PART (A) : PHYSICS

SECTION-I : (SINGLE ANSWER CORRECT TYPE)

This section contains 20 multiple choice questions. Each question has four choices (1), (2), (3) and (4) out of which ONLY ONE is correct.

1. A particle is projected from the top of a tower of height 1,500 m and at a velocity v making an angle of 37° with the horizontal and its vertically downward component is 100 m/s as shown in the figure. The distance from the foot of the tower where it strikes the ground will be (g = 10 m/s²)

   ![Diagram](image)

   (A) 750 m  (B) 1,700 m  (C) 2,000 m  (D) 3,000 m

1. (A)

   \[v_y = 100 \sin 37^\circ \text{ m/s}\]

   \[h = v_y t + \frac{1}{2} gt^2\]

   \[1500 = 100 \ell + 5t^2 \Rightarrow \ell = 10 \text{ s}\]

   \[
   \tan (53) = \frac{v_y}{v_x} \Rightarrow \frac{4}{3} = \frac{100}{v_x}
   \]

   \[\Rightarrow v_x = 75 \text{ m/s}\]

   So Range = \[v_x \times \ell = 75 \times 10\]

   = 750 m

2. Particle A is moving along a straight line with constant velocity v as shown in the figure. Another particle B is moving in a circle with same speed v. The line and the circle are in the same plane. At the moment when A is diametrically opposite to B, the radius of curvature of path of B as seen by A will be (Radius of the circle is R)

   ![Diagram](image)

   (A) R  (B) R/2  (C) 2R  (D) 4R

2. (D)
Acceleration of B is $\frac{v^2}{R}$ (↓)

Speed of B as seen by A at the instant described is 2v, but A will see that acceleration of B is $\frac{v^2}{R}$ (↓).

If $r =$ radius of curvature of path of B as seen by A then $\frac{(2v)^2}{r} = \frac{v^2}{R}$

$\Rightarrow \quad r = 4R$

3. Find the velocity of ring B at the instant shown in the figure. The string is taut and inextensible:

![Diagram](attachment:image.png)

(A) $\frac{1}{2}$ ms$^{-1}$  
(B) $\frac{\sqrt{3}}{4}$ ms$^{-1}$  
(C) $\frac{1}{4}$ ms$^{-1}$  
(D) 1 ms$^{-1}$

3. (D)

$v_B \cos 60^\circ = v_A \cos 60^\circ$

$\Rightarrow \quad v_B = v_A = 1$ ms$^{-1}$

4. The figure shows two blocks A and B connected to an ideal pulley-string system. The system is released and moves without friction.

![Diagram](attachment:image.png)

(A) Acceleration of block A is 1 ms$^{-2}$
(B) Acceleration of block A is 0.5 ms$^{-2}$
(C) Tension in the string connected to block B is 40 N
(D) Tension in the string connected to block B is 80 N

4. (D)

Acceleration of A = a
Acceleration of $P_1 = a$
Acceleration of $P_2 = 2a$

Acceleration of $B = 2 (2a) = 4a$

Tension is $T$, $2T$ and $4T$ in three strings as shown in the figure.

For $A$ : $40 - 4T = 40a$ \hspace{1cm} \ldots \text{(i)}

For $B$ : $T = 10 (4a)$ \hspace{1cm} \ldots \text{(ii)}

(i) – (ii) gives $40 = 5T \Rightarrow T = 80 \text{ N}$

Put this in (ii) to get $a = 2\text{ms}^{-2}$

5. A bead of mass $m$ can slide freely along a taut smooth string as shown in the figure. Given that angular speed $\omega$ = constant, select the correct alternatives.

(A) At equilibrium distance $S = \frac{2g}{\omega^2}$ \hspace{1cm} (B) At equilibrium, $S = \frac{2g}{3\omega^2}$

(C) The equilibrium is stable \hspace{1cm} (D) The equilibrium is unstable

5. (B, D)

$N \sin 30^\circ = mo^2 r$

$N \cos 30^\circ = mg$

$\therefore \tan 30^\circ = \frac{o^2 r}{g}$

$\Rightarrow r = \frac{g}{\sqrt{3} \omega^2}$
\[ S \cos 30^\circ = \frac{g}{\sqrt{3}} \omega^2 \Rightarrow S = \frac{2g}{3\omega^2} \]

If the bead is displaced slightly, say up along the string, the compound moment will become larger than \( mg \sin 30^\circ \) (since \( r \) increases) and this will take the bead further away from its equilibrium position. Such equilibrium is known as unstable.

6. In the arrangement shown, the spring is relaxed and the two identical blocks are made to move always with constant speed \( v \) in opposite directions. If the magnitude of work done by the spring force in first second is 1 J then work done by the spring force in next second will be

\begin{align*}
\text{Smooth horizontal surface} \\
\begin{array}{c}
\text{v} \\
\text{m} \\
\text{m}
\end{array}
\end{align*}

(A) 3 J  
(B) 4 J  
(C) 6 J  
(D) 1 J 

6. (A)

Extension in the spring at the end of first second is \( x_1 = 2vt = 2v \times 1 = 2v \)

Given \( \frac{1}{2} kx_1^2 = 1 \Rightarrow \frac{1}{2} k(2v)^2 = 1 \)  

\( \ldots \) (1)

At the end of 2 seconds, \( x_2 = 2v(2) = 4v \)

Magnitude of work done by the spring in 2 seconds is

\[
\frac{1}{2} kx_2^2 = \frac{1}{2} k(4v)^2 = 4J \quad \text{[using (1)]}
\]

\( \therefore \) work done has a magnitude of 3J in second second.

7. Two blocks A and B, each of mass 10 kg, are sliding on an incline. They are connected by a string. The coefficients of kinetic friction between A and incline is \( \mu_1 = 0.5 \) and that between B and incline is \( \mu_2 = 0.4 \). The tension in string will be

\begin{align*}
\text{(A) Zero} & \quad \text{(B) 2 N} & \quad \text{(C) 8 N} & \quad \text{(D) 4 N} \\
\text{(D)} & \quad \\
\text{Considering A + B together} & \\
20.a = 20.g \cdot \sin 37^\circ - 0.5 \times 10g \cos 37^\circ & - 0.4 \times 10g \cos 37^\circ \\
\Rightarrow & \quad a = 6 - 2 - 1.6 \\
\Rightarrow & \quad a = 2.4 \text{ m/s}^2 \\
\text{Considering only A:} & \\
T + 10 \times g \times \sin 37^\circ - 0.5 \times 10g \cos 37^\circ = 10 \times 2.4 & \\
\Rightarrow & \quad T + 60 - 40 = 24
\end{align*}
8. The velocity-time graph of a linear motion is shown in the figure. The displacement and distance travelled in 8 seconds is:

![Velocity-time graph](image)

(A) 5m, 19m  
(B) 16m, 22m  
(C) 8m, 19m  
(D) 6m, 5m

8. (A)
Displacement = (Area of graph above time axis) – (Area of graph below time axis)
Distance = (Area above time axis) + (Area below time axis)

9. A ball of mass \( m \) is projected with speed \( u \) into the barrel of a spring gun of mass \( M \), initially at rest on a frictionless surface. The mass \( m \) sticks in the barrel at the point of maximum compression of the spring. No energy is lost in friction. What fraction of the initial kinetic energy of the ball is stored in the spring?

![Spring gun diagram](image)

(A) \( \frac{2M}{M+m} \)  
(B) \( \frac{M}{M+m} \)  
(C) \( \frac{M}{2M+m} \)  
(D) None of these

9. (B)
At the point of maximum compression, both \( M \) and \( m \) will have same velocity.

\[ (M + m) v = mu \Rightarrow v = \frac{mu}{M+m} \]

\[ K_i = \frac{1}{2} mu^2; K_f = \frac{1}{2} (M+m)v^2 \]

\[ = \frac{1}{2} (M+m) \left( \frac{mu}{M+m} \right)^2 \]

\[ = \frac{m^2 u^2}{2(M+m)} \]

Loss in KE = Spring PE (U) = \( K_i - K_f \)

\[ = \frac{1}{2} mu^2 \left[ 1 - \frac{m}{M+m} \right] \]

\[ = \frac{1}{2} \left( \frac{mM}{M+m} \right) u^2 \]

\[ \therefore \frac{U}{K_i} = \frac{M}{M+m} \]
10. A cubical vessel with edge L is placed on a cart, which is moving horizontally with an acceleration ‘a’ as shown in figure. The cube is completely filled with an ideal fluid having density $\rho$. It is sealed so that no air remains inside it. The pressure at the centre of the cubical vessel is

\[ P_{\text{centre}} = \rho g \frac{L}{2} + \rho a \frac{L}{2} \]

(A) $\frac{L}{2} \rho g$  (B) $\frac{L}{2} \rho (g + a)$  (C) $\frac{L}{2} \rho a$  (D) $\frac{L}{2} \rho (g - a)$

10. (B)
Pressure at top-right corner = 0
This is because there is no air and the liquid tends to move backward.

\[ P_{\text{centre}} = \rho g \frac{L}{2} + \rho a \frac{L}{2} \]

11. A tube of fine bore AB is connected to a manometer M as shown. The stop cork S controls the flow of air. AB is dipped into and taken out of a liquid whose surface tension is $\sigma$. A film of liquid is formed, which closes the end B. On opening the stop cork for a while, air is forced into the tube and a bubble is formed at B. The manometer level is recorded, showing a difference h in the levels in the two arms. If $\rho$ be the density of manometer liquid and r the radius of curvature of the bubble, then the surface tension $\sigma$ of the liquid is given by

\[ 4 \rho g hr = \frac{r \sigma}{4} \]

(A) $rhg$  (B) $2rhgr$  (C) $4 rhg$  (D) $\frac{rhpg}{4}$

11. (D)
Pressure of air in the tube is higher than atmospheric pressure by $\Delta P = \rho gh$

Excess pressure inside bubble is $\Delta P = \frac{4\sigma}{r}$

The pressure of air inside the bubble and the entire tube is same.

\[ \frac{4\sigma}{r} = \rho gh \Rightarrow \sigma = \frac{\rho gh r}{4} \]
12. A satellite can be in a geostationary orbit around earth at a distance \( r \) from the centre. If the angular velocity of earth about its axis doubles, a satellite can now be in a geostationary orbit around earth if its distance from the centre is

(A) \( \frac{r}{2} \)  
(B) \( \frac{r}{2\sqrt{2}} \)  
(C) \( \frac{r}{(4)^{1/3}} \)  
(D) \( \frac{r}{(2)^{1/3}} \)

12. (C)
Angular speed of Earth doubles. This implies that its time period becomes half (= 12 h).
\[ T^2 \propto r^3 \]
\[ \therefore \left( \frac{T}{T} \right)^2 = \left( \frac{R}{r} \right)^3 \Rightarrow R = \frac{r}{(4)^{1/3}} \]

13. Heat is flowing along the length of a rod of length \( l \) having coefficient of thermal conductivity varying as \( k = \alpha /T \), where \( \alpha \) is a constant. If ends of the rod are kept at temperatures \( T_1 \) and \( T_2 \) \((T_1 > T_2)\) then temperature at a distance \( x \) from the hotter end is

(A) \( T_2 \left( \frac{T_1}{T_2} \right)^{x/l} \)  
(B) \( T_2 \left( \frac{T_1}{T_2} \right)^{x/l} \)  
(C) \( T_1 \left( \frac{T_2}{T_1} \right)^{x/l} \)  
(D) \( T_1 \left( \frac{T_2}{T_1} \right)^{x/l} \)

13. (D)
\[ \frac{dQ}{dt} = -kA \frac{dT}{dx} \]
\[ \Rightarrow \frac{dQ}{dt} \int_0^l dx = -kA \int_{T_1}^{T_2} \frac{dT}{T} \]
\[ \Rightarrow \frac{dQ}{dt} \ell = kA \ln \left( \frac{T_1}{T_2} \right) \]

Now, \( \frac{dQ}{dt} \int_0^l dx = -kA \int_{T_1}^{T_2} \frac{dT}{T} \)
\[ \Rightarrow \frac{dQ}{dt} x = kA \ln \left( \frac{T_1}{T} \right) \]
\[ \Rightarrow \frac{\ell}{x} = \frac{\ln \left( \frac{T_1}{T} \right)}{\ln \left( \frac{T_1}{T_2} \right)} \]
\[ \Rightarrow \ln \left( \frac{T_1}{T} \right) = \ln \left( \frac{T_1}{T_2} \right)^{x/l} \]
\[ \Rightarrow T = T_1 \left( \frac{T_2}{T_1} \right)^{x/l} \]

14. One mole of a monoatomic ideal gas is taken from A to B as shown in the graph. Specific heat of the gas in this process is
14. (B)

\[ C = C_v + \frac{P}{n} \frac{dV}{dT} \]

Here \( P = \frac{2P_0 - P_0}{2V_0 - V_0} V = \frac{P_0}{V_0} V \)

\[ \Rightarrow nRT = \frac{P_0}{V_0} V \]

or \( T = \frac{P_0 V^2}{nR V_0} \Rightarrow \frac{dT}{dV} = \frac{2P_0 V}{nR V_0} \)

\[ \Rightarrow C = C_v + \frac{P}{n} \frac{nRV_0}{2P_0 V} = C_v + PV_0 R \]

\[ \Rightarrow C = \frac{3}{2} R + \frac{1}{2} R = 2R \]

15. A converging and a diverging lens having magnitudes of focal lengths \( f_1 \) and \( f_2 \) respectively are placed at such a separation that a ray, incident parallel to the principal axis, emerges parallel to the principal axis. Separation between the lenses is

\[ \frac{f_1 f_2}{f_1 + f_2} \] (A) \( \frac{f_1 f_2}{f_1 - f_2} \) (B) \( f_1 + f_2 \) (C) \( f_1 - f_2 \) (D) \( f_1 - f_2 \)

16. White light is incident normally on the slits \( S_1S_2 \), where the line \( PO \) divides the slit non-symmetrically, as shown. Distance of the point, from \( O \), on screen, where white fringe will be formed, is

\[ P = 0 = \frac{1}{f} = \frac{1}{f_1} - \frac{1}{f_2} + \frac{d}{f_1 f_2} \Rightarrow f_2 - f_1 + d = 0 \]

or \( d = f_1 - f_2 \)
16. **(B)**

White fringe is obtained at the bisector, where path difference is zero. Here, distance of the bisector from the PQ is

\[ x = \frac{2d}{3} - \frac{d}{2} = \frac{4 - 3d}{6} = \frac{d}{6} \]

17. A radioactive gas is present in a sealed vessel at \( t = 0 \). At time \( T \), some more gas is injected in the vessel. Which of the following graphs does best represent variation of logarithm of active nucleus \( N \) with time?

- **(A)**
- **(B)**
- **(C)**
- **(D)**

18. For the given combination of gates, if the logic states of inputs \( A, B, C \) are as follows \( A = B = C = 0 \) and \( A = B = 1, C = 0 \) then the logic states of output \( D \) are

- **(A)** 0, 0
- **(B)** 0, 1
- **(C)** 1, 0
- **(D)** 1, 1

18. **(D)**

\[
D = \overline{C(A + B)} = \overline{c} + (A + B) = \overline{c} + (A + B)
\]

\[
= \overline{0} + (0 + 0) \text{ and } \overline{0} + (1 + 1)
\]

\[
= 1 + 0 \text{ and } 1 + 1
\]
19. Four equal resistors, each of resistance 10 ohm are connected as shown in the adjoining circuit diagram.
Then the equivalent resistance between points A and B is

\[ R = \frac{20 \times 20}{20 + 20} = 10 \Omega \]

(A) 40 ohm  (B) 20 ohm  (C) 10 ohm  (D) 5 ohm

19. (C)

20. In the circuit shown, the charge on the 3 \( \mu \)F capacitor at steady state will be

(A) 6 \( \mu \)C  (B) 4 \( \mu \)C  (C) 2/3 \( \mu \)C  (D) 3 \( \mu \)C

20. (B)

At steady state, there will be no current in the branches having capacitor only thus equivalent circuit diagram will be as shown in the figure.

\[ \frac{V_{AB} - 1}{1} + \frac{V_{AB} - 2}{2} = 0 \]

\[ \Rightarrow V_{AB} = \frac{4}{3} V \]

Thus \( q = CV_{AB} = 4 \mu C \)

21. A thin uniform rod of negligible mass and length \( \ell \) is attached to the floor by a hinge P. The other end is connected to a spring of force constant \( k \). Rod is in an uniform magnetic field \( B \) pointing inwards the plane of paper. A current \( I \) is passed through the rod. Find the torque acting on the rod due to magnetic force when the rod makes an angle 53° as shown in figure.
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21. (B) 
\[ F = kx = I \ell B \]
\[ \tau = \int d\tau = \int I \ell Bd\ell = \frac{I \ell^2 B}{2} \]

22. You are given a closed circuit with radii a and b as shown in figure, carrying current i. The magnetic dipole moment of the circuit is

![Diagram of a closed circuit with radii a and b and current i.]

(A) \( \pi(a^2 + b^2)i \)  
(B) \( \frac{1}{2} \pi(a^2 + b^2)i \)  
(C) \( \pi(b^2 - a^2)i \)  
(D) \( \frac{1}{2} \pi(b^2 - a^2)i \)

22. (B) 
\[ M = IA = I \left( \frac{\pi a^2}{2} + \frac{\pi b^2}{2} \right) \] magnetism

**SECTION-II : (NUMERICAL VALUE TYPE)**

This section contains 05 questions. The answer to each question is a **NUMERICAL VALUE**. For each question, enter the correct numerical value (e.g. 6.25, 7, 0.33, 30.275, 127.30, −127)

21. Two particles are projected vertically upward from a point on the surface of earth with velocities
\[ v_1 = \sqrt{\frac{2gR}{3}}; v_2 = \sqrt{\frac{4gR}{3}} \] respectively. If the maximum height attained are \( h_1 \) and \( h_2 \), respectively, the ratio of \( h_1/h_2 \) is __________.

21. (0.25) 
Loss of KE = Gain in PE
\[ \Rightarrow \frac{1}{2}mv^2 = mgh \left( \frac{R}{h + R} \right) \]
Case I.
\[
\Rightarrow \frac{1}{2} m \left( \frac{2gR}{3} \right) = mgh_1 \left( \frac{R}{h_1 + R} \right) \quad \text{......... (i)}
\]

Case II.
\[
\Rightarrow \frac{1}{2} m \left( \frac{4gR}{3} \right) = mgh_2 \left( \frac{R}{h_2 + R} \right) \quad \text{......... (ii)}
\]

Solving (i)
\[
\Rightarrow \frac{1}{3} = \frac{h_1}{h_1 + R}
\]
\[
\Rightarrow h_1 = \frac{R}{2} \quad \text{......... (iii)}
\]

Solving (ii)
\[
\Rightarrow \frac{2}{3} = \frac{h_2}{h_2 + R} \quad \text{......... (iv)}
\]
\[
\Rightarrow h_2 = 2R
\]

From (iii) and (iv)
\[
\Rightarrow h_1 = \frac{R}{2} \quad \text{and} \quad h_2 = 2R
\]
\[
\therefore \frac{h_1}{h_2} = \frac{1}{4} = 0.25
\]

22. Two containers filled with two different gases, are maintained at the same temperature. Suppose that the molecular weights of the two gases, \( M_1 \) and \( M_2 \), are in ratio 9 : 4 then, The average momenta (in magnitude) of the molecules are related as \( p_1 = np_2 \). Find the value of \( n \).

22. \( (1.5) \)

Linear momentum, \( p = \sqrt{3k_B T} m \).

But, 
\[
m = \frac{M}{N_A}
\]
\[
\therefore \frac{m_1}{m_2} = \frac{M_1}{M_2}
\]

Since the two gases are maintained at same temperature,
\[
\therefore \frac{p_1}{p_2} = \frac{\sqrt{3m_1k_B T}}{\sqrt{3m_2k_B T}} = \frac{M_1}{\sqrt{M_2}} = \sqrt{\frac{9}{4}} = 1.5
\]
\[
p_1 = 1.5p_2
\]
\[
\therefore n = 1.5
\]

23. The figure shown below is a metre bridge set up with null deflection in the galvanometer. The value of the unknown resistance \( R \) is ________ \( \Omega \).
23. (195)
For balanced metre bridge,
\[
\frac{65}{R} = \frac{25}{75} \Rightarrow R = 195 \Omega
\]

24. An electron in the ground state of hydrogen atom is revolving in anticlockwise direction in a circular orbit of radius R. The atom is placed in a uniform magnetic induction \( \vec{B} \) such that the normal to the plane of electron’s orbit makes an angle of \( 30^\circ \) with the magnetic induction. The torque experienced by the orbiting electron is found to be \( N \frac{e h B}{\pi m_e} \). The value of N is ________.

24. (0.12)
Magnetic moment of electron,
\[
M = \frac{e}{2m_e} \ell = \frac{e}{2m_e} \left( \frac{h}{2\pi} \right) \quad \therefore \quad \ell = \frac{h}{2\pi}
\]
\[
= \frac{e h}{4\pi m}
\]
\[
\vec{\tau} = \vec{M} \times \vec{B}
\]
\[
\Rightarrow |\vec{\tau}| = MB\sin 30^\circ
\]
\[
= \left( \frac{eh}{4\pi m_e} \right) (B) \sin 30^\circ
\]
\[
= \frac{ehB}{8\pi m_e}
\]
\[
= 0.125 \frac{ehB}{\pi m_e}
\]
\[
\therefore \quad \vec{\tau} = 0.12 \frac{ehB}{\pi m_e}
\]
25. A rod of height \( h \) is placed in a beaker of same radius. The height of the beaker is thrice its radius. An observer sees the top end of the rod through a pin–hole (see figure). When the beaker is filled with a liquid \( (\text{R.I.} = \sqrt{\mu}) \) upto a height \( 2h \), he can see the lower end of the rod. Find the value of \( \mu \).

The line of sight of the observer remains constant, making an angle of \( 45^\circ \) with the normal.

\[
\sin \theta = \frac{h}{\sqrt{h^2 + (2h)^2}}
\]

\[
= \frac{1}{\sqrt{5}}
\]

Refractive index of liquid,

\[
\sqrt{\mu} = \frac{\sin 45^\circ}{\sin \theta}
\]

\[
= \frac{\sqrt{2}}{1} = \sqrt{\frac{5}{2}}
\]

\[
\therefore \sqrt{\mu} = \sqrt{2.5} \quad \Rightarrow \mu = 2.5
\]
This section contains 20 multiple choice questions. Each question has four choices (1), (2), (3) and (4) out of which ONLY ONE is correct.

26. The half-time of first–order decomposition of nitramide is 2.1 hour at 15°C.
\[
\text{NH}_2\text{NO}_3(\text{s}) \rightarrow \text{N}_2\text{O(g)} + \text{H}_2\text{O(l)}
\]
If 6.2 g of \( \text{NH}_2\text{NO}_3 \) is allowed to decompose, then time taken for \( \text{NH}_2\text{NO}_3 \) to decompose 99% and volume of dry \( \text{N}_2\text{O} \) produced at this point measured at STP will be

(1) 13.95 hrs, 22.4 L  
(2) 13.95 hrs, 2.217 L  
(3) 2.1 hrs, 22.4 L  
(4) 2.1 hrs, 2.217 L

26. (2)

For a first–order reaction, \( K = \frac{2.303}{t} \log \left[ \frac{[A]_0}{[A]_t} \right] \)

Initial moles of nitramide = \( \frac{6.2}{62} = 0.1 \)

\( \therefore \ t = \frac{2.303 \times 2.1}{0.693} \log \frac{0.1}{0.001} = 13.95 \) hours

Since, the decomposition is 99%, so 99% of the initial moles of \( \text{NH}_2\text{NO}_3 \) would be converted to \( \text{N}_2\text{O} \).

Moles of \( \text{N}_2\text{O} = \frac{0.1 \times 99}{100} \)

Therefore, volume of \( \text{N}_2\text{O} \) at STP = \( \frac{0.1 \times 99 \times 22.4}{100} = 2.217 \) L

27. Calculate the electrode potential at 25°C of \( \text{Cr}^{3+}, \text{Cr}_2\text{O}_7^{2-} \) electrode at \( \text{pOH} = 11 \) in a solution of 0.01 M both in \( \text{Cr}^{3+} \) and \( \text{Cr}_2\text{O}_7^{2-} \), \( E^o \) value for the cell is 1.33 V

\( \text{Cr}_2\text{O}_7^{2-} +14\text{H}^+ + 6\text{e} \rightarrow 2\text{Cr}^{3+} + 7\text{H}_2\text{O} \)

(1) 0.725 V  
(2) 0.936 V  
(3) 0.652 V  
(4) 0.213 V

27. (2)

\[
\begin{align*}
\text{Cr}_2\text{O}_7^{2-} + 14\text{H}^+ + 6\text{e} & \rightarrow 2\text{Cr}^{3+} + 7\text{H}_2\text{O} \\
p\text{OH} &= 11 \\
p\text{H} &= 3 \\
\left[\text{H}^+\right] &= 10^{-3} \\
E_{\text{cell}} &= E^o - \frac{0.0591}{6} \log \frac{\left[\text{Cr}^{3+}\right]^2}{\left[\text{Cr}_2\text{O}_7^{2-}\right] \left[\text{H}^+\right]^4} 
\end{align*}
\]
28. When 0.6 g of urea is dissolved in 100 g of water, the water will boil at ($K_b$ for water = 0.52 kJ mol$^{-1}$ and normal boiling point of water = 100°C):
   (1) 373.052 K  (2) 273.52 K  (3) 372.48 K  (4) 273.052 K

28. (1)
\[
\Delta T_b = T_b - T_0 = \frac{1000K_bW_B}{m_Bw_A} = \frac{1000 \times 0.52 \times 0.6}{60 \times 100}
\]
\[
T_b = 373 = 0.052 \quad T_0 = 373.052
\]

29. A BCC lattice is made up of hollow spheres of B. Spheres of solid A are present in hollow spheres of B. Radius of A is half of radius of B. What is the ratio of total volume of spheres of B unoccupied by A in a unit cell and volume of unit cell?
   (1) \(\frac{7\sqrt{3}\pi}{64}\)  (2) \(\frac{7\sqrt{3}\pi}{128}\)  (3) \(\frac{7\pi}{24}\)  (4) \(\frac{7\sqrt{3}\pi}{64}\)

29. (4)
Number of atoms of B in unit cell = 2
Total volume of B unoccupied by A

In a unit cell = \(2 \times \frac{4}{3}(R^3 - r^3) \times \pi = \frac{7\pi R^3}{3}\)

Volume of unit cell = \(a^3 = \frac{64}{3\sqrt{3}} R^3\)

For BCC \(\sqrt{3}a = 4R\)

\[\rho_{ratio} = \frac{7\pi R^3/3}{64 \frac{R^3}{3\sqrt{3}}} = \frac{7\sqrt{3}\pi}{64}\]

30. Which of the following electrolyte will be most effective in coagulation of gold sol?
   (1) \(\text{NaNO}_3\)  (2) \(K_4[\text{Fe(CN)}_6]\)  (3) \(\text{Na}_3\text{PO}_4\)  (4) \(\text{MgCl}_2\)

30. (2)

31. 25.0 mL of HCl solution gave, on reaction with excess AgNO$_3$ solution 2.125 g of AgCl. The normality of HCl solution is
   (1) 0.25  (2) 0.6  (3) 1.0  (4) 0.75

31. (2)
\[
\text{AgNO}_3 + \text{HCl} \rightarrow \text{AgCl} + \text{H}^+ + \text{NO}_3^-
\]
\[
170 \quad 143.5 \quad \text{g}
\]
\[
\therefore 143.5\text{g AgCl is produced by 170 g of AgNO}_3
\]
\[
\therefore 2.125\text{g AgCl is produced by } \frac{170 \times 2.125}{143.5} = 2.517 \text{ of AgNO}_3
\]
Now for the reaction equivalent of AgNO$_3$ = equivalent of HCl

\[
= 1.33 - \frac{0.0591}{6} \log \left[\frac{0.01}{0.01 \times 10^{-3}}\right]^4 = 0.936 \text{ V}
\]
32. One mole of an ideal monoatomic gas is caused to go through the cycle as shown in figure. Then the change in the interval energy in expanding the gas from a to c along the path abc is

\[
\Delta U = \frac{21RT_0}{2} = 10.5 RT_0
\]

Temperature at ‘a’ = \( T_0 = \frac{PV}{R} \)

At (a) \( T_0 = \frac{P_0V_0}{R} \)  

At (C) \( T_C = \frac{(2P_0)(4V_0)}{R} = 8T_0 \)

\[
\Delta U = nC_v (T_f - T_i) = \frac{3}{2}R(8T_0 - T_0)
\]

33. Which order of acidic strength is wrong?

(1) \( \text{COOH} > \text{SO}_3 \text{H} \)  
(2) \( \text{NO}_2 \text{COOH} > \text{COOH} \)  
(3) \( \text{COOH} > \text{COOH} \)  
(4) \( \text{Cl} \text{COOH} > \text{COOH} \)

33. (3) 
Due to +M of \(-\text{OCH}_3\), it is weak acid.
34. Select the optically active compound among the following:

(1) 

(2) 

(3) 

(4) 

34. (2)

35. 

\[ \text{product.} \]

Major product is:

(1) 

(2) 

(3) 

(4) 

35. (1)

\[ \text{HCO}_3^- \]
36. OH

\[ \text{H}^+ / \text{KMnO}_4 \triangle \]

36. (3)
37. Product is:

\[
\begin{array}{c|c|c|c}
(1) & (2) & (3) & (4) \\
\begin{array}{c}
\text{None of the above}
\end{array} & \begin{array}{c}
\text{None of the above}
\end{array} & \begin{array}{c}
\text{None of the above}
\end{array} & \begin{array}{c}
\text{None of the above}
\end{array} \\
\end{array}
\]

38.
Select incorrect statement:
(1) P can turn blue litmus red
(2) P can not give effervescence of CO₂ with NaHCO₃.
(3) It is Diecmann condensation
(4) Product is a bicyclo compound

39. Which of the following isomeric compounds is most stable?

- (1)
- (2)
- (3)
- (4)

(3) All hydroxyl groups are equatorial.

40. Identify the only paramagnetic species:
(1) Ce⁴⁺   (2) Yb²⁺   (3) Lu³⁺   (4) Nd³⁺

(4) Lu³⁺, Yb²⁺ → 4f¹⁴; Ce⁴⁺ → 4f⁰; Nd³⁺ → 4f³

41. In which of the following complexes the nickel metal is in highest oxidation state?
(1) Ni(CO)₄   (2) K₂NiF₆
(3) [Ni(NH₃)₆(BF₄)₂]   (4) K₄[Ni(CN)₆]
41. \( K_2\left[\text{Ni}^{+4}\text{F}_6\right] \)

42. Which material has been named incorrectly?
   (1) Bauxite : \( \text{Al}_2\text{O}_3.2\text{H}_2\text{O} \)  (2) Corundum : \( \text{Al}_2\text{O}_3 \)
   (3) Cryolite : \( 3\text{NaF.}\text{AlF}_3 \)  (4) Feldspar : \( \text{Be}_3\text{Al}_2\text{Si}_6\text{O}_{18} \)

42. (4)
   Feldspar \( \rightarrow \text{KAlSi}_3\text{O}_8 - \text{NaAlSi}_3\text{O}_8 - \text{CaAl}_2\text{Si}_2\text{O}_8 \)

43. Which metal reacts most vigorously with water?
   (1) Al  (2) Ca  (3) Fe  (4) K

43. (4)

44. Ammonia can be dried by:
   (1) conc. \( \text{H}_2\text{SO}_4 \)  (2) \( \text{P}_4\text{O}_{10} \)  (3) \( \text{CaO} \)  (4) Anhydrous \( \text{CaCl}_2 \)

44. (3)
   NH\(_3\) being basic, should be dried with reagents that are basic. With \( \text{CaCl}_2 \) (anhyd.) \( \Rightarrow \text{CaCl}_2.8\text{NH}_3 \) is formed.

45. Which oxide has the highest melting point?
   (1) \( \text{H}_2\text{O} \)  (2) \( \text{NO}_2 \)  (3) \( \text{SO}_2 \)  (4) \( \text{SiO}_2 \)

45. (4)
   \( \text{SiO}_2 \) is network solid.

46. Which one of the following species is stable in aqueous solution?
   (1) \( \text{MnO}_4^{-} \)  (2) \( \text{MnO}_4^{2-} \)  (3) \( \text{Cu}^{+} \)  (4) \( \text{Cr}^{2+} \)

46. (2)
SECTION-II : (NUMERICAL VALUE TYPE)

This section contains 05 questions. The answer to each question is a NUMERICAL VALUE. For each question, enter the correct numerical value (e.g. 6.25, 7, 0.33, 30.275, 127.30, −127)

46. 1 g of an ideal gas X is introduced into an evacuated flask kept at 298 K. The pressure is found to be 1 atm. If 2 g of another ideal gas Y is added to the same flask, the total pressure becomes 1.5 atm. The molecular mass of Y is __________ times greater than the molecular mass of X.

46. (4)
Pressure of Y = 1.5 − 1 = 0.5 atm
Now,
P\text{X} V = \frac{nRT}{M_X} = \frac{1\times RT}{M_X}
And
P\text{Y} V = \frac{2\times RT}{M_Y}
\therefore \frac{P\text{X}}{P\text{Y}} = \frac{M_Y}{2M_X}
\therefore \frac{M_Y}{M_X} = \frac{2P\text{X}}{P\text{Y}}
\therefore \frac{M_Y}{M_X} = \frac{2\times 1}{0.5} = 4
\therefore M_Y = 4M_X

47. Washing soda is Na\textsubscript{2}CO\textsubscript{3}.xH\textsubscript{2}O. The value of ‘x’ is ________.

47. (10)
Washing soda is decahydrate of sodium carbonate (Na\textsubscript{2}CO\textsubscript{3}.10H\textsubscript{2}O).

48. The sum of coordination number and oxidation number of the metal M in the complex \[ [\text{M(en)}_2(C_2O_4)]\text{Cl} \] is ________.

48. (9)
\[ [\text{M(en)}_2(C_2O_4)]\text{Cl} \]
The complex has two molecules of en and one C\textsubscript{2}O\textsubscript{4}\textsuperscript{2−} ion. Thus, the complex has three bidentate ligands in all.
\therefore Coordination number
= 2 \times \text{number of bidentate ligands} = 2 \times 3 = 6
Let the oxidation number of metal ‘M’ in the complex be ‘x’.
The charge on complex ion, en and C\textsubscript{2}O\textsubscript{4}\textsuperscript{2−} are +1, 0 and −2 respectively.
Hence, x + 0 − 2 = +1
x = 3
.: The sum of coordination number and oxidation number of the metal M = 6 + 3 = 9

49.

\[
\text{OH} + \text{CHCl}_3 + \text{NaOH} \rightarrow \text{ONa} \quad \text{CHO}
\]

The electrophile involved in above reaction has _______ lone pair of electrons on central carbon atom.

49. **(1)**

This is Reimer–Tiemann reaction.

\[
\text{CHCl}_3 + \text{OH}^- \rightarrow \text{CCl}_3 + \text{H}_2\text{O}
\]

\[
\text{CCl}_3 \rightarrow \text{CCL}_2^+ + \text{Cl}^-
\]

\[
\text{OH} \quad \xrightarrow{\text{CHCl}_3, \text{NaOH}} \quad \text{O}^-\text{Na}^+ \quad \text{CHO}
\]

Phenol

The electrophile dichlorocarbene (\(\text{CCL}_2\)) possesses one lone pair of electrons.

50. How many of the following is/are polyester(s)?
   i. Nylon–6, 6
   ii. Terylene
   iii. Polypropene
   iv. Melamine polymer
   v. Polyacrylonitrile

50. **(1)**

Terylene is a polyester.
PART (C) : MATHEMATICS

SECTION-I : (SINGLE ANSWER CORRECT TYPE)

This section contains 20 multiple choice questions. Each question has four choices (1), (2), (3) and (4) out of which ONLY ONE is correct.

51. The plane containing the line \( \frac{x-1}{1} = \frac{y-2}{2} = \frac{z-3}{3} \) and parallel to the line \( \frac{x}{1} = \frac{y}{1} = \frac{z}{4} \) passes through the point

(1) (1, -2, 5) (2) (1, 0, 5) (3) (0, 3, -5) (4) (-1, -3, 0)

51. (2)

The equation of a plane containing \( \frac{x-1}{1} = \frac{y-2}{2} = \frac{z-3}{3} \) is of the form

\[
A (x - 1) + B (y - 2) + C (z - 3) = 0
\]

Where \( A + 2B + 3C = 0 \) ............ (i)

This plane is parallel to \( \frac{x}{1} = \frac{y}{1} = \frac{z}{4} \) if

\[
A + B + 4C = 0 \quad \text{.........}(ii)
\]

From equation (i) and (ii), we get

\[
\frac{A}{8-3} = \frac{B}{3-4} = \frac{C}{1-2} \Rightarrow \frac{A}{5} = \frac{B}{-1} = \frac{C}{-1}
\]

Hence, the equation of the plane is

\[
5(x - 1) - (y - 2) - (z - 3) = 0
\]

\[
5x - y - z = 0
\]

Which passes through \((1, 0, 5)\).

52. Value of \( \sin^4 \frac{\pi}{8} + \sin^4 \frac{3\pi}{8} + \sin^4 \frac{5\pi}{8} + \sin^4 \frac{7\pi}{8} \) is equal to:

(1) \( \frac{3}{2} \) (2) \( \frac{2}{3} \) (3) \( \sqrt{\frac{3}{2}} \) (4) \( \sqrt{\frac{2}{3}} \)

52. (1)

\[
\sin^4 \frac{\pi}{8} + \sin^4 \frac{3\pi}{8} + \sin^4 \frac{5\pi}{8} + \sin^4 \frac{7\pi}{8}
\]

\[
= 2 \left( \sin^4 \frac{\pi}{8} + \sin^4 \frac{3\pi}{8} \right) = 2 \left( \sin^4 \frac{\pi}{8} + \cos^4 \frac{\pi}{8} \right)
\]

\[
= 2 \left( 1 - 2 \sin^2 \frac{\pi}{8} \cos^2 \frac{\pi}{8} \right) = 2 \left( 1 - \frac{1}{2} \sin^2 \frac{\pi}{4} \right)
\]

\[
= 2 \left( 1 - \frac{1}{4} \right) = \frac{3}{2}
\]

53. If \( S_n = \sum_{r=1}^{n} T_r = n(n+1)(n+2)(n+3) \), then \( \sum_{r=1}^{10} \frac{1}{T_r} \) is equal to:

(1) \( \frac{55}{527} \) (2) \( \frac{58}{528} \) (3) \( \frac{59}{528} \) (4) None of these
53. \( T_n = S_n - S_{n-1} = n(n+1)(n+2)(n+3) - (n-1)n(n+1)(n+2) \)
\[ = 4n(n+1)(n+2) \]
\[ \frac{1}{T_r} = \frac{1}{4r(r+1)(r+2) + \frac{r+2-r}{8r(r+1)(r+2)}} \]
\[ = \frac{1}{8} \left[ \frac{1}{r(r+1)} - \frac{1}{(r+1)(r+2)} \right] \]
\[ \frac{1}{T_1} = \frac{1}{8} \left[ \frac{1}{1.2} - \frac{1}{2.3} \right] \]
\[ \frac{1}{T_2} = \frac{1}{8} \left[ \frac{1}{2.3} - \frac{1}{3.4} \right] \]
\[ \vdots \quad \vdots \quad \vdots \]
\[ \frac{1}{T_{10}} = \frac{1}{8} \left[ \frac{1}{10.11} - \frac{1}{11.12} \right] \]
\[ \sum_{r=1}^{10} \frac{1}{T_r} = \frac{1}{8} \left[ \frac{1}{2} - \frac{1}{132} \right] \]
\[ = \frac{65}{1056} \]

54. Let \( f(k) = \frac{k}{2009} \) and \( g(k) = \frac{f^4(k)}{(1-f(k))^4 + f(k)} \), then the sum \( \sum_{k=0}^{2009} g(k) \) is equal:

(1) 2009  (2) 2008  (3) 1005  (4) 1004

54. \( \sum_{k=0}^{2009} g(k) = g(0) + g(1) + g(2) + \ldots + g(2009) = ? \)

Now,
\[ f(k) = \frac{k}{2009} \]
\[ f(2009-k) = \frac{2009-k}{2009} \]
\[ \Rightarrow f(k) + f(2009-k) = 1 \quad \ldots \ldots \quad (i) \]

Again
\[ g(k) = \frac{f^4(k)}{(1-f(k))^4 + f^4(k)} \quad \ldots \ldots \quad (ii) \]
\[ g(2009-k) = \frac{f^4(2009-k)}{(1-f(2009-k))^4 + f^4(2009-k)} \]
\[ = \frac{1-f(k)}{(f(k))^4 + (1-f(k))^4} \quad \ldots \ldots \quad (iii) \]

Eqn. (ii) + eqn. (iii) gives
\[
g(k) + g(2009 - k) = \frac{f^4(k) + (1 - f(k))^4}{(f(k))^4 + (1 - f(k))^4} = 1
\]
\[
g(0) + g(2009) = 1
\]
\[
g(1) + g(2008) = 1
\]
\[
g(2) + g(2007) = 1
\]
\[
\sum_{k=0}^{2009} g(k) + g(1005) = 1
\]
\[
55. \quad \lim_{x \to 1} \frac{nx^{n+1} - (n+1)x^n + 1}{(e^x - e)\sin \pi x}
\]

where \( n = 100 \) is equal to:

(1) \( \frac{5050}{\pi e} \)

(2) \( \frac{100}{\pi e} \)

(3) \( -\frac{5050}{\pi e} \)

(4) \( -\frac{4950}{\pi e} \)

55. (3)

\[
\ell = \lim_{x \to 1} \frac{nx^n (x - 1) - (x^n - 1)}{(e^x - e)\sin \pi x}
\]

Put \( x = 1 + h \) so that as \( x \to 1, h \to 0 \):
\[
\ell = -\lim_{h \to 0} \frac{h.n(1+h)^n - ((1+h)^n - 1)}{e^{(h-1)}\sin \pi h}
\]
\[
n.h \{ 1 + ^nC_1 h + ^nC_2 h^2 + ^nC_3 h^3 + \ldots \}
\]
\[
\ell = -\lim_{x \to 1} \frac{-\{ 1 + ^nC_1 h + ^nC_2 h^2 + ^nC_3 h^3 + \ldots - 1 \}}{\pi e (h^2) \left( \frac{e^h - 1}{h} \right) \left( \frac{\sin \pi h}{\pi h} \right)}
\]
\[
= -\frac{n^2 - ^nC_2}{\pi e} = -\left[ \frac{2n^2 - n(n-1)}{2\pi e} \right]
\]
\[
= -\frac{n^2 + n}{2\pi e} = -\frac{n(n+1)}{2\pi e}
\]

If \( n = 100 \) \( \Rightarrow \ell = -\left( \frac{5050}{\pi e} \right) \)

56. The function \( f(x) = \begin{cases} 5 - 4x & 1 < x < 4 \\ 4 - x & 4 \leq x < \infty \end{cases} \)

(1) Continuous at \( x = 1 \) and \( x = 4 \)

(2) Continuous at \( x = 1 \), discontinuous at \( x = 4 \)

(3) Continuous at \( x = 4 \), discontinuous at \( x = 1 \)
56. Discontinuous at \( x = 1 \) and \( x = 4 \)

57. The value of \( \int \frac{\sin x}{\sin \left( x - \frac{\pi}{6} \right) \sin \left( x + \frac{\pi}{6} \right)} \, dx \) is:

\[
(1) \quad \frac{1}{\sqrt{3}} \left[ \log \tan \left( \frac{x}{2} \right) + \log \tan \left( \frac{x - \pi}{12} \right) \right] + C \\
(2) \quad \frac{1}{\sqrt{3}} \left[ \log \tan \left( \frac{x + \pi}{12} \right) + \log \tan \left( \frac{x}{2} \right) \right] + C \\
(3) \quad \frac{1}{\sqrt{3}} \left[ \log \cot \left( \frac{x + \pi}{12} \right) \right] + C \\
(4) \quad \text{None of the above}
\]

(where \( C \) is the integration constant)

\[
\int \frac{\sin x}{\sin \left( x - \frac{\pi}{6} \right) \sin \left( x + \frac{\pi}{6} \right)} \, dx = \frac{1}{\sqrt{3}} \int \frac{2 \sin x \cos \pi/6}{\sin \left( x - \frac{\pi}{6} \right) \sin \left( x + \frac{\pi}{6} \right)} \, dx
\]

\[
= \frac{1}{\sqrt{3}} \int \frac{\sin \left( x + \frac{\pi}{6} \right) + \sin \left( x - \frac{\pi}{6} \right)}{\sin \left( x - \frac{\pi}{6} \right) \sin \left( x + \frac{\pi}{6} \right)} \, dx
\]

\[
= \frac{1}{\sqrt{3}} \int \left[ \csc \left( x - \frac{\pi}{6} \right) + \csc \left( x + \frac{\pi}{6} \right) \right] \, dx
\]

\[
= \frac{1}{\sqrt{3}} \left[ \log \tan \left( \frac{x}{2} \right) + \log \tan \left( \frac{x - \pi}{12} \right) \right] + c
\]

Where \( c \) is the integration constant.

58. Value of \( \int_{0}^{x} \frac{x - x \cos 2x}{1 + 2 \cos x \sin x} \, dx \) is:

\[
(1) \quad 0 \quad (2) \quad \frac{\pi}{2} \quad (3) \quad \pi \quad (4) \quad 2\pi
\]

58. \( (3) \)

\[
I = \int_{0}^{x} \frac{x (1 - \cos 2x)}{1 + 2 \cos x \sin x} \, dx
\]

\[
\Rightarrow I = 2 \int_{0}^{x} \frac{x \sin 2x}{1 + 2 \cos x \sin x} \, dx \quad \ldots \ldots \text{(i)}
\]

\[
\Rightarrow I = 2 \int_{0}^{x} \frac{(\pi - x) \sin^2 x}{1 + 2 \cos x \sin x} \, dx \quad \ldots \ldots \text{(ii)}
\]

\[
(\therefore \int_{a}^{b} f(x) \, dx = \int_{0}^{b} f(a + b - x) \, dx)
\]

Adding eqns. (i) and (ii),

\[
2I = 2\pi \int_{0}^{\pi/2} \frac{\sin^2 x}{1 + 2 \cos x \sin x} \, dx
\]

\[
\therefore I = \pi \int_{0}^{\pi/2} \frac{\sin^2 x}{1 + 2 \cos x \sin x} \, dx \quad \ldots \ldots \text{(iii)}
\]
\[
\left( \int_0^{2a} f(x) \, dx \right) = 2 \int_0^a f(x) \, dx \text{ where } f(2a-x) = f(x)
\]
\[
\therefore I = 2\pi \int_0^{\pi/2} \frac{\cos^2 x}{1 + 2\sin x \cos x} \, dx \quad \text{............... (iv)}
\]

Adding eqns. (iii) and (iv),
\[
2I = 2\pi \int_0^{\pi/2} \frac{1}{1 + \sin 2x} \, dx
\]
\[
I = \pi \int_0^{\pi/2} \frac{\sec^2 x}{(1 + \tan x)^2} \, dx
\]
\[
= \pi \int_0^\infty \frac{dt}{(1 + t)^2} \quad \text{(put } \tan x = t) \quad \Rightarrow I = \pi
\]

59. If \( f(x) = \begin{cases} e^{x+1} - e^x & x \leq 0 \\ e^{1-x} - 1 & 0 < x < 1 \\ x + \ln x & x \geq 1 \end{cases} \), then:
(1) \( x = 0 \) is point of local maxima, \( x = 1 \) is neither local maxima nor local minima.
(2) \( x = 1 \) is point of local minima, \( x = 0 \) is point of local maxima.
(3) \( x = 0 \) and \( x = 1 \) both are points of local maxima.
(4) \( x = 0 \) and \( x = 1 \) both are points of local minima.

59. (1)

Graph of the function

Hence, \( dx = 0 \) is point of maxima and \( x = 1 \) is neither maxima nor minima.

60. Let \( y = f(x) \) be a real valued function satisfying \( x \frac{dy}{dx} = x^2 + y - 2, f(1) = 1 \), then \( f(3) \) equals:
(1) 0 \quad (2) 3 \quad (3) 5 \quad (4) 8

60. (3)

61. The line segment joining the points \((1, 2)\) and \((-2, 1)\) is divided by the line \(3x + 4y = 7\) in the ratio \(m : n\) where \(m, n \in \mathbb{N}\) then \(\min(m + n) = \)
(1) 7 \quad (2) 13 \quad (3) 12 \quad (4) 3

61. (2)
\[
\frac{m}{n} = \frac{3(1) + 4(2) - 7}{3(-2) + 4(1) - 7} = \frac{4}{9}
\]
\[
\therefore \quad \min(m + n) = 13
\]
62. The combined equation of straight lines through the point (1, 1) and are parallel to the lines represented by the equation \( x^2 - 5xy + 4y^2 + x + 2y - 2 = 0 \) is:

(1) \( x^2 - 5xy + 4y^2 + x + 2y - 3 = 0 \)

(2) \( x^2 - 5xy + 4y^2 + 2x + y - 3 = 0 \)

(3) \( 4x^2 - 5xy + y^2 + x + 2y - 3 = 0 \)

(4) \( x^2 - 5xy + 4y^2 + 3x - 3y = 0 \)

62. (4)

Since the required lines are \( \parallel \) to

\( x^2 - 5xy + 4y^2 + x + 2y - 2 = 0 \)

And through the point (1, 1).

\[ \therefore \quad \text{Their combined equation is} \]

\[ (x-1)^2 - 5(x-1)(y-1) + 4(y-1)^2 = 0 \]

63. \( C_1 \) and \( C_2 \) are circles of unit radius with centres at (0, 0) and (1, 0) respectively. \( C_3 \) is a circle of unit radius, passes through the centres of the circles \( C_1 \) and \( C_2 \) and have its centre above x–axis. Equation of the common tangent to \( C_1 \) and \( C_3 \) which does not pass through \( C_2 \) is:

(1) \( x - \sqrt{3}y + 2 = 0 \)

(2) \( \sqrt{3}x - y + 2 = 0 \)

(3) \( x - y - 2 = 0 \)

(4) \( x + y + 2 = 0 \)

63. (2)

Equation of any circle through (0, 0) and (1, 0) is

\[ (x-0)(x-1)+(y-0)(y-0)+\lambda y = 0 \]

\[ \Rightarrow \quad x^2 + y^2 - x + \lambda y = 0 \]

If it represents \( C_3 \), its radius = 1.

\[ \Rightarrow \quad 1 = (1/4) + (\lambda^2/4) \Rightarrow \lambda = \pm\sqrt{3} \]

As the centre of \( C_3 \), lies above the x–axis, we take

\( \lambda = -\sqrt{3} \) and thus an equation of \( C_3 \) is

\[ x^2 + y^2 - x - \sqrt{3}y = 0 . \]

Since \( C_1 \) and \( C_3 \) intersect and are of unit radius, their common tangents are parallel to the line joining their centres (0, 0) and \( \left( 1/2, \sqrt{3}/2 \right) \).

So, let the equation of a common tangent be

\[ \sqrt{3}x - y + k = 0 \] it will touch \( C_1 \), if

\[ \left| \frac{k}{\sqrt{3}+1} \right| = 1 \quad \Rightarrow \quad k = \pm 2 \]

From the figure, we observes that the required tangent makes positive intercept on the x–axis and hence its equation is \( \sqrt{3}x - y + 2 = 0 \).
64. P is a point on the positive axis of the parabola $y^2 = 4ax$; Q and R are the extremities of its latus rectum, A is its vertex. If PQR is an equilateral triangle lying within the parabola and $\angle AQP = 0$, then $\cos \theta =$

- (1) $\frac{2 - \sqrt{3}}{2\sqrt{5}}$
- (2) $\frac{2 + \sqrt{3}}{2\sqrt{5}}$
- (3) $\frac{2 + \sqrt{3}}{2\sqrt{5}}$
- (4) $\frac{2\sqrt{5} - 1}{2\sqrt{5}}$

64. (1)

![Diagram](attachment:image.png)

Clearly, $QP = 4a$; $AP = AS + SP$ where S is the focus

Since $SP = 2a \cot 30^\circ$.

$\Rightarrow AP = a + 2a\sqrt{3}$

$AQ = \sqrt{4a^2 + a^2} = a\sqrt{5}$

By sine law,

$\sin \theta = \left(\frac{AP \sin 30^\circ}{AQ}\right) / AQ = \frac{2\sqrt{3} + 1}{2\sqrt{5}}$

$\cos^2 \theta = 1 - \left(\frac{2\sqrt{3} + 1}{2\sqrt{5}}\right)^2$

$\Rightarrow \cos \theta = \frac{2 - \sqrt{3}}{2\sqrt{5}}$

65. The radius of the circle passing through the points of intersection of ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ and $x^2 - y^2 = 0$ is:

- (1) $\frac{ab}{\sqrt{a^2 + b^2}}$
- (2) $\frac{\sqrt{2}ab}{\sqrt{a^2 + b^2}}$
- (3) $\frac{a^2 - b^2}{\sqrt{a^2 + b^2}}$
- (4) $\frac{a^2 + b^2}{\sqrt{a^2 - b^2}}$

65. (2)

Two curves are symmetrical about both axes and intersect in four points, so, the circle through their points of intersection will have centre at origin.

Solving $x^2 - y^2 = 0$ and $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$, we get

$x^2 = y^2 = \frac{a^2b^2}{a^2 + b^2}$

Therefore radius of circle

$\frac{2a^2b^2}{\sqrt{a^2 + b^2}} = \frac{\sqrt{2}ab}{\sqrt{a^2 + b^2}}$
66. If \( \alpha, \beta, \gamma \) are the roots of \( x^3 + ax^2 + b = 0 \), then the determinant \( \Delta = \begin{vmatrix} \alpha & \beta & \gamma \\ \beta & \gamma & \alpha \\ \gamma & \alpha & \beta \end{vmatrix} \) equals:

(1) \(-a^3\)  
(2) \(a^3 - 3b\)  
(3) \(a^2 - 3b\)  
(4) \(a^3\)

66. \( \alpha + \beta + \gamma = -a, \Sigma \alpha \beta = 0 \), using \( R_1 \rightarrow R_1 + R_2 + R_3 \)

\[ \Delta = \begin{vmatrix} \alpha + \beta + \gamma & \alpha + \beta + \gamma & \alpha + \beta + \gamma \\ \beta & \gamma & \alpha \\ \gamma & \alpha & \beta \end{vmatrix} \]

\[ \Delta = -a \begin{vmatrix} 1 & 1 & 1 \\ \beta & \gamma & \alpha \\ \gamma & \alpha & \beta \end{vmatrix} \]

Apply \( C_2 \rightarrow C_2 - C_1, C_3 \rightarrow C_3 - C_1 \)

or \( \Delta = -a \begin{vmatrix} 1 & 0 & 0 \\ \beta & \gamma - \beta & \alpha - \beta \\ \gamma & \alpha - \gamma & \beta - \gamma \end{vmatrix} \)

67. If \( A - 2B = \begin{bmatrix} 1 & 5 \\ 3 & 7 \end{bmatrix} \) and \( 2A - 3B = \begin{bmatrix} -2 & 5 \\ 0 & 7 \end{bmatrix} \), then matrix \( B \) is equal to:

(1) \[ \begin{bmatrix} -4 & -5 \\ -6 & -7 \end{bmatrix} \]  
(2) \[ \begin{bmatrix} 0 & 6 \\ -3 & 7 \end{bmatrix} \]  
(3) \[ \begin{bmatrix} 2 & -1 \\ 3 & 2 \end{bmatrix} \]  
(4) \[ \begin{bmatrix} 6 & -1 \\ 0 & 1 \end{bmatrix} \]

67. We, have,

\[ B = (2A - 3B) - 2(A - 2B) = \begin{bmatrix} -4 & -5 \\ -6 & -7 \end{bmatrix} \]

68. In a quadrilateral \( ABCD \), \( \overrightarrow{AC} \) is the bisector of the \( \overrightarrow{AB} \wedge \overrightarrow{AD} \) which is \( \frac{2\pi}{3} \), \( 15|AC| = 3|AB| = 5|AD| \), then \( \cos \left( \overrightarrow{BA} \wedge \overrightarrow{CD} \right) \) is:

(1) \( -\frac{\sqrt{14}}{7\sqrt{2}} \)  
(2) \( -\frac{\sqrt{21}}{7\sqrt{3}} \)  
(3) \( \frac{2}{\sqrt{7}} \)  
(4) \( \frac{2\sqrt{7}}{14} \)

68. Given \( 15|AC| = 3|AB| = 5|AD| \)

\[ D(\vec{d}) \quad C(\vec{c}) \quad \pi/3 \quad \pi/3 \]

\[ A(\vec{a}) \text{ origin} \]

\[ B(\vec{b}) \]

Let \( |AC| = \lambda > 0 \)

\[|AB| = 5\lambda \]
MOCK TEST – 5

Now \( \cos(BA \wedge CD) = \frac{b \cdot (d - c)}{|BA| \cdot |CD|} \)  
\[ \cdots \cdots \text{(i)} \]

By numerator of eqn. \( (i) = b \cdot c - b \cdot d \)
\[ = |b||c| \cos \frac{\pi}{3} - |b||d| \cos \frac{2\pi}{3} \]
\[ = (5\lambda)(\lambda) \frac{1}{2} + 5\lambda (3\lambda) \frac{1}{2} \]
\[ = 5\lambda^2 + 15\lambda^2 = 10\lambda^2 \]

Denominator of eqn. \( (i) = |b||d - c| \)
\[ \text{Denominator of eqn. } \cos(BA \wedge CD) = \frac{10\lambda^2}{\sqrt{7} \lambda} = \frac{2}{\sqrt{7}} \]

69. The equation of the line passing through \((2, 3, -1)\) and lying in the plane \(2x + y - 5z - 12 = 0\) and perpendicular to the line \(2x + y - 5z - 12 = 0 = 4x - 3y + 7z\) is:

(1) \[ \frac{x - 2}{3} = \frac{y - 3}{1} = \frac{z + 1}{1} \]
(2) \[ \frac{x - 2}{4} = \frac{y - 3}{17} = \frac{z + 1}{5} \]
(3) \[ \frac{x - 2}{2} = \frac{y - 3}{1} = \frac{z + 1}{5} \]
(4) \[ \frac{x - 2}{4} = \frac{y - 3}{3} = \frac{z + 1}{7} \]

69. (1)
Any plane passing through first line
\(2x + y + z - 1 + \lambda (3x + y + 2z - 2) = 0\), if it is parallel to second line.
\((2 + 3\lambda)1 + (1 + \lambda)1 + (1 + 2\lambda)1 = 0 \Rightarrow \lambda = \frac{-2}{3}\)
Plane is \(y - z + 1 = 0\)
Distance from \((0, 0, 0) = \frac{1}{\sqrt{2}}\)

70. If \(|z_1| = |z_2|\) and \(\arg(z_1) + \arg(z_2) = \pi/2\), then:

(1) \(z_1, z_2\) is purely real
(2) \(z_1, z_2\) is purely imaginary
(3) \((z_1 + z_2)^2\) is purely real
(4) \(\arg(z_1^{-1}) + \arg(z_2^{-1}) = \frac{\pi}{2}\)
70. (2)
Let \( |z_1| = |z_2| = r \Rightarrow z_1 = r(\cos \theta + i \sin \theta) \)
And \( z_2 = r\left(\cos\left(\frac{\pi}{2} - \theta\right) + i \sin\left(\frac{\pi}{2} - \theta\right)\right) \)
\( \Rightarrow z_1z_2 = r^2i \), which is purely imaginary
\( z_1 + z_2 = r\left[(\cos \theta + i \sin \theta) + i(\cos \theta + \sin \theta)\right] \)
\( \Rightarrow (z_1 + z_2)^2 = 2r^2(\cos \theta + \sin \theta)^2i \)
Which is purely imaginary.
Also \( \arg(z_1^{-1}) + \arg(z_2^{-1}) = -\frac{\pi}{2} \)
Hence, (b) is the correct answer.

SECTION-II : (NUMERICAL VALUE TYPE)

This section contains 05 questions. The answer to each question is a NUMERICAL VALUE. For each question, enter the correct numerical value (e.g. 6.25, 7, 0.33, 30.275, 127.30, -127).

71. Words are formed from the letters of the word BHARAT. If \( p \) and \( q \) represent the number of words having both A’s together and the number of words having two A’s never together respectively, then evaluate \( \frac{p}{q} \).

71. (0.5)
Number of words having both A’s together
(i.e. AA, B, H, R, T) = 5!
\( \Rightarrow p = 120 \)
Number of words having two A’s never together (i.e. 2A’s are placed in 5 gaps created by B, H, R, T) = 4!\times^5C_2
\( \Rightarrow q = 240 \)
\( \Rightarrow \frac{p}{q} = \frac{120}{240} = 0.5 \)

72. The probability of an event happening is the square of the probability of a second event but the odds against the first are the cube of the odds against the second. If \( p_1, p_2 \) are the probabilities of first and second event respectively, then find \( \frac{p_2}{p_1} \).

72. (3)
Given that
\( p_1 = p_2^2 \) \hspace{1cm} ......... (i)
And \( \frac{1-p_1}{p_1} = \left(\frac{1-p_2}{p_2}\right)^3 \) \hspace{1cm} ......... (ii)
(i) and (ii) give
\( \frac{1-p_2}{p_2} = \left(\frac{1-p_1}{p_1}\right)^3 \)
\( \frac{p_2}{p_1} = \left(\frac{p_2}{p_1}\right)^3 \)
\[ \Rightarrow p_2 \left( 1 - p_2^2 \right) = 1 - 3p_2 + 3p_2^2 - p_2^3 \\
\Rightarrow 3p_2^2 - 4p_2 + 1 = 0 \\
\Rightarrow (3p_2 - 1)(p_2 - 1) = 0 \\
\Rightarrow p_2 = \frac{1}{3} \quad \Rightarrow p_1 = \frac{1}{9} \\
\Rightarrow \frac{p_2}{p_1} = \frac{\left( \frac{1}{3} \right)}{\left( \frac{1}{9} \right)} = 3 \\

73. If the 6th term in the expansion of \( \left( \frac{1}{x^3} + x^2 \log_{10} x \right)^8 \) is 5600, find x.

73. (10) 
\[ T_6 = \binom{8}{5} \left( \frac{1}{x^3} \right)^3 \left( x^2 \log_{10} x \right)^5 \\
= 56 \left( \frac{1}{x^3} \right)^3 \left( x^2 \log_{10} x \right)^5 \\
= 56 x^2 \left( \log_{10} x \right)^5 \]
\[ T_6 = 5600 \]
\[ \Rightarrow 56x^2 \left( \log_{10} x \right)^5 = 5600 \]
\[ \Rightarrow x^2 \left( \log_{10} x \right)^5 = 100 \]
\[ \Rightarrow x^2 \left( \log_{10} x \right)^5 = 10^2 \]
\[ \Rightarrow x = 10 \]

74. 
Consider the curve \( y^2 = 4ax \)
Let \( A_1 = \text{area of region OABO} \)
\( A_2 = \text{area of region ABDCA} \)
Find \( \frac{A_2}{A_1} \).

74. (7)
\[ x = at^2, \ y = 2at \]
\[ \Rightarrow dx = 2at \ dt \]

Area \( = A_1 = 2 \int_0^a y \, dx \)
\[ = 2 \int_0^1 (2at)(2at) \, dt \cdots \left[ at^2 = a \Rightarrow t = 1 \right] \]
\[ = 2 \times 4a^2 \left( \frac{t^3}{3} \right)_0^1 = \frac{8a^2}{3} \]

\( at^2 = 4a \Rightarrow t = 2 \)

Area of region OCD \( = 2 \int_0^2 (2at)(2at) \, dt \)
\[ = 8a^2 \left( \frac{t^3}{3} \right)_0^2 = \frac{64a^2}{3} \]
\[ \Rightarrow A_2 = \frac{64a^2}{3} - \frac{8a^2}{3} = \frac{56a^2}{3} \]

\[ \Rightarrow \frac{A_2}{A_1} = \frac{\frac{56a^2}{3}}{\frac{8a^2}{3}} = 7 \]

75. Consider the following frequency distribution.

<table>
<thead>
<tr>
<th>Class</th>
<th>0–10</th>
<th>10–20</th>
<th>20–30</th>
<th>30–40</th>
<th>40–50</th>
</tr>
</thead>
<tbody>
<tr>
<td>Frequency</td>
<td>5</td>
<td>x</td>
<td>10</td>
<td>Y</td>
<td>15</td>
</tr>
</tbody>
</table>

If the sum of the frequencies is 50, and the median is 29, then evaluate \(|x - y|\).

75. \(2\)

<table>
<thead>
<tr>
<th>Class</th>
<th>Frequency</th>
<th>c.f</th>
</tr>
</thead>
<tbody>
<tr>
<td>0–10</td>
<td>5</td>
<td>5</td>
</tr>
<tr>
<td>10–20</td>
<td>x</td>
<td>5 + x</td>
</tr>
<tr>
<td>20–30</td>
<td>10</td>
<td>15 + x</td>
</tr>
<tr>
<td>30–40</td>
<td>y</td>
<td>15 + x + y</td>
</tr>
<tr>
<td>40–50</td>
<td>15</td>
<td>30 + x + y</td>
</tr>
<tr>
<td>Total</td>
<td>N = 30 + x + y</td>
<td></td>
</tr>
</tbody>
</table>

Sum of frequency = 50
\[ \Rightarrow 30 + x + y = 50 \]
\[ \Leftrightarrow x + y = 20 \] \( \ldots \ldots \) \(i\)
Median = 29
\[ \Rightarrow \text{Median lies in the interval} \ 20–30 \]
\[
\text{Median} = \ell_1 + \frac{\left(\frac{N}{2} - c.f\right)}{f} \times h
\]

\[\Rightarrow \text{Median} = \ell_1 + \frac{\left(\frac{N}{2} - c.f\right)}{f} \times h\]

\[\Rightarrow \text{Here } \ell_1 = 20, f = 10, c.f = 5 + x, h = 10, \quad N = 50\]

\[\Rightarrow 29 = 20 + \frac{\left(25 - (5 + x)\right)}{10} \times 10\]

\[\Rightarrow 9 = 20 - x\]

\[\Rightarrow x = 11 \quad \ldots \ldots \text{ (ii)}\]

\[y = 9 \quad \ldots \ldots \text{ [From (i) and (ii)]}\]

\[\Rightarrow |x - y| = |11 - 9| = 2\]