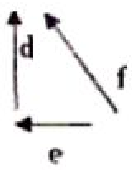
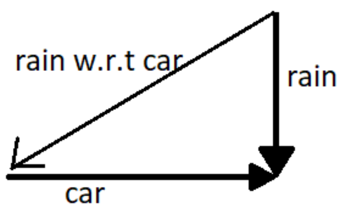
Measurement, Units, Dimensions & Vectors (X^{th})- Exercise Solutions

LEVEL 1:

1. (c) Light year is the unit of length.
2. (d) Density is a derived quantity while others are fundamental (base) quantities.
3. (d) 1 m^3 = 10^6 cm^3
   1000 m^3 = 10^9 cm^3
4. (c) The 7 fundamental units are metre, kilogram, second, candela, ampere, mole, kelvin.
5. (c) SI unit of mass is kg and volume is m^3. Thus, SI unit of density is kg m^3
6. (c) 1 g = 10^{-3} kg; 1 cm^3 = 10^{-6} m^3
   $1\frac{g}{cm^3} = 1000\frac{kg}{m^3}$
7. (c) Power = $\frac{work}{time} = \frac{mass\ x\ acceleration\ x\ displacement}{time}$
   In terms of base units, unit of power is $\frac{kg\ m\ s^{-2}\ m}{s}$, i.e. $kg\ m^2 s^{-3}$
8. (d) Pressure = $\frac{force}{area} = \frac{mass\ x\ acceleration}{area}$
   In terms of base units, unit of pressure is $\frac{kg\ m\ s^{-2}}{m^2}$, i.e. $kg\ m^{-1} s^{-2}$
9. (d) As the equation is dimensionally correct, Dimensions of μ = Dimensions of $\frac{B}{\lambda^2}$
   Then, $M^0L^0T^0 = \frac{B}{M^0L^2T^0}$
   Dimensions of B = $M^0L^2T^0$ which is same as area.
10. (b) $1\frac{km}{hr} = \frac{1000}{3600}\frac{m}{s} = \frac{5}{18}\frac{m}{s}$
11. (a) 1m = 1000 mm; 1m = 100 cm; 1m = 10 dm
    Thus, 7.60 m = 7600 mm; 7.60 m = 760 cm; 7.60 m = 76.0 dm
12. (d) Impulse is a vector quantity.
13. (d) Magnitude of resultant lies in the interval (a-b) ≤ R ≤ (a+b)
14. (d) $F^2 = F_x^2 + F_y^2$; where F_x and F_y are the rectangular components of F
    Then, $F_y = \sqrt{120^2 - (60\sqrt{3})^2} = \sqrt{3600}$ = 60 N
15. (c) Sum of the vectors = $\widehat{8\imath} - 6\hat{\jmath}$
    Magnitude = $\sqrt{8^2 + 6^2}$ = 10
16. (d) By triangle law of vector addition



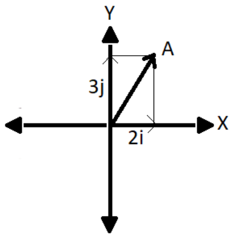
17. (a) Electric current is a scalar quantity.
18. (d) 10, 10, 10 can be zero when all 3 vectors are inclined at 120^0 to each other.
    10, 10, 20 can be zero when two vectors of 10 units each in same direction are arranged parallel and opposite to vector of 20 unit.
19. (c) Magnitude of resultant lies in the interval (a-b) ≤ R ≤ (a+b)
20. (a)



21. (c) $F^2 = F_x^2 + F_y^2$; where F_x and F_y are the rectangular components of F

Then, $F_y = \sqrt{20^2 - (12)^2} = \sqrt{256} = 16$ N

22. (a)

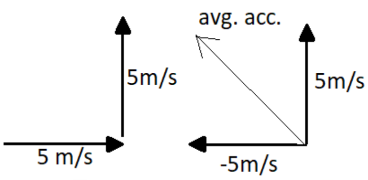


23. (a) $\vec{A} = 3\vec{B}$

24. (d) Resultant $R = \sqrt{A^2 + B^2 + 2AB\cos\theta}$; $(F\sqrt{10})^2 = (2F)^2 + (\sqrt{2}F)^2 + 2(2F)(\sqrt{2}F)\cos\theta$

Then, $\cos\theta = \frac{10-4-2}{4\sqrt{2}} = \frac{1}{\sqrt{2}}$; $\theta = 45^0$

25. (c) Average acceleration = $\frac{change\ in\ velocity}{time} = \frac{\sqrt{5^2+5^2}}{10} = \frac{1}{\sqrt{2}}$



LEVEL 2:

1. (b) Dimensions of F = $[M^1L^1T^{-2}]$

Dimensions of $\sqrt{x}$ = $[M^0L^{\frac{1}{2}}T^0]$

Dimensions of t^2 = $[M^0L^0T^2]$

As the equation is dimensionally correct,

Dimensions of a = $\frac{[M^1L^1T^{-2}]}{\left[M^0L^{\frac{1}{2}}T^0\right]} = \left[M^1L^{\frac{1}{2}}T^{-2}\right]$

Dimensions of b = $\frac{[M^1L^1T^{-2}]}{[M^0L^0T^2]} = [M^1L^1T^{-4}]$

Dimensions of a/b = $\frac{\left[M^1L^{\frac{1}{2}}T^{-2}\right]}{[M^1L^1T^{-4}]} = [M^0L^{\frac{-1}{2}}T^2]$

2. (b) $1\ kg\ m\ s^{-2}$ = 1 N

$1\ g\ cm\ s^{-2}$ = 10^5 N

1 (10 g) (10 cm) $(0.1s)^{-1}$ = $\frac{0.1}{10x10}$ x 10^5 = 100 N

3. (b) Using dimensional analysis,

Dimensions of a = $\frac{[M^0L^1T^{-1}]}{[M^0L^0T^1]} = [M^0L^1T^{-2}]$

Dimensions of b = $[M^0L^1T^{-1}][M^0L^0T^1] = [M^0L^1T^0]$

Dimensions of c = $[M^0L^0T^1]$

4. (c) Force = $\frac{(mass)(length)}{time^2}$; time = $\sqrt{\frac{(mass)(length)}{force}}$

Dimensions of time = $M^{\frac{1}{2}}L^{\frac{1}{2}}F^{\frac{-1}{2}}$

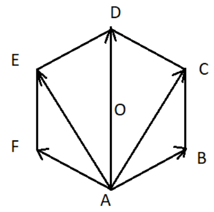
5. (b)

6. (c) Using dimensional analysis, as the physical quantities have different dimensions, (P – Q) cannot be defined.

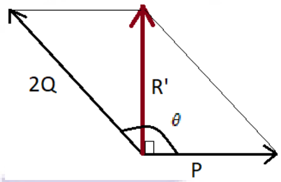
7. (a) SI unit of G is $N\ m^2 kg^{-2}$

CGS unit of G is $dyne\ cm^2 g^{-2}$
1 N = 10^5 dyne; 1 m = 10^2 cm; 1 kg = 10^3 g
Required ratio = $10^{5+4-6} = 10^3$

8. (c) $\overrightarrow{AB} + \overrightarrow{AC} + \overrightarrow{AD} + \overrightarrow{AE} + \overrightarrow{AF} = \overrightarrow{ED} + \overrightarrow{AC} + \overrightarrow{AD} + \overrightarrow{AE} + \overrightarrow{CD}$
$= \overrightarrow{AC} + \overrightarrow{CD} + \overrightarrow{AE} + \overrightarrow{ED} + \overrightarrow{AD}$
$= \overrightarrow{AD} + \overrightarrow{AD} + \overrightarrow{AD}$
$= 3\overrightarrow{AD}$
$= 3(2\overrightarrow{AO})$
$= 6\overrightarrow{AO}$

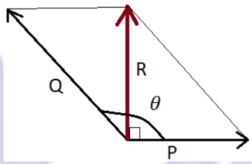


9. (c) In the given figure, $\cos(180 - \theta) = \frac{P}{2Q}$; $P = -2Q\cos\theta$
$R = \sqrt{P^2 + Q^2 + 2PQ\cos\theta}$
$= \sqrt{(-2Q\cos\theta)^2 + Q^2 + 2(-2Q\cos\theta)Q\cos\theta}$
$= \sqrt{4Q^2\cos^2\theta + Q^2 - 4Q^2\cos^2\theta}$
$= Q$

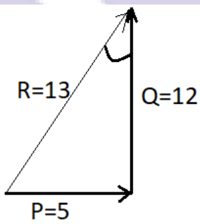


10. (c) For change in velocity, angle between the two vectors will be (180 – 60) i.e. 120^0
$R = \sqrt{V^2 + V^2 + 2VV\cos 120} = \sqrt{2V^2 + 2V^2(-\frac{1}{2})} = V$

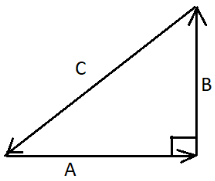
11. (c) Let the angle be θ
In the given figure, $\cos(180 - \theta) = \frac{P}{2Q}$; $P = -Q\cos\theta$
But $P = \frac{Q}{2}$; $\frac{Q}{2} = -Q\cos\theta$; $\cos\theta = -\frac{1}{2}$; $\theta = 120^0$



12. (a) Required angle = $\cos^{-1}(\frac{12}{13})$



13. (d) Since, $|\vec{A}| = |\vec{B}|$ and $|\vec{C}| = \sqrt{2}|\vec{A}|$;
Angle between $|\vec{A}|$ and $|\vec{B}| = 90^0$;
Angle between $|\vec{A}|$ and $|\vec{C}| = 135^0$;
Angle between $|\vec{B}|$ and $|\vec{C}| = 135^0$;



14. (b) $R = \sqrt{P^2 + Q^2 + 2PQ\cos\theta}$
$1 = \sqrt{1^2 + 1^2 + 2(1)(1)\cos\theta}$
$\cos\theta = -\frac{1}{2}$
For difference of two vectors, $R' = \sqrt{P^2 + Q^2 - 2PQ\cos\theta} = \sqrt{1^2 + 1^2 - 2(1)(1)(-\frac{1}{2})} = \sqrt{3}$

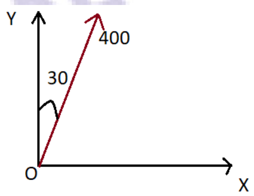
15. (a) Magnitude of change in momentum $= m\sqrt{v^2 + v^2 + 2(v)(v)\cos 2\theta} = mv\sqrt{2(1 + \cos 2\theta)}$
$= mv\sqrt{2\,(2\cos^2\theta)} = 2mv\cos\theta$

SUBJECTIVE QUESTIONS:

3. a) SI unit of energy is J or $kg\ m^2 s^{-2}$
Dimensions of energy = $[M^1 L^2 T^{-2}]$
b) SI unit of Relative density is unitless
Dimensions of relative density = $[M^0 L^0 T^0]$
c) SI unit of Power is W or $kg\ m^2 s^{-3}$

Dimensions of power = $[M^1L^2T^{-3}]$

4. SI unit of K_e is $N\ m^2C^{-2}$
5. Using dimensional analysis,
   Units of x = Units of bt^2
   Unit of b is ms^{-2}
6. Dimensions of length = $[V^1F^0T^1]$
   Power = force x velocity; Dimensions of power = $[V^1F^1T^0]$
7. Dimensions of P = $[M^1L^1T^{-2}]$
   Dimensions of Q = $[M^0L^1T^{-1}]$
   Dimensions of R = $[M^1L^2T^{-3}]$
   But, $P^xQ^yR^z$ is dimensionless
   Thus, $[M^1L^1T^{-2}]^x[M^0L^1T^{-1}]^y[M^1L^2T^{-3}]^z = [M^0L^0T^0]$
   Therefore, $[M^{x+z}L^{x+y+2z}T^{-2x-y-3z}] = [M^0L^0T^0]$
   x + z = 0; x + y + 2z = 0; -2x – y – 3z = 0
   solving them we get,
   x = 1; y = 1; z = -1 as x : y : z = 1 : 1 : -1
8. Dimensions of S = $[M^0L^1T^0]$
   Dimensions of u = $[M^0L^1T^{-1}]$
   As dimensions of S $\neq$ dimensions of u
   The equation is not dimesionally correct.
10. Dimensions of force = $[M^1L^1T^{-2}]$
    Dimensions of density = $[M^1L^{-3}T^0]$
    Dimensions of X = $[M^1L^1T^{-2}][M^1L^{-3}T^0] = [M^2L^{-2}T^{-2}]$
11. x Ns = x $(kg\ m\ s^{-2})$(s) = x $kg\ m\ s^{-1}$
    $1\ g\ cm\ s^{-1} = (10^{-3})(10^{-2})kg\ m\ s^{-1}$
    x = 10^{-5}
12. Power = 1200 MW = 1200 x 10^6 W = 12 x $10^8 kg\ m^2s^{-3}$ = 12 x 10^8 x 10^4 x $(3600)^3 kg\ cm^2hr^{-3}$
    = 559872 x $10^{18} kg\ cm^2hr^{-3}$
    = $\frac{(559872)(10^{18})(2^3)}{(20)(30^2)}(20kg)(30cm)^2(2hr)^{-3}$
    = 248832 x $10^{15}(20kg)(30cm)^2(2hr)^{-3}$
13. Horizontal component = 400 sin 30 = 200 kmh^{-1}
    Vertical component = 400 cos 30 = $200\sqrt{3}kmh^{-1}$



14. Let the two forces be P and Q
    P = $\frac{3}{5}Q$; R = 28 N
    R = $\sqrt{P^2 + Q^2 + 2PQ\cos\theta}$
    28 = $\sqrt{(\frac{3}{5}Q)^2 + Q^2 + 2(\frac{3}{5}Q)(Q)\cos 60}$
    784 = $\frac{9}{25}Q^2 + Q^2 + \frac{3}{5}Q^2$
    19600 = 49 Q^2
    Q = 20 N
    P = 12 N
15. P = Q = R = 1(let)
    R = $\sqrt{P^2 + Q^2 + 2PQ\cos\theta}$
    1 = $\sqrt{1^2 + 1^2 + 2(1)(1)\cos\theta}$
    $\cos\theta = -\frac{1}{2}$
    Thus, $\theta = 120^0$