ and } x^2 - 5x - 36 = 0. \]

Q. 2) Find \( x \in \mathbb{I} \) such that \( 2 > \frac{2 - x}{5} ; \frac{x}{4} > \frac{5x - 2}{3} - \frac{7x - 3}{5} \) and \( x^2 + x^2 - 6 \neq 0. \)

Q. 3) Find \( x \in (1, \infty) \) if \( \frac{x + 4}{x - 2} < 1. \)

Q. 4) Find \( x \in (-3, 2) \) such that \( \frac{x + 4}{x - 6} < 1. \)

Q. 5) Find all pairs of consecutive odd positive integers both of which are smaller than 10 such that their sum is more than 11.

Q. 6) The longest side of a triangle is three times the shortest side and the remaining side is 2 cm shorter than the longest side. If the perimeter of the triangle is at least 61 cm. Find the minimum length of the shortest side.

Q. 7) Vijay wants to invest Rs 30,000 for one year. Part of this will be invested in a stable 5% simple-interest rate account. The remainder will be invested in his father's business, and he says that he'll pay him back with 7% interest. His father knows that he is making these investments in order to pay his child's school tuition with the interest income. What is the least he can invest with his father, and still gets at least Rs 1900 in interest?
9. Linear Inequation

Q. 1) Solve the inequation \[ |x| + |x - 1| \leq 2; \ x \in \mathbb{R}. \]

**HINT:** Take Domain \( R = (-\infty, 0) \cup (0, 1) \cup (1, \infty) \)

Q. 2) Solve the inequation \[ |x - 1| < |x - 2|; \ x \in \mathbb{R}. \]

Q. 3) Find Solution of the inequation \[ |2x - 3| < |x + 2|; \ x \in \mathbb{R}. \]

Q. 4) Food 'A' contains 6 units of vitamins and 12 units of minerals per gram. Food 'B' contains 8 units of vitamins and 6 units of minerals per gram. Daily requirement of vitamins and minerals are 96 units and 120 units respectively. Construct inequations and find the feasible solution graphically.

Q. 5) A manufacturer has 600 liters of a 12% solution of acid. How many liters of a 30% acid solution must be added to it so that the acid content in the resulting mixture will be more than 15% but less than 18%.

Q. 6) An alloy needs to contain between 46% copper and 50% copper. Find the least and greatest amounts of a 60% copper alloy that should be mixed with a 40% copper alloy in order to end up with thirty kilograms of an alloy containing an allowable percentage of copper.

Q. 7) Find solution set of the inequation \[ |x + \frac{1}{x}| > 2; \ x \in \mathbb{R}. \]

Q. 8) Find solution set of the inequation \[ |x + \frac{1}{x}| < 4; \ x \in \mathbb{R}. \]
10. Determinants

Q. 1) If the determinant
\[
\begin{array}{ccc}
\cos 2x & \sin^2 x & \cos 4x \\
\sin^2 x & \cos 2x & \cos^2 x \\
\cos 4x & \cos^2 x & \cos 2x \\
\end{array}
\]
is expanded powers of \(\sin x\), then the constant term in the expansion is....

a) \(-2\)  

b) 2  

c) \(-1\)  

d) 1

Q. 2) If
\[
\begin{array}{ccc}
x & 2 & x \\
x^2 & x & 6 \\
x & x & 6 \\
\end{array}
\]
= \(\alpha x^4 + \beta x^3 + \gamma x^2 + \delta x + \lambda\) then \(\lambda = \ldots\)

a) \(-11\)  

b) 0  

c) \(-16\)  

d) 16

Q. 3) The value of the determinant
\[
\begin{array}{ccc}
\log_a(x/y) & \log_a(y/z) & \log_a(z/x) \\
\log_a(y/z) & \log_a(z/x) & \log_a(x/y) \\
\log_a(z/x) & \log_a(x/y) & \log_a(y/z) \\
\end{array}
\]
is...

a) 1  

b) \(-1\)  

c) \(\log_a x y z\)  

d) 0

Q. 4) Value of the determinant
\[
\begin{array}{ccc}
a & b & c \\
b & c & a \\
c & a & b \\
\end{array}
\]
is....

a) \(-\frac{1}{2}(a+b+c)((a-b)^2 + (b-c)^2 + (c-a)^2)\)  

b) \(\frac{1}{2}(a+b+c)((a-b)^2 + (b-c)^2 + (c-a)^2)\)  

b) \(-\frac{1}{2}(a+b+c)((a+b)^2 + (b+c)^2 + (c+a)^2)\)  

d) \(\frac{1}{2}(a+b+c)((a+b)^2 + (b+c)^2 + (c+a)^2)\)

Q. 5) If \(x, y, z\) are positive numbers then value of
\[
\begin{array}{ccc}
1 & \log_x y & \log_x z \\
\log_y x & 3 & \log_y z \\
\log_z x & \log_z y & 5 \\
\end{array}
\]
is...

a) 16  

b) 1  

c) 8  

d) \(\log x \cdot \log y \cdot \log z\)

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Q. 6) The value of
\[
\begin{vmatrix}
(-1)^n a & (-1)^{n+1} b & (-1)^{n+1} c \\
a + 1 & 1 - b & 1 + c \\
a - 1 & b + 1 & 1 - c
\end{vmatrix}
\]
+ \[
\begin{vmatrix}
(-1)^{n+1} a & a + 1 & a - 1 \\
(-1)^n b & 1 - b & b + 1 \\
(-1)^{n+2} c & 1 + c & 1 - c
\end{vmatrix}
\]
is...

a) 1  

b) - 1  

c) 0  

d) 2

Q. 7) If a, b, c > 0 and x, y, z \in \mathbb{R} then value of the determinant.

\[
\begin{vmatrix}
(a^x + a^{-x})^2 & (a^x - a^{-x})^2 & 1 \\
(b^y + b^{-y})^2 & (b^y - b^{-y})^2 & 1 \\
(c^z + c^{-z})^2 & (c^z - c^{-z})^2 & 1
\end{vmatrix}
\]
is...

a) 0  

b) a^2  

c) - a^2  

d) 2

Q. 8) Value of the determinant

\[
\begin{vmatrix}
a & b & c \\
b & c & a \\
c & a & b
\end{vmatrix}
\]
is

a) a + b + c  

b) abc

b) abc

d) (a^3 + b^3 + c^3) - 3abc  

d) 3abc - (a^3 + b^3 + c^3)
10. Determinants

Q. 1) If \(x, y, z\) are integers between 0 and 9 which are in A.P. then find the value of the determinant

\[
\begin{vmatrix}
5 & 4 & 3 \\
5^1 & 4^1 & Z^1_x \\
x & y & z
\end{vmatrix}
\]

Q. 2) If \(f(x) = \begin{vmatrix}
1 & x & x+1 \\
2x & x(x-1) & x(x+1) \\
3x(x-1) & x(x-1)(x-2) & x(x^2-1)
\end{vmatrix}\) then find the value of \(f(200)\).

Q. 3) If A, B, C are angles of triangle then prove that

\[
\begin{vmatrix}
\cos A & \cos B & \cos C \\
\cos A + \cos B & \cos A + \cos C & \cos A + \cos B + \cos C \\
-1 + \cos B & \cos C + \cos B & \cos B
\end{vmatrix} = 0.
\]

Q. 4) If \(3^n\) is a factor of the determinant \(nC_1 (n+3)C_1 (n+6)C_1 nC_2 (n+3)C_2 (n+6)C_2\) then show that the maximum value of 'n' is 3.

Q. 5) If\( A + B + C = \pi \) then find value of the determinant

\[
\begin{vmatrix}
\sin(A+B+C) & \sin B & \cos C \\
-\sin B & 0 & \tan A \\
\cos(A+B) & -\tan A & \cos \left(\frac{A+B+C}{2}\right)
\end{vmatrix}
\]

Q. 6) If \(f(x) = \begin{vmatrix}
a & -1 & 0 \\
ax & a & -1 \\
ax^2 & ax & a
\end{vmatrix}\) and \(f(2x) - f(x) = 0\) then prove that \(x = 0\) or \(x = -2a/3\).

Q. 7) If \(\begin{vmatrix}
x^2 + x & x + 1 & x - 2 \\
2x^2 + 3x - 1 & 3x & 3x - 3 \\
x^2 + 2x + 3 & 2x - 1 & 2x - 1
\end{vmatrix} = Ax - 12\) then find the value of A.
10. Determinants

Marks - 3

Q. 1) If a, b, c are positive and not all equal then show that \[ \begin{vmatrix} a & b & c \\ b & c & a \\ c & a & b \end{vmatrix} \] is negative.

Q. 2) If n is a fixed positive integer and \[ \Delta = \frac{n! (n+1)! (n+2)!}{(n+1)! (n+2)! (n+3)!} \] then prove that \[ \frac{\Delta}{(n!)^3 - 4} \] is divisible by n.

Q. 3) If a, b, c are sides of triangle and \[ \begin{vmatrix} a^2 & b^2 & c^2 \\ (a+1)^2 & (b+1)^2 & (c+1)^2 \\ (a-1)^2 & (b-1)^2 & (c-1)^2 \end{vmatrix} = 0 \] then prove that \( \Delta ABC \) is an Isosceles triangle.

Q. 4) If \( D_k = \begin{vmatrix} 1 & n & n \\ 2k & n^2 + n + 1 & n^2 + n \\ 2k-1 & n^2 & n^2 + n + 1 \end{vmatrix} \) and \( \sum_{k=1}^{n} D_k = 56 \) then find the value of n.

Q. 5) The digits A, B, C are such that three digit numbers A88, 6B8, 86C are divisible by 72 then show that determinant \[ \begin{vmatrix} A & A & 8 \\ 8 & 8 & 6 \\ 8 & 8 & C \end{vmatrix} \] is divisible by 72.

Q. 6) If \[ \begin{vmatrix} a & 1 & 1 \\ 1 & b & 1 \\ 1 & 1 & c \end{vmatrix} = 0 \] then show that \( \frac{1}{1-a} + \frac{1}{1-b} + \frac{1}{1-c} = 1 \).

Q. 7) Prove that determinant \[ \begin{vmatrix} a^2 + x^2 & ab & ac \\ ab & b^2 + x^2 & bc \\ ac & bc & c^2 + x^2 \end{vmatrix} \] is divisible by \( x^4 \).
Q. 8) If $a_1, a_2, a_3, a_4 \ldots \ldots \ldots a_n$ are in G.P. then find the value of

\[
\frac{a_n}{a_{n+1}} \quad \frac{a_{n+1}}{a_{n+2}} \quad \frac{a_{n+2}}{a_{n+3}} \quad \frac{a_{n+3}}{a_{n+4}} \quad \frac{a_{n+4}}{a_{n+5}} \quad \frac{a_{n+5}}{a_{n+6}} \quad \frac{a_{n+6}}{a_{n+7}} \quad \frac{a_{n+7}}{a_{n+8}}
\]

Q. 9) Show that

\[
\begin{vmatrix}
2bc - a^2 & c^2 & b^2 \\
c^2 & 2ac - b^3 & a^2 \\
b^2 & a^2 & 2ab - c^2
\end{vmatrix} = (a^3 + b^3 + c^3 - 3abc)^2.
\]
10. Determinants

Q. 1) If \( \alpha \neq 0 \) and
\[
\begin{vmatrix}
 a & b & a\alpha + b \\
 b & c & b\alpha + c \\
a\alpha + b & b\alpha + c & 0
\end{vmatrix} = 0
\]
then prove that a, b, c are in G.P. or '\( \alpha \)' is root of the equation \( ax^2 + 2bx + c = 0 \).

Q. 2) Prove that
\[
\begin{vmatrix}
(n + 2)\binom{n}{n} & (n + 3)\binom{n + 1}{n} & (n + 4)\binom{n + 2}{n+1} \\
(n + 3)\binom{n+1}{n} & (n + 4)\binom{n + 2}{n+1} & (n + 5)\binom{n + 3}{n+2} \\
(n + 4)\binom{n+2}{n+1} & (n + 5)\binom{n + 3}{n+2} & (n + 6)\binom{n + 4}{n+3}
\end{vmatrix} = -1
\]
HINT : \( n\epsilon_n = n\epsilon_{n-r} \).

Q. 3) If
\[
\begin{vmatrix}
 x^k & x^{k+2} & x^{k+3} \\
 y^k & y^{k+2} & y^{k+3} \\
z^k & z^{k+2} & z^{k+3}
\end{vmatrix} = (x-y)(y-z)(z-x)\left(\frac{1}{x} + \frac{1}{y} + \frac{1}{z}\right)
\]
then find 'k'.

Q. 4) a, b, c, d are in A.P. and
\[
\begin{vmatrix}
 x+a & x+b & x+a-c \\
x+b & x+c & x-1 \\
x+c & x+d & x-b+d
\end{vmatrix} = -16
\]
then prove that the common difference of the A.P. is \( \pm 2 \).

Q. 5) If \( f(n) = \alpha^n + \beta^n \) and
\[
\begin{vmatrix}
 3 & 1 + f(1) & 1 + f(2) \\
1 + f(1) & 1 + f(2) & 1 + f(3) \\
1 + f(2) & 1 + f(3) & 1 + f(4)
\end{vmatrix} = k(1 - \alpha^2)(1 - \beta^2)(\alpha - \beta)^2
\]
then find the value of 'k'.

Q. 6) If
\[
\begin{vmatrix}
 (b+c)^2 & a^2 & a^2 \\
b^2 & (c+a)^2 & ba^2 \\
c^2 & c^2 & (a+b)^2
\end{vmatrix} = \lambda \ ab\ (a+b+c)^3
\]
then find \( \sum_{r=1}^{10} \lambda^r \).

Q. 7) If
\[
\begin{vmatrix}
 f(x-2) & f(x+5) & f(x-2) \\
5 & 4 & -5 \\
10 & 12 & 30
\end{vmatrix}
\]
is a singular matrix \( \forall x \in \mathbb{R} \), then prove that \( f(x) \) is a periodic function. Also find its period.

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11. Matrice

Q. 1) \( A = \begin{bmatrix} 1 & -1 \\ 2 & -1 \end{bmatrix} \) then which of the following is true.
   a) \( A^2 = I \)  
   b) \( A^2 = -I \)  
   c) \( A^2 = 2I \)  
   d) \( A^2 = -2I \)

Q. 2) \( A = \begin{bmatrix} x & 1 \\ 1 & -x \end{bmatrix} \) and \( A^2 \) is identity matrix then value of \( x \) is.....
   a) 1  
   b) -1  
   c) 0  
   d) -2

Q. 3) \( A = \begin{bmatrix} 4 & x + 2 \\ 2x - 3 & x + 1 \end{bmatrix} \) is symmetric matrix then value of \( x \) is
   a) 3  
   b) 5  
   c) 2  
   d) 4

Q. 4) \( A = \begin{bmatrix} i & 0 \\ 0 & i \end{bmatrix} \) & \( B = \begin{bmatrix} 0 & -i \\ -i & 0 \end{bmatrix} \) then \( (A + B)(A - B) = .... \)
   a) \( A^2 - B^2 \)  
   b) \( A^2 + B^2 \)  
   c) \( A^2 - B^2 + BA + AB \)  
   d) \( A^2 + B^2 + BA + AB \)

Q. 5) If \( A \) is \( 3 \times 3 \) matrix and \( \det(3A) = k \det(2A) \) then value of \( k \) is ......
   a) 1  
   b) 27/8  
   c) 27  
   d) 8/27

Q. 6) If \( A = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ a & b & -1 \end{bmatrix} \) then \( A^2 = \)
   a) \( A \)  
   b) \(-A\)  
   c) Identity Matrix  
   d) Null Matrix

Q. 7) If \( A \) and \( B \) are square matrices of order 3 such that \( \det(A) = -1 \) and \( \det(B) = 3 \) then find the value of \( \det(3AB) \).
   a) 81  
   b) -81  
   c) 9  
   d) -9
11. Matrice

Marks - 2

Q. 1) If A, B are symmetric matrices of order n then prove that AB - BA is skew symmetric matrix.

Q. 2) If A is a square matrix of order n and AA\(^T\) = I then find |A|.

Q. 3) If A, B are square matrices of same order such that A\(^2\) = A, B\(^2\) = B & AB = BA = 0 then prove that (A + B)\(^2\) = A + B.

Q. 4) If A is a square matrix of order 3, |A| = 6, B = 5A\(^2\) then find |B|.

Q. 5) A = [a\(_{ij}\)] is a 2 x 2 matrix where a\(_{ij}\) = i\(^2\) - j\(^2\). Prove that A is a skew symmetric matrix.
11. Matrice

Marks - 3

Q. 1) \( A = \begin{bmatrix} 4 & 2 & -3 \\ 1 & 3 & -6 \\ -5 & 0 & -7 \end{bmatrix} \) express A as the sum of symmetric and skew-symmetric matrix.

Q. 2) \( A = \begin{bmatrix} \alpha & 0 \\ 1 & 1 \end{bmatrix} ; B = \begin{bmatrix} 1 & 0 \\ 2 & 1 \end{bmatrix} \) find \( \alpha \) if \( A^2 = B \).

Q. 3) \( b, c \) are real numbers such that \( bc \neq 0 \) and the matrix \( A = \begin{bmatrix} a & b \\ c & d \end{bmatrix} \) satisfies the equation \( x^2 + p = 0 \) then prove that \( a + d = 0 \) and \( |A| = p \).

Q. 4) \( A = \begin{bmatrix} 1 & 3 & \lambda + 2 \\ 2 & 4 & 8 \\ 3 & 5 & 10 \end{bmatrix} , B = \begin{bmatrix} 3 & 2 & 4 \\ 3 & 2 & 5 \\ 2 & 1 & 4 \end{bmatrix} \) find \( \lambda \) if \( 2A + 3B \) is singular.

Q. 5) \( M = \begin{bmatrix} 1 & 2 \\ 2 & 3 \end{bmatrix} \) and \( M^2 - \lambda M - I_2 = 0 \). Find the value of \( \lambda \).

Q. 6) \( a, b, c \) are real numbers different from 0 such that \( \frac{1}{a} + \frac{1}{b} + \frac{1}{c} = 0 \). Prove that
\[ \begin{bmatrix} 1+a & 1 \\ 1 & 1+b \\ 1 & 1+c \end{bmatrix} \] is a non-singular matrix.

Q. 7) If \( \begin{bmatrix} 3 & -1+x & 2 \\ 3 & -1 & x+2 \\ x+3 & -1 & 2 \end{bmatrix} \) is a singular matrix then find the value of \( x \) in \([-5, -1]\)
Q. 1) \[ A = \begin{bmatrix} 0 & 2\beta & \gamma \\ \alpha & \beta & -\gamma \\ \alpha & \beta & \gamma \end{bmatrix} \] find values of \( \alpha, \beta, \gamma \) if \( AA^T = I \).

Q. 2) \( a_r = r(7c_r), b_r = (7 - r)(7c_r) \) \[ A_r = \begin{bmatrix} a_r & 0 \\ 0 & b_r \end{bmatrix} \] and \( A = \sum_{r=1}^{7} A_r = \begin{bmatrix} a & 0 \\ 0 & b \end{bmatrix} \). Find \( a + b \).

Q. 3) If \( a, b, c \) are sides of \( \triangle ABC \) and \[ \begin{bmatrix} 1 & a & b \\ 1 & c & a \\ 1 & b & c \end{bmatrix} = 0 \] then find the value of \[ 64\{\sin^2 A + \sin^2 B + \sin^2 C\} \].

Q. 4) \( a, b, c \in \mathbb{R} \) such that \( abc = 2 \) & \( A = \begin{bmatrix} 2a & b & c \\ b & 2c & a \\ c & a & 2b \end{bmatrix} \) and \( AA^T = 2I \).

Find the value of \( (a^3 + b^3 + c^3)^4 \).

Q. 5) If matrix \( A \) satisfies the equation \( A^2 - 5A + 7I = 0 \) and if \( A^5 = aA + bI \) then find the value of \( 2a - 3b \).

Q. 6) \( A = \begin{bmatrix} a & b & c \\ b & c & a \\ c & a & b \end{bmatrix} \) such that \( AA^T = I \). Find the value of \( a^3 + b^3 + c^3 \) if \( abc = 1 \).

Q. 7) \( A = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix} \) is the unit matrix of order 2 and \( a, b \) are arbitrary constants then prove that \((aI + bA)^2\) is equal to \(a^2I + 2abA\).
Q. 1) Choose the correct answer out of the four options given below each question:

1) A set which does not contain any element is called ............
   a) Empty set    b) subset    c) Non empty set    d) Infinite set

2) Two sets A & B are said to be disjoint if .......
   a) A \cup B = \phi    b) A \cap B = \phi    c) A = B    d) A \subseteq B

3) A set which contains only one element is called .............
   a) Infinite set    b) Singleton set    c) Subset    d) Empty set

4) If A is any set, A \cup U = ............
   a) A    b) U    c) \phi    D) Non of the above

5) If A & B are any two sets and A \subseteq B, then A \cup B = ..............
   a) A    b) B    c) \phi    d) U

6) A function defined by \( f(x) = k \), for all values of x, where k is a constant, is called ..............
   a) One One function    b) Constant function    c) Even function    d) Odd function

7) If A & B are any two sets and A \subseteq B, then A \cap B = ............
   a) A    b) B    c) \phi    d) U

8) If \( U \) is the universal set and A, B are subsets of U them A \cup A' = .............
   a) U    b) \phi    c) A    d) A' 

9) If A = \{ 1, 2, 3, 4, 5 \}, B = \{ 3, 4, 5, 6, 7, 8 \}, then the number of elements in the power set of B - A is ............
   a) 3    b) 4    c) 8    d) 6

10) If A = \{ 3x / x \in N \} & B = \{ 6x / x \in N \} Then A \cap B is .............
    a) \{ 3x / x \in N \}    b) \{ 6x / x \in N \}    c) \{ x / x \in N \}    d) \{ 2x / x \in N \}
12. Sets, Relations And Functions

Marks - 2

1) If \( A = \{1, 2, 3, 4\} \) and \( B = \{3, 4, 5, 6\} \) and Universal set \( x = \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10\} \), find \( A \cap B \).

2) If \( A = \{1, 2, 3, 4\} \), write down the power set of \( A \).

3) If \( A = \{2x \mid x \in N\} \) and \( B = \{2x + 1 \mid x \in N\} \) are subsets of the universal set \( X = N \). Find \( A \cup B \).

4) If \( A = \{a, b, c\} \) and \( B = \{X, y\} \). Find \( A \times B \) and \( B \times A \).

5) If \( f(x) = 2x^2 - 3x - 1 \), find \( f(x + 2) \).

6) Find 'a' if \( f(x) = ax + 5 \) and \( f(1) = 8 \).

7) If \( f(x) = (x - 1)(2x + 1) \) then find \( f(1) \) and \( f(-3) \).

8) Let \( A = \{1, 2, 3, 4\} \), \( B = \{4, 5, 6\} \), \( C = \{5, 6\} \). Find \( A \times (B \cap C) \).

9) If \( (x - 1, y + 4) = (1, 2) \). Find the values of \( x \) and \( y \).

10) Let \( R = \{(a, b) \mid b = a + 1, a \in I, 0 < a < 5\} \). Find the range of \( R \).
12. Sets, Relations And Functions

Marks - 3

1) If \( A = \{1, 2, 3, 4\} \) & \( B = \{3, 4, 5, 6\} \). Final \( n(A \cup B) \) & \( n(A \cap B) \).

2) If \( X = \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10\} \) is the universal set & \( A = \{1, 2, 3, 4\}, B = \{2, 4, 6, 8\}, \)
\( C = \{3, 4, 5, 6\} \) verify the following.
   a) \( A \cup (B \cup C) = (A \cup B) \cup C \)
   b) \( A \cap (B \cup C) = (A \cap B) \cup (A \cap C) \)
   c) \((A')' = A\)

3) Let \( A = \{-1, 2, 3, 4\}, B = \{4, 5, 6\}, C = \{5, 6\} \) Find \( A \times (B \cup C) \)

4) Find the range of the following function.
   \( f(x) = 3x - 4 \) for \( 1 \leq x \leq 3 \).

5) Let \( f \) and \( g \) be two real valued functions defined by \( f(x) = x + 1 \) and \( g(x) = 2x - 3 \). Finds \( f + g, f - g, f \cdot g \).

6) Let \( f \) and \( g \) be two real valued functions defined by \( f(x) = 2x + 1 \) and \( g(x) = x - 3 \). Find \( f \circ g \).

7) If \( f(x) = 3x + a \) and \( f(1) = 7 \) Find \( 'a' \) \( f(4) \).

8) If \( f(x) = ax^2 + bx + 2 \) and \( f(1) = 3, f(4) = 42 \), Find \( 'a' \) and \( 'b' \).

9) Find \( f \circ g \) if \( f(x) = x^2 + 5, g(x) = x - 8 \).

10) Find \( g \circ f \) if \( f(x) = 3x - 2, g(x) = x^2 \).
12. Sets, Relations And Functions

Marks - 4

1) If A and B are the subsets of the universal set x and n (x) = 50, n (A) = 35, n (B) = 20, n \((A' \cap B')\) = 5. Find (i) n \((A \cup B)\) (ii) n \((A \cap B)\) (iii) n \((A' \cap B)\) (iv) n \((A \cap B')\)

2) From amongst 2000 literate individuals of a town, 70% read Marathi newspapers, 50% read English newspapers and 32.5% read both Marathi & English newspapers. Find the numbers of individuals who read.
   (i) at least one of newspapers.
   (ii) neither marathi nor english newspaper.
   (iii) Only one of the newspapers.

3) Find the range of the function \(f(x) = 9 - 2x^2\) for \(-5 \leq x \leq 3\).

4) Examine whether the following function is one one, onto or not:
   \(f : R \to R\) given by \(f(x) = x^3 + 5\) for all \(x \in R\).

5) Find \(fog\) & \(gof\), where \(f(x) = \frac{1}{x}\), \(g(x) = \frac{x-2}{x+2}\).

6) If \(f(x) = \frac{3x+2}{4x-1}\), \(g(x) = \frac{x+2}{4x-3}\), Prove that \((fog)(x) = (gof)(x) = x\).

7) If \(f(x) = \frac{2x+3}{3x-2}\), prove that \(fog\) is identity function.

8) If \(f = \{(2, 4), (3, 6) (4, 8), (5, 10), (6, 12)\}\) and \(g = \{(4, 13), (6, 19), (8, 25), (10, 31), (12, 37)\}\). Find \(gof\).

9) show that \(f : R \to R\) given by \(f(x) = 3x - 4\) is one-one and onto Find inverse function.

10) Functions \(f : R \to R\) and \(g : R \to R\) are given by \(f(x) = 2x + 3\) and \(g(x) = 3x^2 - 4x + 5\).
   Find (i) \(fog\) (ii) \(gog\)
13. LOGARITHM

Q. 1) The Value of $\log_{25}125$ is ...
   a) 2           b) 3           c) \( \frac{1}{2} \)           d) \( \frac{3}{2} \)

Q. 2) If $\log_{4}2 = \log_{3}x$ Then $x = ......$
   a) 4           b) 6           c) 9           d) 16

Q. 3) If $\log_{6}(6) = \log_{x}(x) + \log_{2} + \log_{3}$ then $x = .....$
   a) 0           b) 1           c) 2           d) 3

Q. 4) Which of the following is positive?
   a) $\log_{2}\left(\frac{1}{2}\right)$           b) $\log_{4}\left(\frac{1}{2}\right)$           c) $\log_{\frac{1}{3}}2$           d) $\log_{4}2$

Q. 5) If positive numbers $a$ and $b$ are reciprocals of each other then ....
   a) $\log a = \log a b$           b) $\log a + \log a b = 1$
   c) $\log a + \log a b = 0$           d) $\log a x \log a b = 0$

Q. 6) If $\log\left(\frac{2}{3}\right) = \log 2 + \log 3 - k$ then $k = ....$
   a) $\log 9$           b) 0           c) 3           d) $\log 6$

Q. 7) If $\log_{5}5 = x$ and $\log_{5}5 = y$ then $\log_{6}5 =.........$
   a) $x + y$           b) $xy$           c) $\frac{x + y}{xy}$           d) $\frac{xy}{x + y}$

Q. 8) If $x, y$ are positive real numbers such that $2 \log (x - 2y) = \log x + \log y$ then $\frac{x}{y} = ....$
   a) 1           b) 2           c) 3           d) 4
13. LOGARITHM

Q. 1) Show that \( \log 360 = 3 \log 2 + 2 \log 3 + \log 5 \)

Q. 2) If \( \log_2 4 = x \) and \( \log_9 3 = y \) then find the value of \( xy \)
   
   a) 4  
   b) 6  
   c) 9  
   d) 16

Q. 3) Simplify \( \frac{\log 4^7}{\log 4^5} - \frac{\log 9^5}{\log 9^7} \)

Q. 4) If \( \log x = 4 \) then find the value of \( \log 4^x \) ....

Q. 5) Prove that \( \log 540 = 2 \log 2 + 3 \log 3 + \log 5 \)

Q. 6) Find the value of \( \log \sqrt[6]{2^4} \)

Q. 7) Arrange the following in ascending order: \( \log e, \log \pi, \log 10, \log 1 \)

Q. 8) Simplify \( 3.1342 + 2.3214 - 1.2341 \)

Q. 9) Find \( x \) if \( 3.1342 - 2.2311 = x \)

Q. 10) Using logarithmic table evaluate \( \sqrt[23]{2^3} \)
13. LOGARITHM

Q. 1) Show that \( \log (\sqrt{x^2+1}+x) + \log (\sqrt{x^2+1} - x) = 0 \)

Q. 2) Simplify \( \log (\log x^4) - \log (\log x) \)

Q. 3) Show that \( \log y^3 \cdot \log x^4 \cdot \log z^5 = 60 \)

Q. 4) Using logarithmic table evaluate \( \frac{28.45 \times 0.3254}{32.43 \times 0.3046} \)

Q. 5) If \( a^2 + a^2 = 7ab \) then show that \( \log \left( \frac{a + b}{3} \right) = \frac{1}{2} \log a + \frac{1}{2} \log b \)

Q. 6) Find \( x \) if \( \log_8 x + \log_4 x + \log_2 x = 22 \)

Q. 7) Show that \( \log_6 7 = \frac{\log_2 7}{1 + \log_2 3} \)

Q. 8) Prove that \( \frac{1}{\log_{24} 24} + \frac{1}{\log_{12} 24} + \frac{1}{\log_8 24} = 2 \)

Q. 9) If \( \log_2 x + \log_4 x + \log_{16} x = \frac{21}{4} \) then find the value of \( x \)
Q. 1) Given $\pi = 3.142$, $r = 2.307$, $h = 8.5$ find the value of $\pi r^2h$ using logarithm table.

Q. 2) If \( \frac{\log a}{4} = \frac{\log b}{6} = \frac{\log c}{3K} \) then find the value of $K$.

Q. 3) Using logarithm tables find the number of digits in $2^{64}$.

Q. 4) Using definition of logarithm prove that $\log_b a = \frac{\log a}{\log b}$.

Q. 5) If \( x = \frac{e^y - e^{-y}}{e^y + e^{-y}} \) then show that $y = \frac{1}{2} \log \left( \frac{1 + x}{2 + x} \right)$.

Q. 6) If $x = 1 + \log_a bc$, $y = 1 + \log_b ca$, $z = 1 + \log_c ab$

then prove that $xy + yz + zx = xyz$.

Q. 7) If $a^x = b^y = c^z$ and $b^2 = ac$ then prove that $y$ is the harmonic mean of $x$ and $z$.

Q. 8) Show that $\log 2^3$ is not rational.
14. Complex Number

Marks - 1

1) If \( z = 5 \) then \( z = \)
   a) \( 5 \)  
   b) \( -5 \)  
   c) \( \pm 5 \)  
   d) 0

2) \( z \bar{z} = \)
   a) Purely Imaginary Number  
   b) Purely real Number  
   c) 2 \( z \)  
   d) 0

3) If \( x = \frac{-1+i\sqrt{3}}{2}, \ y = \frac{-1-i\sqrt{3}}{2} \) then \( x^2. \)
   a) \( x \)  
   b) \( y \)  
   c) \( xy \)  
   d) \( y^2 \)

4) If \( \omega \) is ille complex cube root of unity then the value of \( (1 - \omega + \omega^2)^5(1 + \omega - \omega^2)^5 \)
   a) 32  
   b) \(-32\)  
   c) 0  
   d) 22

5) Out of these complex number \( z_1 = 2 - 3i, \ z_2 = \frac{1}{2} + 2i \) and \( z_3 = -1 + 4i \), in a complex
   plane which is nearest to origin.
   a) \( z_1 \)  
   b) \( z_2 \)  
   c) \( z_3 \)  
   d) \( z_1 \) & \( z_3 \)

6) If \( z \) be a complex number then the angle between \( z \) and \( iz \) is.
   a) \( \pi \)  
   b) 0  
   c) \(-\frac{\pi}{2}\)  
   d) \( \frac{\pi}{2} \)

7) The value of \( (1 + i)^4 (1 + 1/i)^4 \) is
   a) 12  
   b) 2  
   c) 8  
   d) 16

8) If \( \bar{z} \) is the conjugate of the complex number \( z \) then which of the following relation is false.
   a) \( |z| = |\bar{z}| \)  
   b) \( z \cdot \bar{z} = |z|^2 \)  
   c) \( \frac{z_1 + z_2}{z_1 + z_2} = \frac{\bar{z}_1 + \bar{z}_2}{\bar{z}_1 + \bar{z}_2} \)  
   d) \( \arg(z) = \arg(\bar{z}) \)

9) The conjugate of a complex number is \( \frac{1}{i-1} \) then that complex number is
   a) \( \frac{1}{i-1} \)  
   b) \( \frac{-1}{i-1} \)  
   c) \( \frac{1}{i+1} \)  
   d) \( \frac{-1}{i+1} \)

10) The Value of \( (1 + i)^5(1 - i)^5 \) is
    a) -8  
    b) 8i  
    c) 8  
    d) 32

(60)
14. Complex Number

Marks - 2

1) If \((1 + 2i)(1 + 3i)(2 + i)^{-1} = a + bi\) then find the value of \(a + b\).

2) For \(z = -2 - 3i\) Verity that \(\overline{z} = |z|^2\)

3) Show that \((-1 + \sqrt{3}i)^3\) is a real number.

4) Find the least positive integer \(n\) for which \(\left(\frac{1+i}{1-i}\right)^n\) is real number.

5) If \(\alpha\) is a cube root of unity and is not real then find the value of \(\alpha^{3n+1} + \alpha^{3n+3} + \alpha^{3n+5}\)

6) If \(z\) is a complex number then find \(\arg(\overline{z})\).

7) Find the real part of \(\frac{(1+i)^2}{3-i}\).

8) Find the value of \(|z - 5|\) if \(z = x + yi\)

9) If \(\arg(z) = \theta\) then find \(\arg(\overline{z})\).

10) The complex number \(\frac{1+2i}{1-i}\) lies in which quadrant.
14. Complex Number

Marks - 3

1) For what value of $\theta$, \[ \frac{3 + 2i \sin \theta}{1 - 2i \sin \theta} \] is purely real?

2) If $\alpha$ and $\beta$ are complex cube roots of unity then find the value of $\alpha^2 + \beta^2 + \alpha \beta$.

3) If $\sqrt{8i} = a + bi$ then find the values of $a + b$.

4) Represent the complex number $z = -2 - 2i$ in the Polar form.

5) If $z(2 - 2\sqrt{3}i)^2 = i(\sqrt{3} + i)^4$ then find the amplitude of $Z$.

6) Show that \[ \left| \frac{1}{2} (z_1 + z_2) + \sqrt{z_1 z_2} \right| + \left| \frac{1}{2} (z_1 + z_2) - \sqrt{z_1 z_2} \right| = |z_1| + |z_2| \].

7) If $z = \frac{1 - \sqrt{3}i}{1 + \sqrt{3}i}$ then find arg $(z)$.

8) If $x + yi = \frac{a + bi}{c + di}$ then find $(x^2 + y^2)^2$.

9) If $(1 - i)^n = 2^n$ then find the value of $n$.

10) If \[ \begin{vmatrix} 6i & -3i & 1 \\ 4 & 3i & -1 \\ 20 & 3 & i \end{vmatrix} = x + yi \] then find the value of $(x, y)$.
14. Complex Number

Marks - 4

1) The imaginary part of $\frac{2z+1}{iz+1}$ is -2 then find the locus of the points representing $z$ in the complex plane.

2) If $w$ is a complex cube root of unity then find the value of

$$\begin{vmatrix} x+1 & w & w^2 \\ w & x+w^2 & 1 \\ w^2 & 1 & x+w \end{vmatrix}$$

3) If $x = -5 + \sqrt{-4}$ then find the value of $x^4 + 9x^3 + 35x^2 - x - 4$.

4) Convert the complex number $z = \frac{i - 1}{\cos \frac{\pi}{3} + i \sin \frac{\pi}{3}}$ in the polar form.

5) Solve the quadratic equation $x^2 - (7 - i)x + (18 - i) = 0$.

6) If $z_1 = 2 + 3i$, $z_2 = 1 - 2i$, Verity that

i) $z_1 \cdot z_2 = z_1 \cdot \overline{z_2}$

ii) $\left( \frac{z_1}{z_2} \right) = \frac{\overline{z_1}}{\overline{z_2}}$

7) Final the square root of $2 + 2\sqrt{3}i$.

8) Find the number of solution of the equation $z^2 + z = 0$.

9) If $a = \cos \theta + i \sin \theta$ then find the value of $\frac{1+a}{1-a}$.

10) If $i = \sqrt{-1}$ then find the value of

$$4 + 5 \left( \frac{-1}{2} + \frac{i\sqrt{3}}{2} \right)^3 + 3 \left( \frac{-1}{2} + \frac{i\sqrt{3}}{2} \right)^3.$$
15. Arithmetic Progression & Geometric Progression

Marks - 1

Q.1) Select and write the correct answer from the given alternatives in each of the following questions.

1) Determine 25th term of an A.P. whose 9th term is - 6 and common difference $\frac{5}{4}$.
   a) 16  b) 18  c) 12  d) 14

2) The third term of a G.P. is 1. The sum of third and fifth term is 10. Find the common ratio of the G.P.
   a) 2, - 3  b) 3, - 3  c) 1, - 3  d) 5, - 3

3) If an A.P. given by $k, \frac{2k}{3}, \frac{k}{3}, 0$ then the 6th term is
   a) $\frac{3}{4} - k$  b) $\frac{2}{3} - k$  c) $\frac{3}{2}k$  d) $\frac{3}{2} - k$

4) Which term of the AP is 181 in the given sequence 5, 13, 21, ......, 181.
   a) 21st  b) 22nd  c) 23rd  d) 24th

5) If $\frac{1}{1^2} + \frac{1}{2^2} + \frac{1}{3^2} + \ldots \ldots \ldots$ upto infinite terms $= \frac{\pi^2}{6}$ then the value of $\frac{1}{1^5} + \frac{1}{3^5} + \frac{1}{5^5} + \ldots \ldots \ldots$ upto $\infty$ terms =
   a) $\frac{\pi^2}{4}$  b) $\frac{\pi^2}{6}$  c) $\frac{\pi^2}{8}$  d) $\frac{\pi^2}{12}$

6) If for a G.P. $t_8 = 640, a = 2$ then the common ratio is...
   a) 3  b) 5  c) 9  d) - 3

7) If for G.P., $a = 5, t_6 = \frac{1}{625}$ then the common ratio is...
   a) 5  b) $\frac{1}{5}$  c) 25  d) $\frac{1}{25}$

8) The sum of first 'n' natural number is equal to...
   a) $\frac{n(n+1)}{2}$  b) $n^2$  c) 2n  d) $\frac{n(n+1)}{6}$
9) If for the H.P. \( \frac{1}{3}, \frac{1}{5}, \frac{1}{7}, \frac{1}{9}, \ldots \) then its 10th term is...
   a) \( \frac{1}{10} \)  
   b) \( \frac{1}{15} \)  
   c) \( \frac{1}{25} \)  
   d) \( \frac{1}{21} \)

10) The H.M. of 5 and 45 is...
   a) 9  
   b) 25  
   c) 15  
   d) 30

Q.2) Solve the following, : 2 marks each

1) For a G.P, if \( a = 5 \) and \( t_6 = \frac{1}{625} \) then find \( t_{10} \).

2) If for a sequence \( t_n = \frac{2(n-2)}{5(n-3)} \) then show that the sequence is G.P.

3) Show that \( \frac{1}{2}, \frac{1}{5}, \frac{1}{8}, \frac{1}{11} \ldots \ldots \ldots \ldots \ldots \) are in H.P. hence find the 10th term.

4) Find H.M. of two positive number whose A.M and G.M are 16 and 9 respectively.

5) How many terms are there in the G.P. 2, 4, 8, ..., 128 ?

6) If lengths of sides of a right angle triangle are in A.P. then find the sine of the acute angle.

7) Find \( t_n \) and \( S_n \) for the A.P. \( \frac{14}{3}, \frac{16}{3}, 6 \ldots \ldots \)

8) If for a G.P. \( t_8 = 640, r = 2 \) then Find \( a \).

9) If for a given G.P. \( a = 729 \) and 7th term is 64. then determine \( S_7 \).

10) If for a G.P. \( t_4 = 24, t_9 = 768. \) then find \( S_8 \).
Q.3) Solve the following, :  

1) Insert two numbers between 3 and 81 such that resulting sequence is in G.P.

2) Prove that \( \log(1 + 2 + 3) = \log 1 + \log 2 + \log 3. \)

3) Find \( 9 + 99 + 999 + 9999 + \ldots \) upto \( n \) terms.

4) Find the sum \( 5^2 + 6^2 + 7^2 + \ldots + 30^2. \)

5) If for an A.P. \( t_3 = 17, t_7 = 37 \), find \( S_{16}. \)

6) If \( a, b, c, d \) are in G.P then prove that \( a + b, b + c, c + d \) are also in G.P.


8) Express the recurring decimals \( 0.23 \) as a rational number (i.e in the form of \( \frac{p}{q} \)).

9) The sum of an infinite G.P. is \( \frac{80}{9} \) and its common ratio is \( \frac{-4}{5} \). Find its first term.

10) Find the following sums \( \sum_{r=1}^{n} (6r^2 - 2r + 6). \)

Q.4) Solve the following, :  

1) Find the sum \( 8 + 88 + 888 + 8888 + \ldots \) upto \( n \) terms.

2) Find three numbers in G.P. such that their sum is 28 and product is 512.

4) Determine the number of terms in a G.P. If \( a_1 = 3, a_n = 96 \) and \( S_n = 189. \)

5) Find the sum of \( n \) terms of the sequence 0.5, 0.55, 0.555, \ldots

6) For a sequence, If \( S_n = \frac{4^n - 3^n}{3^n} \) then Find the \( n^{th} \) term. Hence show that it is a G.P. Also find the common ratio.

7) The sum of an infinite geometric progression is 15 and the sum of the squares of these
terms is 45. Find the G.P.

8) Find the two numbers whose A.M exceeds their G.M by 30 and H.M by 48.

9) Find the sum of all natural numbers from 1 to 200 which are divisible by 3.

10) The A.M of two numbers exceeds their G.M by \( \frac{3}{2} \) and their G.M. exceeds their H.M. by \( \frac{6}{5} \). Find the numbers.
16. Permutation and Combination

Marks - 1

Q.1) Select and write the correct answer from the given alternatives in each of the following questions.

1) How many lines can be drawn through 21 points on a circle?
   a) 310  b) 210  c) 410  d) 570

2) How many integers between 1000 and 10000 have no digits other than 4, 5 or 6.
   a) 91  b) 51  c) 81  d) 71

3) The number of diagonals in a decagon is...
   a) 25  b) 35  c) 45  d) 15

4) There are 6 candidates for 3 posts. In how many ways can the posts be filled?
   a) 120  b) 130  c) 100  d) 110

5) How many ways can 5 sportsman be selected from a group of 10?
   a) 272  b) 282  c) 252  d) 242

6) How many two digit odd numbers can be formed from the digit 1, 3, 4, 5 & 8 if repetition of digits is allowed.
   a) 5  b) 15  c) 35  d) 25

7) If \( \binom{n}{8} = \binom{n}{6} \) then value of \( n \) is....
   a) 14  b) 18  c) 10  d) 12

8) In a football championship 153 matches were played, one match with each other. The numbers of teams participating in the championship is...
   a) 18  b) 16  c) 9  d) 10

Q.2) Solve the following : 2 marks each

1) If \( \binom{x+y}{2} = 56 \) and \( \binom{x-y}{2} = 12 \) then find \( x \) and \( y \).

2) In how many ways can the letters of the word STORY be arranged so that T and Y are always together?

3) If \( \binom{14}{5} + \binom{14}{6} + \binom{15}{7} + \binom{16}{8} = \binom{17}{x} \) then find \( x \).

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4) If $^nP_3 : ^nP_6 = 1 : 210$ find 'n'.

5) Find 'n' and 'r' if $^nP_r = 720$ and $^nC_r = 120$.

6) In how many ways can a man invite 6 friends to a party so that two or more of them remain present?

Q.3) Solve the following : 3 marks each

1) There are 5 red, 4 white and 3 green marbles in a bag. If all are drawn, determine the number of different arrangement.

2) In how many ways can 18 objects be divided into 3 groups containing 9, 6, 3 object respectively.

3) Determine the number of parallelogram that can be formed from a set of 4 parallel lines intersecting another set of 5 parallel lines.

4) If $^{10}C_{r+2} : ^{10}C_r = 10 : 21$ then find r.

5) At the end of certain meeting, after everyone had shaken hands with every one else, It was found that 45 handshakes were exchanged. How many members were present at the meeting?

6) Find x if $\frac{1}{4!} + \frac{3}{6!} = \frac{x}{8!}$

7) How many different words can be formed with the letters of the word DIPAWALI ? In how many of these P and D are never together ?

8) How many chords can drawn through 21 points on a circle ?

9) Find the difference between the greatest value of $^{13}C_r - ^{11}C_r$.

10) In how many ways can a team of 3 boys and 2 girls be selected from 6 boys and 5 girls.
Q.4) Solve the following : 4 marks each

1) Four cards are drawn from a pack of 52 playing cards. In how many different ways can this be done? How many selection will contain
   a) Exactly one card of each suit.
   b) All cards of the same suit.

2) In how ways can 6 persons be selected from 4 doctors and 8 lawyers, if at least one doctor is included in the selection.

3) How many different arrangements can be made, with the letters of the word MATHEMATICS? In how many of these arrangements vowels occur together?

4) Find the numbers of the straight lines obtained by joining 10 points on a plane such that
   a) No three of them are collinear.
   b) Four points are collinear.

5) If \( n^r_P = 6652800 \) and \( n^r_C = 165 \). then find \( (r+3)C(r+4) \).

6) Among 22 cricket players, there are 3 wicketkeepers and 6 bowlers. In how many ways can the team of 11 players be chosen so as to include exactly one wicketkeeper and at least 4 bowlers?
17. Method of Mathematical Induction & Binomial Thm

Marks - 1

Q.1) Solve. (for ONE Mark)

1) Method of mathematical induction is used for statements involving.
   a) Complex numbers   b) Natural numbers   c) Real numbers   d) All of these

2) The coefficients of Binomial theorem forms
   a) Binary coefficient   b) Pascal's triangle   c) Even coefficient   d)Odd coefficient

3) The sum 1 + 2 ......... + n = ..................
   a) n (n+1)   b) $\frac{n(n+1)}{2}$   c) n (2n + 1)   d) $\frac{n(n+1)(2n+1)}{6}$

4) The Number of terms in the expansion of $(a+b)^n$ are is....
   a) n   b) $\binom{n+3}{2}$   c) n + 1   d) None of these

5) $C_0 + C_1 + C_2 + \cdots + C_n =$
   a) 2   b) $n^{n+1}$   c) $2^n$   d) $2^{n-1}$

6) $C_0 + C_1 + \cdots + C_{10} =$
   a) 2024   b) 1024   c) 512   d) 1028

7) In each term of $(a+b)^n$ sum of Indices of a and b is....
   a) n + 1   b) n   c) zero   d) None of these

8) The $3^{rd}$ term in the expansion of is $\left( x + \frac{5}{x} \right)^4$ is ....
   a) $150x^2$   b) $50x^2$   c) 150   d) $125x^3$

9) If P (n) is a statement $(n \in N)$ such that if P(k) is true, P(k+1) is true for $k \in N$ then P (n) is true true for...
   a) for all n   b) for all n > 1
   c) for all n > 2   d) Nothing can be said

10) The coefficient of $x^4$ in the expansion of $\left( \frac{x}{2} - \frac{3}{x^2} \right)^{10}$ is
    a) $\frac{504}{259}$   b) $\frac{450}{263}$   c) $\frac{405}{256}$   d) None of these
Q.2) Attempt following. (2 Marks for each)

1) Using binomial theorem expand \((\sqrt{3} - 1)^4\).

2) Evaluate \(\binom{0}{0} + \binom{1}{1} + \ldots + \binom{15}{15} =\)

3) Find \((2.02)^3\)

4) Find the coefficient of \(x^9\) in the expansion of \(\left(\frac{2}{x} - x^3\right)^{18}\).

5) Find the fourth term in the expansion of \(\left(x^2 - \frac{2}{x}\right)^7\).

6) Prove that \(\binom{1}{1} + 2\binom{2}{2} + 3\binom{3}{3} + 4\binom{4}{3} + \ldots + n\binom{n}{n} = n \cdot 2^{n-1}\).

7) Without expanding find the value of

\[
(2x + 3)^4 + 4(2x + 3)^3(3x - 3) + 6(2x + 3)^2(3x - 3)^2 + 4(2x + 3)(3x - 3)^3 + (3x - 3)^4
\]

8) Find the value of

\[
(5x + 9)^5 - 5(5x + 9)^4(5x + 5) + 10(5x + 9)^3(5x + 5)^2 - 10(5x + 9)^2
\]

\[
(5x + 5)^3 + 5(5x + 5)^4 - (5x + 5)^5
\]

9) Find \((0.9)^6\) correct up to 4 decimal places.

10) Show that \(\binom{1}{1} + \binom{2}{2} + \ldots + \binom{10}{10} = 1023\).
Q.3) Attempt following. (3 Marks for each)

1) Expand \((\sqrt{3} - \sqrt{5})^4\).

2) Find the constant term in the expansion of \(\left(3x - \frac{1}{2x^2}\right)^9\).

3) Find the tenth term in the expansion of \(\left(2\sqrt{x} - \frac{3}{x}\right)^{12}\).

4) Find the first three terms in the expansion of \((2 - 3x)^{14}\).

5) Evaluate \(\sqrt{217}\) upto 4 decimal places.

6) Find first four terms in the expansion of \((a - b)^{-1/5}\) if \(|a| > |b|\).

7) Find first 3 terms in the expansion of \((2 - x^2)^4\).

8) Find value of \(\sqrt{42}\) upto 3 decimal places.

9) Expand \((2x^3 - 3y^2)^4\).

10) If the coefficient of \(x^{16}\) in the expansion of \((x^2 + ax)^{10}\) is 3360, find a.

Q.4) Attempt following. (4 Marks for each)

1) Prove by the method of induction

\[1^2 + 4^2 + 7^2 + \ldots (3n - 2)^2 = \frac{n}{2} (6n^2 - 3n - 1) \quad \text{For all } n \in N\]

2) Prove the statement by method of induction: \((5^{2n} - 2^{3n})\) is divisible by 3, for all \(n \in N\).

3) If the coefficient of \(x^2 \text{ and } x^3\) in the expansion of \((3 + kx)^9\) are equal, find k.

4) Show that there is no term containing \(x^6\) in the expansion of \(\left(x^2 - \frac{3}{x}\right)^{11}\).

5) Find the middle term in the expansion of \(\left(\frac{4x^3}{5} + \frac{5}{2x^2}\right)^{10}\).
6) If the middle term in the expansion of \( \left( x + \frac{b}{x} \right)^6 \) is 160, find the value of \( b \).

7) Find the first 3 terms in expansion of \( (3 - 5x)^{-1/2} \).

8) Find the coefficient of \( x^{40} \) in the expansion of \( \left( \frac{1}{x^2} - x^4 \right)^{18} \).

9) Prove that \( 15^{2n-1} + 1 \) is divisible by 16 for all \( n \in \mathbb{N} \), by method of induction.

10) Find the first four terms in the expansion of \( (1 + 3x)^{-5} \).
Q.1) Solve. (1 Marks for each)

1) If \( f(x) = \begin{cases} x, & x < 0 \\ 1, & x = 0 \\ x^2, & x > 0 \end{cases} \) then \( \lim_{x \to 0} f(x) = \ldots \)
   a) 1  
   b) Zero  
   c) does not exist  
   d) None of these

2) \( \lim_{x \to 0} \frac{\tan x - \sin x}{x^3} \) is equal to...
   a) 0  
   b) 1  
   c) 1/2  
   d) -1/2

3) The value of \( \lim_{x \to 0} \left[ \frac{1 - \cos(1 - \cos x)}{x^4} \right] \) is
   a) \( \frac{1}{8} \)  
   b) \( \frac{1}{2} \)  
   c) \( \frac{1}{4} \)  
   d) \( \frac{1}{16} \)

4) The value of \( \lim_{x \to 0} \left( \tan \left( \frac{\pi}{4} + x \right) \right)^{1/x} \) is....
   a) \( e \)  
   b) \( e^2 \)  
   c) \( e^3 \)  
   d) \( e^{-2} \)

5) \( \lim_{x \to 4} \frac{1}{(x-3)^{x-\pi}} \) is ...
   a) -e  
   b) -2e  
   c) e  
   d) 2e

6) The value of \( \lim_{x \to 0} \frac{a^x - 1}{1 - b^x} \) is....
   a) \( \frac{\log a}{\log b} \)  
   b) \( \frac{\log b}{\log a} \)  
   c) \( -\frac{\log a}{\log b} \)  
   d) \( \log \left( \frac{a}{b} \right) \)

7) \( \lim_{x \to 3} \frac{1x - 31}{x - 3} \)
   a) 1  
   b) -1  
   c) 0  
   d) does not exist

8) \( \lim_{x \to 0} \frac{\sin x}{\sqrt{x}} \)
   a) 0  
   b) \( \frac{1}{2} \)  
   c) 1  
   d) does not exist

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9) If \( G(x) = -\sqrt{25 - x^2} \) then \( \lim_{x \to 1} \frac{G(x) - G(1)}{x - 1} = \)

a) \( \frac{1}{24} \)  

b) \( \frac{1}{5} \)  

c) \( -\sqrt{24} \)  

d) \( \frac{1}{\sqrt{24}} \)

10) \( \lim_{x \to 0} \frac{\sin 1x}{x} \) is

a) 1  

b) 0  

 c) infinity  

does not exist

Q.2) Solve. (2 Marks for each)

1) Evaluate \( \lim_{x \to 0} \frac{\log_e(1 + x)}{3^x - 1} \).

2) Evaluate \( \lim_{x \to 1} [\log_2(2x)] \log_2 x \).

3) Evaluate \( \lim_{x \to 1} \frac{x^2 + x \log x - \log x - 1}{x^2 - 1} \).

4) Evaluate \( \lim_{x \to 2} \frac{3^{x^2} - 81}{9^x - 9^2} \).

5) If \( \lim_{x \to 1} \frac{x + x^2 + x^3 + x^4 + x^5 - 5}{x - 1} = 15k \) then find the value of \( k \).

6) If \( \lim_{x \to 2} \frac{x^n - 2^n}{x - 2} = 1024 \) then find the value of \( n \).

7) Evaluate \( \lim_{x \to \infty} \left( \sqrt{x + \sqrt{x + \sqrt{x}} - \sqrt{x}} \right) \).

8) Evaluate \( \lim_{x \to \infty} \left[ \frac{(2 + x)^4(4 + x)^5}{(2 - x)^{45}} \right] \).

9) Evaluate \( \lim_{x \to 0} \frac{x \cdot 2^x - x}{1 - \cos x} \).

10) Evaluate \( \lim_{x \to 0} \frac{\tan x - \sin x}{x^3} \).

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Q.3) Solve. (3 Marks for each)

1) Evaluate \( \lim_{x \to 0} \frac{(4^x - 1)^3}{\sin(x^2 / 4) \log(1+3x)} \)

2) If \( f(x) = \sqrt{x + 2} \) then find \( \lim_{x \to 2} \frac{f(x) - f(2)}{x - 2} \)

3) Evaluate \( \lim_{x \to \frac{\pi}{4}} (1 + \cos 2x)^{4 \sec 2x} \).

4) Evaluate \( \lim_{x \to 0} \left[ \frac{\sqrt{a^2 - ax + x^2}}{\sqrt{a + x}} - \frac{\sqrt{a^2 + ax + x^2}}{\sqrt{a - x}} \right] \)

5) Evaluate \( \lim_{x \to 0} \frac{\log(1+2h) - \log(1+h)}{h^2} \).

6) Evaluate \( \lim_{x \to 0} \frac{\log(1+x+x^2) + \log(1-x+x^2)}{\sec x - \cos x} \).

7) Evaluate \( \lim_{x \to 0} \frac{27^x - 9^x - 3^x + 1}{\sqrt{5} - \sqrt{4 + \cos x}} \).

8) Evaluate \( \lim_{x \to 0} \frac{\sin a - \tan a}{\sin^3 a} \).

9) Evaluate \( \lim_{x \to \infty} \left[ \frac{1}{n^2} + \frac{2}{n^2} + \frac{3}{n^2} + \ldots + \frac{1}{n^2} \right] \).

10) Evaluate \( \lim_{x \to \infty} \frac{(x+1)^{10} + (x+2)^{10} + \ldots + (x+100)^{10}}{(x^{10} + 10^{10})} \).

Q.4) Solve. (4 Marks for each)

1) If \( f(x) = x^2 - 3 \), \( 2 < x < 3 \)

\[ = 2x + 5 \], \( 3 < x < 4 \)

then find the equation whose roots are \( \lim_{x \to 3^-} f(x) \) and \( \lim_{x \to 3^+} f(x) \).
2) Evaluate \( \lim_{h \to 0} \frac{\sin(a+3h)-3\sin(a+2h)+3\sin(a+h)-\sin a}{h^3} \)

3) If \( \lim_{x \to a} \frac{\sin x - \sin a}{\cos x - \cos a} = -1 \) then find the value of \( a \).

4) Evaluate \( \lim_{x \to 0} \frac{5\sin x - 7\sin 2x + 3\sin 3x}{x^2 \sin x} \)

5) Evaluate \( \lim_{x \to 0} \frac{8}{x^2} \left[ 1 - \cos(x^2/2) - \cos^2(x^2/4) + \cos^2(x^2/2) \cos^2(x^2/4) \right] \)

6) Evaluate \( \lim_{y \to 0} \frac{(x+y) \sec(x+y) - x \sec x}{y} \)

7) Find the value of constant \( \alpha \) & \( \beta \) such that

\[
\lim_{x \to \infty} \left[ \frac{x^2 + 1}{x + 1} - \alpha x - \beta \right] = 0.
\]

8) Evaluate \( \lim_{x \to 2} \frac{\sqrt{1+\sqrt{2+x}} - \sqrt{3}}{x-2} \)

9) If \( f(x) = \cot^{-1}\left( \frac{3x-x^2}{1-3x^2} \right) \) and \( g(x) = \cos^{-1}\left( \frac{1-x^2}{1+x^2} \right) \) then

\[
\lim_{x \to a} \frac{f(x) - f(a)}{g(x) - g(a)}, \quad 0 < a < \frac{1}{2}.
\]

10) Evaluate \( \lim_{n \to 0} \left[ \frac{1}{1.3} + \frac{1}{3.5} + \frac{1}{5.7} + \ldots + \frac{1}{(2n-1)(2n+1)} \right] \)
Q.1) Solve. (1 Marks for each)

1) If \( f \) is a differentiable function \( x \) then \( \lim_{h \to 0} \frac{[f(x+h)]^2 - [f(x)]^2}{2h} = \)

a) 0  
   b) \( f'(x) f(x) \)  
   c) \( f''(x) \cdot f'''(x) \)  
   d) 1

2) If \( f(0) = 0 = g(0) \) and \( f'(0) = 6 = g'(0) \) then

\( \lim_{x \to 0} \frac{f(x)}{g(x)} \) is

a) 1  
   b) 0  
   c) 12  
   d) -1

3) If \( y = e^{x+3 \log x} \) then \( \frac{dy}{dx} = \ldots \)

a) \( e^{x+3hx} \)  
   b) \( e^x \cdot x^2 \cdot (x+3) \)  
   c) \( e^x \cdot x^3 \)  
   d) \( e^x \cdot \frac{1}{x^3} \)

4) \( \frac{d}{dx} \left[ \frac{\sin x + \cos x}{\sqrt{1 + \sin 2x}} \right] \), \( 0 < x < \frac{\pi}{4} \) =

a) 1  
   b) 0  
   c) -1  
   d) None of these

5) If \( \frac{d}{dx} \left[ \frac{\tan x - \cot x}{\tan x + \cot x} \right] = \)

a) 2 \( \sin 2x \)  
   b) -2 \( \sin 2x \)  
   c) 2 \( \cos 2x \)  
   d) -2 \( \cos 2x \)

6) If \( y = x \sin^{-1} x + \sqrt{1-x^2} \) then \( \frac{dy}{dx} = \)

a) \( \sqrt{1-x^2} \)  
   b) -\( \sqrt{1-x^2} \)  
   c) \( \frac{1}{\sqrt{1-x^2}} \)  
   d) \( \sin^{-1} x \)

7) \( \frac{d}{dx} (\sin^{-1} x + \cos^{-1} x) = \)

a) \( \pi/2 \)  
   b) 0  
   c) \( \frac{2}{\sqrt{1-x^2}} \)  
   d) 1
8) The derivative of an odd function is always.
   a) an odd function       b) an even function       c) does not exist       d) none of these

9) Given \( f(x) = \frac{1}{2x-1} \) then for \( x = 0 \).
   a) \( f'(x) \) vanishes       b) \( f'(x) = f(x) \)       c) \( f'(x) > 0 \)       d) \( f'(x) < 0 \)

10) If \( f'(a) \) exist then \( \lim_{x \to a} \frac{xf(a) - af(x)}{x - a} \) is
    a) \( f(a) - af'(a) \)       b) \( f'(a) \)       c) \( -f'(a) \)       d) \( f(a) + af'(a) \)

Q.2) Solve.  
(2 Marks for each)

1) If \( y = \sqrt{1 + 2 \sec x \tan x + 2 \tan^2 x} \) then show that \( \frac{dy}{dx} = y \sec x \).

2) If \( f(x) = \left( \frac{x^a}{x^b} \right)^{a+b} \left( \frac{x^b}{x^c} \right)^{b+c} \left( \frac{x^c}{x^a} \right)^{c+a} \) then find \( f'(x) \).

3) If \( y = 4 \log_2 (\sin x) + 9 \log_3 (\cos x) \) then find \( \frac{dy}{dx} \).

4) If \( y = \frac{e^x + 1}{e^x - 1} \) then find the value of \( \frac{y^2}{2} + \frac{dy}{dx} \).

5) If \( y = (\sin x + \cos x)^2 \) then find \( \frac{dy}{dx} \).

6) If \( y = x^2 \cos x \log x \), find \( \frac{dy}{dx} \).

7) If \( f(x) = 3x^2 - 2x + 1 \), find \( 3f'(0) + 2f'(1) \).

8) If \( f(x) = 4 \sin x - 5 \cos x \) find \( \frac{f'(\frac{\pi}{2}) + f(0)}{f'(\frac{\pi}{2}) - f(0)} \).

9) If \( f(x) = x^2 \tan x \) find \( f'(\frac{\pi}{2}) \).

10) If \( y = \frac{\log x}{1 + \log x} \) find \( \frac{dy}{dx} \) at \( x = e \).
Q.3) Solve. (3 Marks for each)

1) Evaluate \( \frac{d}{dx} \left[ \frac{(x+h)^4 - x^4}{h} \right] \)

2) If \( y = \sqrt{x + \sqrt{y + \sqrt{x + \sqrt{y + \ldots}}}} \) then find \( \frac{dy}{dx} \).

3) If \( x\sqrt{1+y} + y\sqrt{1+x} = 0 \) then find \( \frac{dy}{dx} \).

4) If \( f(x) = \frac{x - 4}{2\sqrt{x}} \) then show that \( f'(0) \) does not exist.

5) Let \( f(x) = e^x \cdot g(x), \ g(0) = 4, \ g'(0) = 2 \) then find \( f'(0) \).

6) If \( f(2) = 4, \ f'(2) = 4 \), then find \( \lim_{x \to 2} \frac{xf(2) - 2f(x)}{x-2} \).

7) If \( f(x) = 9 \) and \( f'(9) = 3 \) then find the value of \( \lim_{x \to 9} \frac{\sqrt{f(x)} - 3}{\sqrt{x} - 3} \).

8) If \( y = 9 \log_3 \cos \frac{x}{2} - 16 \log_4 \sin \frac{x}{2} \) find \( \frac{dy}{dx} \).

9) If \( \tan y = \frac{\sqrt{1+\sin 2x} - \sqrt{1-\sin 2x}}{\sqrt{1+\sin 2x} + \sqrt{1-\sin 2x}} \) then find \( \frac{dy}{dx} \).

10) If \( y = \log a^x + \log x^a + \log x^a + \log x^a \) then find \( \frac{dy}{dx} \).

Q.4) Solve. (4 Marks for each)

1) If \( g(x) = x^2 + 2x + 3f(x), \ f(0) = 5 \) and \( \lim_{x \to 0} \frac{f(x)-5}{x} = 4 \) then find \( g'(0) \).

2) Find the derivative of \( f(x) = 2x^2 + 3x - 5 \) at \( x = -1 \). Also prove that \( f'(0) + 3f'(-1) = 0 \).

3) If \( y = \frac{x\sin x}{\sin x + \cos x} \) find \( \frac{dy}{dx} \).

4) If \( y = \left( \frac{1}{\sqrt{x}} - 3x^3 \right) \log x \) find \( \frac{dy}{dx} \).

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5) If \( y = \sqrt{1 - 2 \cos cx \cot x + 2 \cot^2 x} \), show that \( \frac{dy}{dx} = y \csc X \).

6) If \( y = \frac{1 + \sin 2x}{\sqrt{1 - \sin 2x}} \) show that \( \frac{dy}{dx} \) at \( x = \frac{\pi}{2} \) is 2.
20. Integration

Marks - 1

1) \[ \int 7^x \, dx = \ldots \ldots \ldots \]
   a) \( 7^x \log 7 + c \)
   b) \( x \cdot 7^{x-1} + c \)
   c) \( \frac{7^{x+1}}{x+1} + c \)
   d) \( \frac{7^x}{\log 7} + C \)

2) \[ \int 4^x e^x \, dx = \ldots \ldots \ldots \]
   a) \( 4^x e^x \log 4 + c \)
   b) \( (4^x + e^x) \log 4 + c \)
   c) \( 4^x e^x + c \)
   d) \( \frac{4^x e^x}{1 + \log 4} + c \)

3) \[ \int \frac{1}{\sin^2 x} \, dx = \ldots \ldots \ldots \]
   a) \( \sec^2 x + c \)
   b) \( \tan^2 x + c \)
   c) \( -\cot x + c \)
   d) \( \cos^2 x + c \)

4) \[ \int x^{-1} \, dx = \ldots \ldots \ldots \]
   a) \( x^{-1} + c \)
   b) \( x + c \)
   c) \( \log x + c \)
   d) \( \log x \cdot 1 + c \)

5) \[ \int \sin(2x + 3) \, dx = \ldots \ldots \ldots \]
   a) \( -\frac{1}{2} \cos (2x + 3) + c \)
   b) \( 2 \cos (2x + 3) + c \)
   c) \( \frac{1}{2} \cos (2x + 3) + c \)
   d) \( \cos(2x + 3) + c \)

6) \[ \int \tan^2 x \, dx = \ldots \ldots \ldots \]
   a) \( \sec^2 x + c \)
   b) \( \log(\sin x) + c \)
   c) \( \log(\sec x) + c \)
   d) \( \tan x - x + c \)

7) \[ \int 3x^2 + \sin x \, dx = \ldots \ldots \ldots \]
   a) \( x^3 - \cos x + c \)
   b) \( x^3 + \cos x + c \)
   c) \( \frac{x^3}{3} - \cos x + c \)
   d) \( \frac{x^3}{3} + \cos x + c \)

8) \[ \int (x + 1)(x - 1) \, dx = \ldots \ldots \ldots \]
   a) \( x^2 - 1 + c \)
   b) \( x^3 - x + c \)
   c) \( \frac{x^2}{2} - x + c \)
   d) \( \frac{x^3}{3} - x + c \)

9) \[ \int \frac{x + 1}{x^2} \, dx = \ldots \ldots \ldots \]
   a) \( \log x + x + c \)
   b) \( \log x - x + c \)
   c) \( \log x - \frac{1}{x} + c \)
   d) \( \log x + \frac{1}{x} + c \)
10) \[ \int (2x + 3)^3 \, dx = \ldots \ldots \ldots \]

\[ a) \frac{1}{2} (2x + 3) + c \quad b) 3(2x + 3) + c \quad c) \frac{(2x + 3)^4}{8} + c \quad d) \frac{(2x + 3)^4}{4} + c \]

**Q.2) Evaluate.**

(2 Marks for each)

1) \[ \int 1 + \sin (3x) \, dx \]

2) \[ \int (x + 1)(x + 2) \, dx \]

3) \[ \int 2^x + x^2 + 2^2 \, dx \]

4) \[ \int \frac{1}{1 + \sin x} \, dx \]

5) \[ \int \frac{1}{1 - \sin x} \, dx \]

6) \[ \int \sin 10x \cdot \sin 6x \, dx \]

7) \[ \int \sqrt{5 + 4x} + \frac{1}{\sqrt{5 - 4x}} \, dx \]

8) \[ \int \sin^{-1}(\cos 2x) \, dx \]

9) \[ \int \cos^{-1}(\sin 2x) \, dx \]

10) \[ \int 1 + \cos(2x - 3) \, dx \]

**Q.3) Solve.**

(3 Marks for each)

1) \[ \int (\sin x + \cos x)^2 \, dx \]

2) \[ \int (\tan x + \cot x)^2 \, dx \]

3) \[ \int (x + 1)(x + 2)(x + 3) \, dx \]
4) \[ \int \frac{\sec x}{\sec x + \tan x} \, dx \]

5) \[ \int \sqrt{1 + \sin 2x} \, dx \]

6) \[ \int \sin \alpha \sin x + \cos \alpha \cos x \, dx \]

7) \[ \int 3\sin x + 4\cos x + e^x \, dx \]

8) \[ \int \left( x - \frac{1}{x} \right)^2 \, dx \]

9) \[ \int 3\tan^2(4x) \, dx \]

10) \[ \int \sin^2(3x) \, dx \]

Q.4) Solve. (4 Marks for each)

1) \[ \int \frac{1}{\sqrt{x + a}} - \frac{1}{\sqrt{x - a}} \, dx \]

2) \[ \int (e^x - 1)^3 \, dx \]

3) \[ \int \tan^{-1}\left( \frac{\sin 2x}{1 + \cos 2x} \right) \, dx \]

4) \[ \int \tan^{-1}\left( \frac{\sin 2x}{1 - \cos 2x} \right) \, dx \]

5) \[ \int \sin^3 x \, dx \]

6) \[ \int \cos^3(2x) \, dx \]

7) \[ \int \sin x \sin 3x \sin 5x \, dx \]

8) \[ \int \frac{\sin^4 2x + \cos^4 2x}{\sin^2 2x \cos^2 2x} \, dx \]
9) \[ \int \frac{\tan x}{\sec x + \tan x} \, dx \]

10) If \( f(0) = 1, \) \( f(1) = 4, \) and \( f'(x) = 4x^3 - 3x^2 + 2x + k \) then find \( f(x) \). Hence find \( f(2) \).
21. Statistics

Q. 1) Select and write the correct answer from the given alternatives in each of the following question. (1 marks each)

1) A student obtained 75%, 80% and 85% marks in three subjects. If the marks of another subject is added then his average cannot be less than
   a) 60%          b) 65%          c) 80%          d) 90%

2) If mean of set of observations $x_1$, $x_2$, $x_3$, .... $x_{10}$ is 40 then the mean of $x_1 + 4$, $x_2 + 8$, $x_3 + 12$, .... $x_{10} + 40$ is....
   a) 62          b) 52          c) 38          d) 42

3) 1, 2, 4, 6, 8, 9 transformed into another series whose mean is 12. Two middle terms of the transformed series are
   a) 7, 8          b) 10, 14          c) 9, 15          d) 4, 13

4) The standard deviation of the numbers 2, 4, 5 and 6 is $\alpha$, the standard deviation of the number 4, 6, 7, 8 is...
   a) $\alpha + 2$          b) $2 \alpha$          c) $\alpha$          d) $\sqrt{2\alpha}$

5) The simplest measure of dispersion to indicate the extreme variation in the data is
   a) Range          b) Q.D          c) M.D          d) S.D

6) If a group of 20 men have A.M 5 and a group of 30 women have A.M 6. then combined A.M of the entire group is...
   a) 5          b) 6          c) 5.6          d) 8

7) The mode of 5, 9, 10, 15, 20...
   a) does not exist          b) each value is a mode
   c) The middle value 10 in this case          d) The A.M in this case

8) In case of extreme values the best measure of central tendency is...
   a) A.M          b) median          c) Mode          d) None of the above.

9) The mean deviation is minimum when it is taken about...
   a) Mean          b) Median          c) Mode          d) None of the above.

10) The standard deviation is based on...
    a) A.M          b) mean          c) Mode          d) All of the observations
Q.2) Solve the following : 2 marks each

1) Find the variance of first 20 natural numbers.

2) For two data sets, each of size 5. The variance are given to be 4 and 5 the corresponding means are given to be 2 and 4 respectively. Find the variance of the combined data.

3) Find the mean deviation about the median for the following data 5, 10, 2, 17, 3.

4) Calculate the standard deviation of the following marks obtained by 5 students in a tutorial group.
   Marks 8, 12, 13, 15, 22.

5) The Coefficient of variance of two groups are respectively 30% and 40% and their standard deviation are 4 and 5. Find their arithmetic mean.

6) The following are the prices (in ₹) of shares of a company for six days of a week. Calculate the range.
   200, 210, 208, 160, 220, 250.

7) From the following distribution of the daily expenditure of 100 families, calculate Range

<table>
<thead>
<tr>
<th>daily expenditure (in ₹)</th>
<th>10-20</th>
<th>20-30</th>
<th>30-40</th>
<th>40-50</th>
</tr>
</thead>
<tbody>
<tr>
<td>No. of families</td>
<td>20</td>
<td>30</td>
<td>25</td>
<td>15</td>
</tr>
</tbody>
</table>

8) Calculate the Quartile for the following frequency distribution

<table>
<thead>
<tr>
<th>x</th>
<th>2</th>
<th>4</th>
<th>6</th>
<th>8</th>
<th>10</th>
<th>11</th>
</tr>
</thead>
<tbody>
<tr>
<td>f</td>
<td>3</td>
<td>8</td>
<td>12</td>
<td>14</td>
<td>11</td>
<td>08</td>
</tr>
</tbody>
</table>

9) Compute the lower and upper quartiles for the following data,
   7, 2, 11, 19, 20, 5, 6, 25.

10) If \( n = 10, \bar{x} = 12, \sum x^2 = 1530 \). Find the coefficient of variation.

Q.3) Solve the following : 3 marks each

1) The mean of five observations is 4 and their variance is 5.2. If three of these observations are 1, 2 and 6 then Find the other two observations.

2) Find the missing frequency for the following frequency distribution table.
If the total frequency is 685 and median is 42.6.

3) Calculate the quartile deviation for the following frequency distribution table.

<table>
<thead>
<tr>
<th>$x_i$</th>
<th>2</th>
<th>4</th>
<th>6</th>
<th>8</th>
<th>10</th>
<th>11</th>
</tr>
</thead>
<tbody>
<tr>
<td>$f_i$</td>
<td>3</td>
<td>8</td>
<td>12</td>
<td>14</td>
<td>11</td>
<td>8</td>
</tr>
</tbody>
</table>

4) From the following distribution of the monthly pocket money of the students, calculate the range and coefficient of range.

<table>
<thead>
<tr>
<th>Student no.</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
<th>8</th>
<th>9</th>
<th>10</th>
</tr>
</thead>
<tbody>
<tr>
<td>Pocket Money (Rs.)</td>
<td>100</td>
<td>50</td>
<td>70</td>
<td>200</td>
<td>250</td>
<td>80</td>
<td>800</td>
<td>200</td>
<td>250</td>
<td>300</td>
</tr>
</tbody>
</table>

5) The monthly sales for the first 11 months of the year of a certain salesman were Rs.12000/- but due to his illness during the last month the average sales for the whole year came down to Rs.11375/-. Find the value of the sale during the last month.

6) Mean of 10 terms is 50 and standard deviation is 14. Find the sum of the squares of all terms.

7) Coefficient of variation of two groups are respectively 40% and 50% and their standard deviations are 16 and 20. Find their arithmetic mean.

Q.4) Solve the following: 4 marks each

1) Calculate the mean deviation about mean for the following frequency distribution using step deviation method.

<table>
<thead>
<tr>
<th>Class Intervals</th>
<th>2-6</th>
<th>6-10</th>
<th>10-14</th>
<th>14-18</th>
</tr>
</thead>
<tbody>
<tr>
<td>Frequency</td>
<td>6</td>
<td>8</td>
<td>4</td>
<td>2</td>
</tr>
</tbody>
</table>

2) Find the standard deviation from the following frequency distribution.

<table>
<thead>
<tr>
<th>Weight in Kg</th>
<th>20-29</th>
<th>30-39</th>
<th>40-49</th>
<th>50-59</th>
<th>60-69</th>
<th>70-79</th>
</tr>
</thead>
<tbody>
<tr>
<td>No.of Persons</td>
<td>5</td>
<td>12</td>
<td>15</td>
<td>20</td>
<td>18</td>
<td>10</td>
</tr>
</tbody>
</table>
3) Calculate the mean deviation about mean and its coefficient for the following data.

<table>
<thead>
<tr>
<th>Marks</th>
<th>5</th>
<th>15</th>
<th>25</th>
<th>35</th>
<th>45</th>
</tr>
</thead>
<tbody>
<tr>
<td>No. of Students</td>
<td>5</td>
<td>8</td>
<td>15</td>
<td>16</td>
<td>6</td>
</tr>
</tbody>
</table>

4) Find the mean deviation from the median of the following frequency distribution.

<table>
<thead>
<tr>
<th>Variance (x)</th>
<th>10</th>
<th>20</th>
<th>30</th>
<th>40</th>
<th>50</th>
<th>60</th>
</tr>
</thead>
<tbody>
<tr>
<td>Frequency (f)</td>
<td>9</td>
<td>18</td>
<td>25</td>
<td>27</td>
<td>14</td>
<td>7</td>
</tr>
</tbody>
</table>

5) Find the quartile deviation about median from the following data.

<table>
<thead>
<tr>
<th>Wages</th>
<th>5-15</th>
<th>15-25</th>
<th>25-35</th>
<th>35-45</th>
<th>45-55</th>
</tr>
</thead>
<tbody>
<tr>
<td>No. of workers</td>
<td>22</td>
<td>38</td>
<td>46</td>
<td>35</td>
<td>20</td>
</tr>
</tbody>
</table>

6) Find the standard deviation for the following frequency distribution.

<table>
<thead>
<tr>
<th>$x_i$</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
</tr>
</thead>
<tbody>
<tr>
<td>$f_i$</td>
<td>9</td>
<td>29</td>
<td>39</td>
<td>49</td>
<td>59</td>
</tr>
</tbody>
</table>
22. Probability

Marks - 1

1) A die and a coin are tossed simultaneously. The number of possible outcomes is .......
   a) 2  b) 6  c) 12  d) 18

2) A coin is tossed. If it shows a tail, we draw a ball from a bag containing 3 white balls; If it shows head we throw a die. The number of possible outcomes is .......
   a) 9  b) 18  c) 36  d) 54

3) For the experiment of tossing a coin, the number of possible events is ...... 
   a) 4  b) 2  c) 8  d) 16

4) A die is rolled. What is the probability that it shows a prime number ?
   a) \( \frac{2}{3} \)  b) \( \frac{1}{3} \)  c) \( \frac{1}{2} \)  d) \( \frac{1}{6} \)

5) A die is rolled. What is the probability that the outcome is a factor of 100 ?
   a) \( \frac{1}{2} \)  b) \( \frac{2}{3} \)  c) \( \frac{1}{4} \)  d) \( \frac{1}{5} \)

6) A two digit number is selected at random. What is the probability that the sum of its digits is 9 ?
   a) \( \frac{1}{9} \)  b) \( \frac{1}{10} \)  c) \( \frac{2}{5} \)  d) \( \frac{10}{91} \)

7) What is the Probability that a leap year selected at random will contain 53 Mondays ?
   a) \( \frac{1}{7} \)  b) \( \frac{2}{7} \)  c) \( \frac{3}{7} \)  d) \( \frac{53}{366} \)

8) What is the Probability that a non- leap year selected at random will contain 53 Sundays?
   a) \( \frac{2}{7} \)  b) \( \frac{3}{7} \)  c) \( \frac{1}{7} \)  d) \( \frac{53}{365} \)

9) Letters in the word BOMB are rearranged. What is the Probability that two B’s are not together ?
   a) \( \frac{2}{3} \)  b) \( \frac{1}{2} \)  c) \( \frac{3}{4} \)  d) \( \frac{1}{3} \)

10) Two letters are selected at random from 26 English letters. What is the Probability that the selection contains a vowel and a consonant ?
    a) \( \frac{13}{25} \)  b) \( \frac{1}{2} \)  c) \( \frac{21}{65} \)  d) \( \frac{23}{26} \)
Q.2) Solve the following : 2 marks each

1) A coin is tossed repeatedly until tail comes up. Write the sample space for this experiment.

2) If \( P(A) = \frac{1}{4} \), \( P(B) = \frac{2}{5} \) and \( P(A \cup B) = \frac{1}{2} \) then find \( P(A \cap B) \).

3) If the chance of A winning a race is \( \frac{1}{6} \) and that of B is \( \frac{1}{8} \), what is the chance that both will win ?

4) If events A and B are independent then show that \( P(A \cap B) = P(A) \cdot P(B) \).

5) If \( P(A) = 0.3 \) and \( P(B) = 0.4 \) then find \( P(A') + P(B') \).

6) A card is drawn from a pack of 52 cards. What is the probability that it is not a face card?

7) Two dice are rolled. What is the probability that they show the same number ?

8) What is the Probability that a month in a Gregarian Calendar contains 31 days ?

9) A ball is selected from a bag containing 3 red, 4 white and 5 black balls. What is the probability that it is not black ?

10) Two coins are tossed. What is the probability of getting at most one head ?

Q.3) Solve the following : 3 marks each

1) A coin is tossed repeatedly until tail comes up for the first time. What is the probability that we get tail in the second toss ?

2) If \( P(A) = \frac{1}{4} \), \( P(B) = \frac{2}{5} \) and \( P(A \cap B) = \frac{3}{20} \) then find \( P(A' \cup B') \).

3) A card is drawn from a pack of 52 cards. Find the probability of it being a heart or a black card.

4) If \( P(A \cup B) = \frac{5}{6} \), \( P(A \cap B) = \frac{1}{3} \) and \( P(B') = \frac{1}{3} \) then find \( P(A) \).

5) Two dice are rolled. What is the Probability that the score is a prime number ?

6) Three boys and two girls are arranged in straight line for a photograph. What is the probability that girls are not together ?

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7) A man and two women are arranged in straight line for a photograph. What is the probability that women are together?

8) A pair of dice is thrown. If the score is an even number, what is the probability that it is a perfect square?

9) If \( P(A) = 0.6 \), \( P(B) = 0.2 \), \( P(A \cap B) = 0.5 \). Examine whether A and B are independent.

10) A speaks 80% truth, B speaks 60% truth. Find the probability that they contradict each other.

Q.4) Solve the following :  

1) If A and B are independent events then show that events A’ and B’ are also independent.

2) Two week days are selected randomly. What is the probability that selection contains Sunday?

3) \( P(A) = 0.38 \), \( P(A \cup B) = 0.69 \). If events A and B are independent then find \( P(B) \).

4) Each coefficient in the equation \( ax^2 + bx + c = 0 \) is determined by throwing an ordinary die. Find the probability that the equation will have equal roots.

5) From each of three married couples one of the partners is selected at random. Find the probability that the selection consists of two men and a woman.

6) A man is known to speak truth 3 out of 4 times. He throws a die and reports that it is four. Find the probability that it is actually four.

7) A ticket is drawn from a set of 20 tickets numbered from 1 to 20 and kept a side. Then another ticket is drawn. Find the probability that both the tickets show even numbers.

8) An urn contains 10 red, 5 white and 5 black balls. Two balls are drawn at random. Find the probability that they are not of the same colour.

9) A card is drawn from a pack of 52 cards. What is the probability that it is a heart or a queen?

10) Letters in the word EQUATION are arranged in row. Find the probability that all vowels are together.

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