SOLUTIONS

LEVEL1

1. Divisibility rule of 11 is that add up the alternates starting from the last then subtract the remaining.
   2+6+1-4-5-3= -3
   So, 3 is the smallest number that should be added.

2. To check 48 divisibility, the number must be divisible by 8 & 6.
   For 8, last 3 digits must be easily divisible by 8. So, 25y must be multiple of 8.It means y must be equal to 6(256 is a multiple of 8)
   For 6, the number must be divisible by 2 & 3.
   Last digit is even because y=6, so number is divisible by 2.
   Sum of all digits must be divisible by 3 & sum=5+6+6+8+x+2+5+6=38+x
   For least value , x must be equal to 1
   (x+y)=1+6=7

3.

   $$N = 16 + 2k(2k+2)(2k+4)(2k+6) = 16[1 + k(k+1)(k+2)(k+3)],$$
   $$= [4(k^2 + 3k + 1)]^2$$

   hence, N is a perfect square, and is clearly divisible by 16.

   When k is odd, $k^2 + 3k + 1$ is odd, hence 16 is the highest factor containing 2 by which N is divisible (thus eliminating 32 and 64)

   You may verify this for $k = 1$ - The first four of these 2, 4, 6, 8 product with 16 added gives 400 which is neither divisible by 32 nor 64

4. Prime numbers greater than 5 can be represent in the form of 6k$\pm$1.
   So,cube of (6k+1) & (6k-1) leave the remainder of 1 & 5, while cubes of 2,3, & 5 leave the remainder of 2,3 & 5 respectively.
   sum=1+5+2+3=11

5. Two numbers 698 and 450 when divide by certain divisor leave remainder of 9 and 8 respectively.
   H.CF[(698-9),(450-8)]=H.C.F(689,442)=13

6.

68488 and 67516 on dividing with N leaves same remainder.
N is a three digit number.

$68488 - 67516 = 972$. On dividing by 972, 68488 and 67516 leaves the same remainder.

Now, three digit factors of 972 is 486, 324, 243, 162, 108.

So there are six values of N.

7.

In case of division by 7 and 5 the remainder is **divisor-4**

LCM (5,7)=35

So this number=LCM-4=35–4=31

Thus number N=35*p-4……………(1) (p is any integer)

Now the number on division by 6 leaves remainder 5

Thus N=6q+5……………..(2)

From (1) and (2)

6q+5=35p-4

35p=6q+9

35p=3*(2q+3)

OR p*35=3*(2q+3)

On comparison we get that

35 is not divisible by 3 So p=3 and 2q+3=35

So q=32/2=16

From(2) using q=16 we get

N=6*16+5=101

101 is the smallest 3 digit required number

8. **Incorrect options**
number=L.C.M(9,11)K - 4=99k-4
Smallest 4 digit number would be at k=1089-4=1085

9. number=l.c.m(7,9,11)k +5=693k + 5
Largest number will be at k=14

10. Number of soldiers= L.C.M(8,15,20)K+1=120K+1
Number of soldiers=L.C.M(9,13)z+4=117z+4
At k=1,z=1, number of soldiers =121

11. dimensions=870 X 638=$58^2 X 15 X 11$
Least number of tiles will be 15*11=165(of dimensions,58x58)

12.

Four blocks of chocolates of weights $6\frac{1}{8}$kg, $10\frac{1}{2}$kg, $8\frac{3}{4}$kg and $3\frac{15}{16}$kg

H.C.F$[\frac{49}{8},\frac{21}{2},\frac{35}{4},\frac{63}{16}] = \frac{7}{16}$

Least number of pieces which can be distributed=$\frac{\frac{49}{8}+\frac{21}{2}+\frac{35}{4}+\frac{63}{16}}{7/16}$=67

13. H.C.F=6, Product=4320
Let the number be 6a,6b (a & b are coprimes)
L.C.M=6ab
H.C.F * L.C.M=4320
36ab=4320, ab=120
(a,b)=(3,40),(5,24),(8,15),
Only 3 pairs will satisfy above conditions

14. H.C.F=7, L.C.M=196
Let the numbers be 7a & 7b (a>b)
L.C.M=7ab=196,
ab=28…..(1)
difference=7(a-b)=21
$\Rightarrow a - b = 3$….(2)
From (1) & (2)

a=7,b=4
Largest number=7a=49

15. $x = 0.7\underline{54}$
Multiply both sides by 10
$\Rightarrow 10x = 7.\underline{54}$…..(1)
Multiply both sides by 100
⇒1000x=754.$\underline{54}$…..(2)
Subtract equation 1 from 2
990x=747
x=$\frac{747}{990}$

x=0.$69\underline{2}$
Multiply both sides by 100
$\Rightarrow 100x = 69.\underline{2}$…(3)
Multiply both sides by 10
$\Rightarrow 1000x = 692.\underline{2}$…..(4)
Subtract 3 equation from 4
900x=623
x=$\frac{623}{900}$
$0.7\overline{54} + 0.69\overline{2}$
=$\frac{747}{990} + \frac{623}{900}$=$\frac{14323}{9900}$

16. INCORRECT Q

17.
$$5+6\times\frac{1}{3}\text{of}9-\left\{4-\frac{5}{8}+2\frac{7}{8}+\frac{3}{4}\right\}$$
=5+6X$\frac{1}{3}X$9-{4-$\frac{5}{8} + \frac{23}{8} + \frac{3}{4}$}
=$5 + 18 - \{7\}$
=16

18. $75^3 - 50^3 - 25^3 = 5^3k$
In all cubes, 5 comes.So, the resultant must be the multiple of 125.
For 125 divisibility, last three digits must be divisible by 125.
Only,281250 satisfy this.

19. Square root of 123456...a……..321 is always equal to11….atimes
Square root of 12345654321 is 111111

Level 2

1. On dividing 546789 by 7, we get 5 as remainder.So, 5 is the number must be subtracted

2. 64A3B6C is divisible by 360.
   So, C must be 0.
   360 is also divisible by 8.So, check divisiblity test on 64A3B60.
   B60 must be divisible by 8, B=1,3,5,7,9
   The number is divisible by 3 & 9 both.
   sum=19+A+B
   B=1,A=7
   B=3,A=5
   B=5,A=3
   B=7,A=1
   B=9,A=8

3. $X^4 + 2X^3 + 3X^2 + 4X + 36$
   X must be a factor of 36. 36 has nine factors

4. abcde-acdbe=10000a+1000b+100c+10d+e-10000a-1000c-100d-10b-e
   =990b-900c-90d=90(11b-10c-d)

5. All prime numbers greater than 5 can be represent in form of $6k\pm1$
   Squares of 2,3 &5 leaves remainders of 4 , 3 & 1 respectively, while others in form of $6k\pm1$ when squaring leaves 1 as remainder.
   sum=1+4+3=8

6. **Error in q**

7. H.C.F[(971-3),(852-5)]=121

8. NUMBER=L.C.M(5,6,7)k-3=210k-3…(1)
   NUMBER=47z-6 …(2)
   On comparing 1 & 2
   210k-3=47z-6
   210k+3=47z
   k=2 satisfy
   Number=210x2-3=417

9. BY chinese remainder theorem
   number=9x+6, numbers are 6,15,24,**33**,42,51,60,69,78,,,
   number=7y+5, numbers are 5,12,19,26,**33**,40,47,,,
   number=l.c.m(7,9)m+least common(=33)
   =63m+33
   63m+33<1000
   63m<967
   m<15.34

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m=15

number=63x15+33=978

10. Take the L.C.M of the times which will be the time at which all will meet at the starting point.

L.C.M(360,200,360,450)=1800

11. p, q and r be distinct positive integers that are odd

a)$pq^2r^2 = (odd)(odd)^2(odd)^2 = odd$

b)$(p+q)^2r^2 = (odd+odd)^2(odd)^2 = evenXodd = even$

c)$(p-q+r)^2(q+r) = (even+odd)^2(odd+odd) = even$

d)$(2n-1)(2n+1)(2n+3) = (4n^2-1)(2n+3) = 8n^3+12n^2-2n-3$

n=1, remainder=3

n=2, remainder=1

So,not sure about the remainder

12. Product of first three prime numbers=2431=11*13*17

Product of last three prime numbers=4199=13*17*19

Largest =19

13. Sum of n natural numbers=n(n+1)/2

14. $x^2-y^2=255$

$$(x-y)(x+y)=5*51$$

x-y=5, x+y=51

$$\Rightarrow x=28, y=23$$

$$(x-y)(x+y)=15*17$$

x-y=15,x+y=17

$$\Rightarrow x=16, y=1$$

$$(x-y)(x+y)=1*255$$

⇒x-y=1,x+y=255

$$\Rightarrow x=128, y=127$$

$$(x-y)(x+y)=3*85$$

$\Rightarrow x-y=$3,x+y=85

$\Rightarrow x=44, y=41$

15. N is the least integer which leaves remainder of 5, 6, 7 when divided by the divisors 7, 8, 9 Respectively.

N=L.C.M(7,8,9)k -2=504k -2

Least=502

502 leaves 9 as remainder when divided by 17.