Section A

Q.No.1 Select and write the correct answer :

i) If $A = \{2, 3, 4, 5, 6\}$, then which of the following is ‘not’ true?
   (a) $\exists x \in A$ such that $x + 3 = 8$  
   (b) $\exists x \in A$ such that $x + 2 < 8$
   (c) $\exists x \in A$ such that $x + 2 < 9$  
   (d) $\exists x \in A$ such that $x + 6 \geq 9$

ii) If the vector $\vec{i} - 2\vec{j} + 7\vec{k}$, $a\vec{i} - 5\vec{j} + 3\vec{k}$ and $5\vec{i} - 9\vec{j} + 4\vec{k}$ are coplanar, then the value of $a$ is
   (a) 3  
   (b) -3  
   (c) 2  
   (d) -2

iii) The vector equation of a line passes through the point with position vector $4\vec{i} - \vec{j} + 2\vec{k}$ and is in the direction of $-2\vec{i} + \vec{j} + \vec{k}$ is
   (a) $\vec{r} = (4\vec{i} - \vec{j} + 2\vec{k}) + \lambda (2\vec{i} + \vec{j} + \vec{k})$
   (b) $\vec{r} = (4\vec{i} - \vec{j} + 2\vec{k}) + \lambda (-2\vec{i} + \vec{j} + \vec{k})$
   (c) $\vec{r} = (4\vec{i} - \vec{j} + 2\vec{k}) + \lambda (2\vec{i} - 2\vec{j} + \vec{k})$
   (d) $\vec{r} = (2\vec{i} - 2\vec{j} + \vec{k}) + \lambda (4\vec{i} - \vec{j} + 2\vec{k})$

iv) The acute angle between the two planes $x + y + 2z = 3$ and $3x - 2y + 2z = 7$ is
   (a) $\sin^{-1}\left(\frac{5}{\sqrt{102}}\right)$  
   (b) $\cos^{-1}\left(\frac{5}{\sqrt{102}}\right)$  
   (c) $\sin^{-1}\left(\frac{15}{\sqrt{102}}\right)$  
   (d) $\cos^{-1}\left(\frac{15}{\sqrt{102}}\right)$

v) If $\sec\left(\frac{x + y}{x - y}\right) = a^2$, then $\frac{d^2y}{dx^2} =$ ......  
   (a) $y$  
   (b) $x$  
   (c) $\frac{y}{x}$  
   (d) 0

vi) Equation of tangent to the curve $y = 3x^2 - x + 1$ at $P(1, 3)$ is ........
   (a) $5x - y = 2$  
   (b) $x + 5y = 16$  
   (c) $5x = y$
vii) The area of the ellipse \( \frac{x^2}{a^2} + \frac{y^2}{b^2} = 1 \) is
(a) \( ab \) sq.units (b) \( \pi a \) sq.units (c) \( \pi ab \) sq.units (d) \( \pi b \) sq.units

viii) If \( X \sim B(n, p) \) and \( E(X) = 12 \), \( \text{Var}(X) = 4 \), then the value of \( n \) is .............
(a) 3 (b) 48 (c) 18 (d) 36

Q.No.2 Answer the following:

i) Find the principal solution of \( \cot x = -\sqrt{3} \)

ii) Find \( k \), if the equation \( 2x^2 + xy - y^2 + x + 4y - 3 = 0 \) represents a pair of straight lines.

iii) Find the area of the region bounded by the curve \( y = \sin x \), the line \( x = -\frac{\pi}{2}, x = \frac{\pi}{2} \) and \( X - \text{axis} \).

iv) The probability that a bomb will hit a target is 0.8. Find the probability that out of 10 bombs dropped, exactly 4 will hit the target.

Section B

Attempt any Eight:

Q.No.3 Write the dual of the following statement:

i) \( \sim p \land (q \lor c) \) ii) "Shweta is a doctor or Seema is a teacher."

Q.No.4 If \( A = \begin{bmatrix} 2 & -3 \\ 3 & 5 \end{bmatrix} \) then find \( A^{-1} \) by adjoint method.

Q.No.5 In \( \triangle ABC \), prove that \( ac \cos B - bc \cos A = a^2 - b^2 \)

Q.No.6 Find the value of \( 'k', \) if sum of the slopes of the lines represented by \( x^2 + kxy - 3y^2 = 0 \) is twice their product.

Q.No.7 If \( \frac{c}{a} = \frac{3a - 2b}{c}, \) then prove that \( \begin{bmatrix} a \\ b \\ c \end{bmatrix} = 0 \).

Q.No.8 Find the direction ratios of a vector perpendicular to the two lines whose direction ratio are \( -2, 1, -1 \) and \( -3, -4, 1 \).

Q.No.9 If \( y = x^5 \), find \( \frac{dy}{dx} \)

Q.No.10 Verify Rolle’s Theorem for the function \( f(x) = x^2 - 5x + 9 \) on \([1, 4]\)

Q.No.11 Evaluate: \( \int e^{x}(1+x) \cos^2(xe^{x}) \, dx \)

Q.No.12 Find the area of the region bounded by the parabola \( y^2 = 16x \) and the line \( x = 3 \).

Q.No.13 Let the p.m.f. of a random variable \( X \) be
\[ P(x) = \frac{3-x}{10}, \quad \text{for } x = -1, 0, 1, 2 \]
\[ = 0, \quad \text{otherwise} \]
Then find \( E(X) \).

Q.No.14 The probability that a certain kind of component will survive a check test is 0.5. Find the probability that exactly two of the next four component tested will survive.

Section C
Q.No.15  Show that \( \sin^{-1}\left(\frac{5}{13}\right) + \cos^{-1}\left(\frac{3}{5}\right) = \tan^{-1}\left(\frac{63}{16}\right) \)

Q.No.16  If the angle between the lines represented by \( ax^2 + 2hxy + by^2 = 0 \) is equal to the angle between the lines \( 2x^2 - 5xy + 3y^2 = 0 \), then show that \( 100(h^2 - ab) = (a + b)^2 \)

Q.No.17  If \( A(a) \) and \( B(b) \) be any two points in the space \( R(r) \) be a point on the line segment \( AB \) dividing it internally in the ratio \( m : n \), then prove that \( r = \frac{mb + na}{m+n} \)

Q.No.18  If a line makes angles \( \alpha, \beta, \gamma \) with co-ordinate axes, prove that \( \cos 2\alpha + \cos 2\beta + \cos 2\gamma + 1 = 0 \)

Q.No.19  The equation of a line is \( 2x - 2 = 3y + 1 = 6z - 2 \), find its direction ratios and also find the vector equation of the line.

Q.No.20  Show that the points \( (1, -1, 3) \) and \( (3, 4, 3) \) are equidistance from the plane \( 5x + 2y - 7z + 8 = 0 \).

Q.No.21  Discuss the continuity of the function
\[
f(x) = \frac{\log(2 + x) - \log(2 - x)}{\tan x}, \quad for \ x \neq 0 \\
\]
\[
= 1 , \quad for \ x = 0 \\
\]

Q.No.22  If \( u \) and \( v \) are differentiable function of \( x \), then prove that:
\[
\int uv \, dx = u \int v \, dx - \int \left[ \frac{du}{dx} \int v \, dx \right] \, dx
\]

Q.No.23  Find the area of the sector of the circle bounded by \( x^2 + y^2 = 16 \) and the line \( y = x \) in the first quadrant.

Q.No.24  Solve: \( \frac{dy}{dx} = \cos(x + y) \)

Q.No.25  Discuss the continuity of the following function, at \( x = 0 \).
\[
f(x) = \frac{x}{|x|}, \quad for \ x \neq 0 \\
\]
\[
= 1 , \quad for \ x = 0 \\
\]

Q.No.26  For the following probability density function (p.d.f.) of \( X \), find:

\( i) P(X < 1), \quad ii) P(|X| < 1) \) if
\[
f(x) = \frac{x^2}{18}, \quad -3 < x < 3 \\
\]
\[
= 0 , \quad otherwise \\
\]

Section D
Q.No.27 Find the symbolic form of the given switching circuit. Construct its switching table and interpret your result.

Q.No.28 Express the following function in matrix form and solve them by the method of reduction: \( x - y + z = 4 \), \( 2x + y - 3z = 0 \) and \( x + y + z = 2 \).

Q.No.29 Using the Sine rule, prove that Cosine rule.

Q.No.30 Maximize: \( Z = 6x + 4y \)
Subject to \( x \leq 2 \), \( x + y \leq 3 \), \( -2x + y \leq 1 \), \( x \geq 0 \), \( y \geq 0 \).
Also find the maximum value of \( Z \).

Q.No.31 If the function is continuous in the interval \([-2, 2]\), find the values of \( a \) and \( b \)
\[
f(x) = \begin{cases} 
-\sin \frac{ax}{x} - 2, & \text{for } -2 < x < 0 \\
2x + 1, & \text{for } 0 < x < 1 \\
2b\sqrt{x^2 + 3} - 1, & \text{for } 1 < x < 2
\end{cases}
\]

Q.No.32 \( f(x) = (x - 1)(x - 2)(x - 3) \), \( x \in [0, 4] \), find ‘c’ if LMVT can be applied.

Q.No.33 Prove that:\[
\int \frac{dx}{\sqrt{x^2 + a^2}} = \log \left| x + \sqrt{x^2 + a^2} \right| + c
\]

Q.No.34 A body is heated at 110\(^\circ\)C and placed in air at 10\(^\circ\)C. After 1 hour its temperature is 60\(^\circ\)C. How much additional time is required for it to cool to 35\(^\circ\)C?