ELECTROSTATICS (PART I)

Electrostatics deals with the study of static electric charges.

It includes the forces between them and the effects produced in the form of electric fields and electric potentials.

ELECTRIC CHARGES

Electric charge is a basic property of elementary particles of which matter is made of. These elementary particles are proton, neutron and electron.

Proton is considered to be positively charged and electron to be negatively charged. Neutron is electrically neutral i.e. it has no charge.

An atomic nucleus is made up of protons and neutrons and hence positively charged. Negatively charged electrons surround the nucleus so as to make an atom electrically neutral.

A body can be charged by three methods as follows:

Charging by friction

When certain dissimilar substances are rubbed against each other, electrons get transferred to other substance making them charged.

The substance receiving electron becomes negatively charged while the other left with equal amount of positive charge.

Charging by conduction

Charging by conduction involves the contact of a charged object to a neutral object.

Hence when an uncharged conductor is brought in contact with a charged conductor, charge is shared between the two conductors and hence the uncharged conductor gets charged.
Charging by induction

An uncharged object is charged when placed very close to a charged conductor without touching.

The nearer end acquires a charge opposite to the charge on the charged conductors and the two bodies attract.

In induction there is no transfer of charges between the charged body and conductor. So when the charged body is moved away from the conductor, the charges in the conductor are free again.

BASIC PROPERTIES OF ELECTRIC CHARGES

Additive Nature of Charge

The total electric charge on an object is equal to the algebraic sum of all charges distributed over object.

While taking algebraic sum the sign (positive or negative) must be taken into account.

Quantization of charge

The charge on an object is in discrete form rather than in a continuous number.

The minimum value of charge is \( e = 1.6 \times 10^{-19} \) C.

The charge on an object must be integral multiples of \( e \). This is known as quantization of charge.

\[ q = \pm ne, \]  where \( n \) is an integer.

Conservation of charge

In any given physical process, charge may get transferred from one part of the system to the another, but total charge in the system remains constant.

In other words, total charge of an isolated system is always conserved.
Forces between charges

Unlike charges attract each other whereas like charges repel each other.

COULOMB’S LAW

Statement

The force of attraction or repulsion between two point charges at rest is directly proportional to the product of magnitude of charges and inversely proportional to the square of the distance between them.

This force acts along the line joining the two charges.

Scalar form of Coulomb’s Law

Let $q_1$ and $q_2$ be two point charges at rest and $r$ be the distance between them.

The magnitude of force $F$ is given by,

$$ F \propto \frac{q_1 q_2}{r^2} $$

$$ F = K \frac{q_1 q_2}{r^2} \quad \text{...(1)} $$

where, $K$ is the constant of proportionality.

Relative Permittivity or Dielectric Constants

The medium affects the force between two charges.

The force between two charges placed in a medium is written as

$$ F_{\text{med}} = \frac{1}{4\pi \varepsilon} \left( \frac{q_1 q_2}{r^2} \right) \quad \text{...(2)} $$

The force between the same two charges placed in vacuum is written as
\[ F_{\text{vac}} = \frac{1}{4\pi \varepsilon_0} \left( \frac{q_1 q_2}{r^2} \right) \quad \text{...}(3) \]

where, \( \varepsilon \) is the permittivity of medium, \( \varepsilon_0 \) is the permittivity of free space or vacuum.

Dividing Eq. (2) by (1),

\[ \frac{F_{\text{med}}}{F_{\text{vac}}} = \frac{\frac{1}{4\pi \varepsilon_0} \left( \frac{q_1 q_2}{r^2} \right)}{\frac{1}{4\pi \varepsilon_0} \left( \frac{q_1 q_2}{r^2} \right)} = \frac{\varepsilon}{\varepsilon_0} \]

The ratio \( \frac{\varepsilon}{\varepsilon_0} \) is the relative permittivity or dielectric constant and is denoted by \( \varepsilon_r \) or \( k \).

\[ k \text{ or } \varepsilon_r = \frac{\varepsilon}{\varepsilon_0} \quad \text{...}(4) \]

The force between two charges placed in a medium of relative permittivity is written as

\[ F = \frac{1}{4\pi \varepsilon_0 k} \left( \frac{q_1 q_2}{r^2} \right) \quad \text{...}(5) \]

**Definition of unit charge**

We know that,

\[ F = \frac{1}{4\pi \varepsilon_0} \left( \frac{q_1 q_2}{r^2} \right) \]

\[ \varepsilon_0 = 8.85 \times 10^{-12} \text{ C}^2/\text{Nm}^2 \text{ (constant), if } q_1 = q_2 = 1 \text{C and } r = 1 \text{m} \]

then, \( F \approx 9 \times 10^9 \text{N} \).

One coulomb is the amount of charge which, when placed at a distance of one metre from the another charge of same magnitude in vacuum, experiences a force of \( 9 \times 10^9 \text{N} \).

Conversions of unit of charge:

\[ 1\mu\text{C} = 10^{-6}\text{C} \quad 1\text{nC} = 10^{-9}\text{C} \quad 1\text{pC} = 10^{-12}\text{C} \]

**Coulomb's law in vector Form**

Let \( q_1 \) and \( q_2 \) be two similar point charges situated at points A and B.

Let \( \vec{r}_{12} \) be the position vector of \( q_1 \) w.r.t. \( q_2 \) and

\[ \vec{r}_{21} \text{ be the position vector of } q_2 \text{ w.r.t. } q_1 \]
\( \vec{F}_{12} \) denotes force exerted on \( q_1 \) by \( q_2 \) is given by

\[
\vec{F}_{12} = \frac{1}{4\pi\varepsilon_0} \left( \frac{q_1 q_2}{r_{12}^2} \right) \hat{r}_{12}
\]...

\( \vec{F}_{21} \) similarly exerted on \( q_2 \) by \( q_1 \) is given by

\[
\vec{F}_{21} = \frac{1}{4\pi\varepsilon_0} \left( \frac{q_1 q_2}{r_{21}^2} \right) \hat{r}_{21}
\]...

The unit vectors \( \hat{r}_{12} = -\hat{r}_{21} \)

\( \therefore \vec{F}_{12} = -\vec{F}_{21} \)

**PRINCIPLE OF SUPERPOSITION**

**Statement**

The principle of superposition states that when a number of charges are interacting, the resultant force on a particular charge is given by the vector sum of the forces exerted by individual charges.

**Expression**

Consider a system contains number of point charges \( q_1, q_2, q_3, \ldots \) kept at points \( A_1, A_2, A_3, \ldots \) respectively.

The total force \( \vec{F}_1 \) on charge \( q_1 \) due to rest of the charges is

\[
\vec{F}_1 = \vec{F}_{12} + \vec{F}_{13} + \vec{F}_{14} + \ldots \ldots
\]

\[
\vec{F}_1 = \frac{1}{4\pi\varepsilon_0} \left[ \left( \frac{q_1 q_2}{r_{12}^2} \right) \hat{r}_{12} + \left( \frac{q_1 q_3}{r_{13}^2} \right) \hat{r}_{13} + \left( \frac{q_1 q_4}{r_{14}^2} \right) \hat{r}_{14} + \ldots \right]
\]

Where, \( \hat{r}_{12}, \hat{r}_{13}, \hat{r}_{14} \) be the unit vectors directed to \( q_1 \) from \( q_2, q_3, \ldots \) respectively.

Similarly total forces on charge \( q_2, q_3, \ldots \) due to other charges can be found using the principle.
ELECTRIC FIELD (ELECTRIC INTENSITY)

Electric field is defined as force experienced per unit charge.

\[ E = \frac{F}{q_o} \]  

...(8)

In vector form, \( \vec{E} = \frac{\vec{F}}{q_o} \)

The unit of electric intensity is N/C or V/m and dimension is [L^1M^1T^{-3}I^{-1}]

Electric Field Intensity due to a point charge

Consider charge \( q \) is placed in the medium of dielectric constant \( k \). To determine electric intensity at a distance of \( r \) at point \( P \), let test charge \( q \) is placed at that point. Let \( \hat{r} \) be the unit vector in the direction of intensity.

\[ \vec{F} = \frac{1}{4\pi\varepsilon_0 k} \left( \frac{q \ q_o}{r^2} \right) \hat{r} \]

By the definition of electric intensity,

\[ \vec{E} = \frac{\vec{F}}{q_o} \]

\[ \therefore \vec{E} = \frac{1}{4\pi\varepsilon_0 k} \left( \frac{q}{r^2} \right) \hat{r} \] . In magnitude \( E = \frac{1}{4\pi\varepsilon_0 k} \left( \frac{q}{r^2} \right) \)  

...(9)

Practical way of calculating electric field

A pair of parallel plates is connected as shown in fig below.

A Potential difference \( V \) is applied between two parallel plates separated by distance \( d \). The electric field between them is director from plate A to plate B as shown.
A Charge \(+q\) placed between the plates experience a force \(F\) due to the electric field. If we have to move the charge against the direction of field, i.e., towards the positive plate, we have to do some work on it. If we move the charge \(+q\) from the negative plate \(B\) to the positive plate \(A\), the work done against the field is \(W = Fd\); where \(d\) is the separation between the plates. The potential difference \(V\) between the two plates is given by

\[
W = Vq \quad \text{but} \quad W = Fd
\]

\[
\therefore Vq = Fd \quad \therefore \frac{F}{q} = \frac{V}{d} = E
\]

\[
\therefore E = \frac{V}{d} \quad \text{...(10)}
\]

**Electric Lines of Force**

The path along which the unit positive charge moves is called a line of force.

A line of force is defined as a curve such that the tangent at any point to this curve gives the direction of electric field at that point.

The density of field lines indicates the strength of electric field at the given point in space.

**Uniform electric field:** A uniform electric field is a field whose magnitude and direction is same at all points.

**Non uniform electric field:** A field whose magnitude and direction is not the same at all points.
Characteristics of electric lines of force:

1) The line of force originates from a positively charged object and end on a negatively charged object.
2) The lines of force neither intersect nor meet each other, as it will mean that electric field has two directions at a single point.
3) The lines of force leave or terminate on a conductor normally.
4) The lines of force do not pass through conductor i.e. electric field inside a conductor is always zero, but they pass through insulators.
5) Magnitude of the electric field intensity is proportional to the number of lines force per unit area of the surface held perpendicular to the field.
6) Electric lines of force are crowded in a region where electric intensity is large.
7) Electric lines of force are widely separated from each other in a region where electric intensity is small.
8) The lines of force of uniform electric field are parallel to each other and are equally spaced.

Representation of lines of force for the following charges

Positive charge  negative charge  Unlike charges  Like charges