PART (A) : PHYSICS

SECTION-I : (SINGLE ANSWER CORRECT TYPE)

This section contains 20 multiple choice questions. Each question has four choices (1), (2), (3) and (4) out of which ONLY ONE is correct.

1. Two stones are thrown up simultaneously from the edge of a cliff 240 m high with initial speed of 10 m/s and 40 m/s respectively. Which of the following graph best represents the time variation of relative position of the second stone with respect to the first? (Assume stones do no rebound after hitting the ground and neglect air resistance, take $g = 10 \text{ m/s}^2$) (The figures are schematic and not drawn to scale)

1. (3) 
1st stone
$0 \leq t \leq 8 \text{ sec}$
$v_r = 40 - 10$
$= 30 \text{ m/s}$
$a_r = 0$

$S_r = v_r \times t = 30 \times 8 = 240 \text{ m}$

$8 \text{ sec} < t \leq 12 \text{ sec}$
$v_r$ increases in magnitude and relative acceleration is $g$ downwards
2. The period of oscillation of a simple pendulum is \( T = 2\pi \sqrt{\frac{L}{g}} \). Measured value of L is 20.0 cm known to 1 mm accuracy and time for 100 oscillations of the pendulum is found to be 90 s using a wrist watch of 1 s resolution. The accuracy in the determination of \( g \) is:

   (1) 1%  
   (2) 3%  
   (3) 2%  
   (4) 5%

2. \[
\frac{1}{2} \frac{d^2g}{g} = \frac{1}{2} \frac{dL}{L} + \frac{dT}{T}
\]

\[
= \frac{1}{2} \times \frac{0.1}{20} + \frac{1/100}{90/100} = \frac{1}{400} + \frac{1}{90}
\]

\[
\frac{1}{2} \frac{d^2g}{g} = \frac{1}{400} + \frac{1}{90}
\]

\[
\frac{dg}{g} = \left( \frac{490}{400 \times 90} \right) \times 2
\]

\[
= \left( \frac{490}{200 \times 90} \right) = .0272
\]

\[
dg/g \times 100 \approx 2.72\% \approx 3\%
\]

3. Distance of the centre of mass of a solid uniform cone from its vertex is \( Z_0 \). If the radius of its base is \( R \) and its height is \( h \) then \( Z_0 \) is equal to:

   (1) \( \frac{5h}{8} \)  
   (2) \( \frac{3h}{4} \)  
   (3) \( \frac{5h}{8} \)  
   (4) \( \frac{3h^2}{8R} \)

3. \( Z_0 = \frac{3h}{4} \)

(From class theory)

4. From a solid sphere of mass \( M \) and radius \( R \) a cube of maximum possible volume is cut. Moment of inertia of cube about an axis passing through its centre and perpendicular to one of its faces is:

   (1) \( \frac{MR^2}{32\sqrt{2}\pi} \)  
   (2) \( \frac{4MR^2}{3\sqrt{3}\pi} \)  
   (3) \( \frac{4MR^2}{9\sqrt{3}\pi} \)  
   (4) \( \frac{MR^2}{16\sqrt{2}\pi} \)

4. \( \frac{4MR^2}{9\sqrt{3}\pi} \)
I = \frac{\text{M}x^2}{6}

Edge length : (x)
2R = \sqrt{3}x
x = \frac{2R}{\sqrt{3}}

Now,
Mass of cube :
\[
m = \frac{\text{M}}{\left(\frac{4}{3}\pi R^3\right)^\frac{2}{3}}\left(\frac{2R}{\sqrt{3}}\right)^3
\]
\[
\left(\frac{3\text{M}}{4\pi R^3}\right)\left(\frac{8R^3}{3\sqrt{3}}\right)
\]
\[
m = \frac{2\text{M}}{\sqrt{3}\pi}
\]
\[
I = \frac{1}{3}\left(\frac{2\text{M}}{\sqrt{3}\pi}\right)\left[\frac{4R^2}{3}\right]
\]
\[
= \frac{4\text{MR}^2}{9\sqrt{3}\pi}
\]

5. From a solid sphere of mass \(\text{M}\) and radius \(\text{R}\), a spherical portion of radius \(\text{R}/2\) is removed, as shown in the figure. Taking gravitational potential \(V = 0\) at \(r = \infty\), the potential at the centre of the cavity thus formed is:

\((\text{G} = \text{gravitational constant})\)

\[
(1) \quad \frac{-2\text{GM}}{3\text{R}} \quad (2) \quad \frac{-\text{GM}}{\text{R}} \quad (3) \quad \frac{-\text{GM}}{2\text{R}} \quad (4) \quad \frac{-2\text{GM}}{\text{R}}
\]

5. (2)
Solid sphere is of mass \(\text{M}\), radius \(\text{R}\). Spherical portion removed have radius \(\text{R}/2\), therefore its mass is \(\text{M}/8\).
Potential at the centre of cavity = \(V_{\text{solid sphere}} + V_{\text{removed part}}\)

\[
= \frac{-\text{GM}}{2\text{R}^3}\left[3\text{R}^2 - \left(\frac{\text{R}}{2}\right)^2\right] + \frac{3\text{G}(\text{M}/8)}{2(\text{R}/2)} = \frac{-\text{GM}}{\text{R}}
\]
6. Consider a spherical shell of radius R at temperature T. The black body radiation inside it can be considered as an ideal gas of photons with internal energy per unit volume $u = \frac{U}{V} \propto T^4$ and pressure $P = \frac{1}{3} \left( \frac{U}{V} \right)$. If the shell now undergoes an adiabatic expansion the relation between T and R is:

(1) $T \propto e^{-R}$
(2) $T \propto \frac{1}{R^3}$
(3) $T \propto \frac{1}{R}$
(4) $T \propto e^{-3R}$

6. (3)

$$u = \frac{U}{V} \propto T^4$$

$$P = \frac{1}{3} \left( \frac{U}{V} \right)$$

Adiabatic expansion

$$TV^{-\gamma-1} = K$$

$$TV^2 = C$$

$$\gamma - 1 = \frac{\gamma}{4}$$

$$\frac{3\gamma}{4} = 1$$

$$\gamma = 3$$

$$TV^\gamma = C$$

$$TV^{\frac{1}{3}} = C$$

$$T \left( \frac{4}{3} \pi R^3 \right)^{\frac{1}{3}} = C$$

$$T \propto \frac{1}{R}$$

7. For a simple pendulum, a graph is plotted between its kinetic energy (KE) and potential energy (PE) against its displacement d. Which one of the following represents these correctly? (graphs are schematic and not drawn to scale)

(1)  
(2)  
(3)  
(4)  

7. (2)
KE<sub>max</sub> at mean position
\[ V = \text{max} \]
\[ \nu = 0 \]

PE<sub>min</sub> at mean position

8. A uniformly charged solid sphere of radius \( R \) has potential \( V_0 \) (measured with respect to \( \infty \)) on its surface. For this sphere the equipotential surfaces with potentials \( \frac{3V_0}{2}, \frac{5V_0}{4}, \frac{3V_0}{4}, \text{and} \ \frac{V_0}{4} \) have radius \( R_1, R_2, R_3, \text{and} \ R_4 \) respectively. Then

(1) \[ R_1 = 0 \text{ and } R_2 > (R_4 - R_3) \]

(2) \[ R_1 \neq 0 \text{ and } (R_2 - R_1) > (R_4 - R_3) \]

(3) \[ R_1 = 0 \text{ and } R_2 < (R_4 - R_3) \]

(4) None of these

8. (3, 4)

\[ R_1 = \frac{3V_0}{2}; R_2 = \frac{5V_0}{4}; R_3 = \frac{3V_0}{4}; R_4 = \frac{V_0}{4} \]

\[ \therefore \ r < R \ \ \ V = \frac{KQ}{2R^2}(3R^2 - r^2) \]

\[ \nu = \frac{3V_0}{2}, \ R_1 = 0 \]

\[ \frac{5V_0}{4} = \frac{KQ}{2R^2}(3R^2 - R_2^2) \]

\[ \therefore \ R_2 = \frac{R}{\sqrt{2}} \]

\[ r > R \]

\[ \frac{3V_0}{4} = \frac{KQ}{R_3} \]

\[ R_3 = \frac{4KQ}{3V_0} = \frac{KQ \times R}{3 \times KQ} = \frac{R}{3} \]

\[ \frac{V_0}{4} = \frac{KQ}{R_4} \]

\[ \therefore \ R_4 = \frac{4KQ}{V_0} = \frac{4KQ}{KQ} \times R = 4R \]

On comparing we get
9. In the given circuit, charge $Q_2$ on the 2µF capacitor changes as $C$ is varied from 1µF to 3µF. $Q_2$ as a function of ‘C’ is given properly by: (figure are drawn schematically and are not to scale)

![Diagram of the circuit]

(1) & (2)

(3) & (4)

9. 2

$q = \left( \frac{3C}{C+3} \right) E \quad q = CV$

$q_2 = \left( \frac{3C}{C+3} \right) E \left( \frac{2}{3} \right)$

$q_2 = \left( \frac{2C}{C+3} \right) E$

$q_2 = \left( \frac{2}{1 + \frac{3}{C}} \right) E \quad q = CV$

$C \uparrow \quad q_2 \uparrow$

If $C \to \infty$, $q = \text{constant value}$.

10. In the circuit shown, the current in the 1Ω resistor is:
10. (3)
Take \( V_p = 0 \) & \( V_q = x \)

\[
x + 9 + \frac{x - 6}{3} + \frac{x}{1} = 0
\]

\[
\Rightarrow 3x + 27 + 5x - 30 + 15x = 0
\]

\[
\Rightarrow x = \frac{3}{23} \quad \text{V} \quad \text{&} \quad i = \frac{3}{23} \quad \text{A}
\]

From Q to P

11. Two long current carrying thin wires, both with current I, are held by insulating threads of length L and are in equilibrium as shown in the figure, with threads making an angle \( \theta \) with the vertical. If wires have mass \( \lambda \) per unit length then the value of I is:

\( (g = \text{gravitational acceleration}) \)

(1) \( \sqrt{\frac{rgL}{\mu_0} \tan \theta} \)

(2) \( 2 \sin \theta \sqrt{\frac{\pi \lambda gL}{\mu_0 \cos \theta}} \)

(3) \( \sin \theta \sqrt{\frac{\pi \lambda gL}{\mu_0 \cos \theta}} \)

(4) \( \sqrt{\frac{\pi \lambda gL}{\mu_0} \tan \theta} \)

11. (2)
Forces per unit length are taken

\[ \tan \theta = \frac{\mu_0 i}{2\pi d} \]

\[ i = \frac{\lambda g \sin \theta}{\mu_0 \cos \theta} \quad [d=2L \sin \theta] \]

\[ i = 2 \sin \theta \frac{\lambda g \pi L}{\mu_0 \cos \theta} \]

12. A rectangular loop of sides 10 cm and 5 cm carrying a current I of 12 A is placed in different orientation as shown in the figures below:

If there is a uniform magnetic field of 0.3 T in the positive z direction, in which orientations the loop would be in (i) stable equilibrium and (ii) unstable equilibrium?

(1) (a) and (b), respectively
(2) (b) and (c), respectively
(3) (b) and (d), respectively
(4) (a) and (c), respectively

12. (3)

For equilibrium \( \vec{\tau} = 0 \)

\[ \vec{\tau} = MB \sin \theta \hat{n} \]

If, \( \sin \theta = 0; \quad \vec{\tau} = 0 \)

If angle between \( \vec{M} \) and \( \vec{B} \) is zero, then stable equilibrium

If angle between \( \vec{M} \) and \( \vec{B} \) is \( \pi \), then unstable equilibrium

13. An inductor (\( L = 0.03 \text{ H} \)) and a resistor (\( R = 0.15 \text{ k}\Omega \)) are connected in series to a battery of 15V EMF in a circuit shown below. The key \( K_1 \) has been kept closed for a long time. Then at \( t = 0 \), \( K_1 \) is opened and key \( K_2 \) is closed simultaneously. At \( t = 1 \text{ ms} \), the current in the circuit will be \( (e^i \approx 150) \)
13. According to given conditions:

\[ i_0 = \frac{V}{R} \]
\[ = \frac{15}{0.15 \times 10^3} \]
\[ = 0.1 \text{ A} \]

\[ i = i_0 e^{\frac{-15 \times 10^3}{0.03}} \]
\[ = 0.1 \times e^{\frac{-150}{5}} = 0.1 \times e^{-5} = \frac{0.1}{150} = 0.67 \text{ mA} \]

14. Monochromatic light is incident on a glass prism of angle \(A\). If the refractive index of the material of the prism is \(\mu\), a ray, incident at an angle \(\theta\), on the face AB would get transmitted through the face AC of the prism provided:

\[ \theta > \sin^{-1}\left[\mu \sin \left(A - \sin^{-1}\left(\frac{1}{\mu}\right)\right)\right] \quad (1) \]
\[ \theta < \cos^{-1}\left[\mu \sin \left(A - \sin^{-1}\left(\frac{1}{\mu}\right)\right)\right] \quad (2) \]
\[ \theta > \cos^{-1}\left[\mu \sin \left(A + \sin^{-1}\left(\frac{1}{\mu}\right)\right)\right] \quad (3) \]
\[ \theta > \sin^{-1}\left[\mu \sin \left(A - \sin^{-1}\left(\frac{1}{\mu}\right)\right)\right] \quad (4) \]
\[
\sin^{-1} \left[ \mu \left[ \sin \left( A - \sin^{-1} \left( \frac{1}{m} \right) \right) \right] \right] < 0
\]

15. A signal of 5 kHz frequency is amplitude modulated on a carrier wave of frequency 2000 kHz. The frequencies of the resultant signal is/are:
   (1) 2 MHz only
   (2) 2000 kHz and 1995 kHz
   (3) 2005 kHz, 2000 kHz and 1995 kHz
   (4) 2005 kHz and 1995 kHz

15. (3)
   Frequencies are \( F_c, F_c \pm F_s \)

16. An LCR circuit is equivalent to a damped pendulum. In an LCR circuit the capacitor is charged to \( Q_0 \) and then connected to the L and R as shown below:

   ![Diagram of LCR circuit](image)

If a student plots graphs of the square of maximum charge \( Q_{max}^2 \) on the capacitor with time (t) for two different values \( L_1 \) and \( L_2 \) \((L_1 > L_2)\) of L then which of the following represents this graph correctly?
(Plots are schematic and not drawn to scale)

(1) \( Q_{max}^2 \) vs. t
(2) \( Q_{max}^2 \) vs. t for both \( L_1 \) and \( L_2 \)
(3) \( Q_{max}^2 \) vs. t
(4) \( Q_{max}^2 \) vs. t

16. (1)
IR + L \frac{dI}{dt} = \frac{q}{C} = 0

L \frac{d^2q}{dt^2} = -R \frac{dq}{dt} + \frac{q}{C}

Comparing with equation of damped oscillation

\frac{d^2y}{dt^2} = -\gamma \frac{dy}{dt} - ky

The equation of amplitude is \( y = Ae^{-bt} \)

Where \( b = \frac{\gamma}{2m} = \frac{R}{2L} \)

\therefore q_{max} = q_0 e^{\frac{R}{2L}}

\therefore q^2 = q_0^2 e^{-\frac{R}{2L}}

\therefore \text{time constant } \tau = \frac{R}{L}

Since \( L_1 > L_2 \)

\( \tau_1 < \tau_2 \)

Hence correct graph is 3.

**Alternative solution**

The value of \( Q_{max} \) reduces because of energy dissipation in resistor. As the value of inductor increases the time taken for capacity to discharge or charge increases therefore heat dissipation time decreases.

Hence correct graph is 3.

17. Given the figure are two blocks A and B of weight 20 N and 100 N, respectively. These are being pressed against a wall by a force \( F \) as shown. If the coefficient of friction between the blocks is 0.1 and between block B and the wall is 0.15, the frictional force applied by the wall on block B is

\( F \)

A

B

(1) 100 N (2) 150 N (3) 120 N (4) 80 N

17. (3)

Assume the system is in equilibrium. Net gravitational force must be balanced by friction force from the wall.

Force of friction = 120 N
18. Assuming human pupil to have a radius of 0.25 cm and a comfortable viewing distance of 25 cm, the minimum separation between two objects that human eye can resolve at 500 nm wavelength is:
   (1) 100 µm  (2) 30 µm  (3) 1 µm  (4) 300 µm

18. (2)
   \[ y = 1.22 \frac{\lambda D}{d} = 1.22 \times \frac{500 \times 10^{-9} \times 25 \times 10^{-2}}{2 \times 0.25 \times 10^{-2}} \]
   \[ \Rightarrow y = 30 \mu m \]

19. A uniform disc of mass \( m = 2 \) kg and radius \( R = 5 \) cm is pivoted smoothly at its centre of mass. A light spring of stiffness \( k \) is attached with the disc tangentially (as shown in figure). The angular frequency of torsional oscillation of the disc is \( \boxed{\text{(1) 2.23}} \) rad/s. (Take \( k = 5N/s, \sqrt{5} = 2.23 \))

![Disc Diagram]

19. (1)

If disc is twisted clockwise for a small angle \( \theta \), deformation in spring \( x = R \theta \)
   \[ \Rightarrow F_s = kx = k (R \theta) \]

   \[ \tau_{res} = -F_s R \]
   \[ = -kR^2 \theta \quad \ldots \ldots \text{(i)} \]

   Also \( \tau = I_c \alpha \quad \ldots \ldots \text{(ii)} \)

   From (i) & (ii)
   \[ \Rightarrow \alpha = \frac{kR^2 \theta}{I_c} \]
   \[ = -\frac{2k}{m} \theta \quad \ldots \ldots \left[ : I_c = \frac{mR^2}{2} \right] \]

   Comparing with standard equation,
   \[ \omega^2 \theta = \frac{-2k}{m} \theta \]
   \[ \therefore \omega = \sqrt{\frac{2k}{m}} \]
\[ \omega = \sqrt{\frac{2 \times 5}{2}} \]
\[ \therefore \omega = 2.23 \text{ rad/s} \]

20. In the figure shown below, the charges on the left plate of the 10 \( \mu F \) capacitor is \(-30 \mu C\). The charge on the right plate of the 6 \( \mu F \) capacitor is _______ \( \mu C \).

![Capacitor Diagram]

(1) 6 \( \mu C \) (2) \(-6 \mu C\) (3) 18 \( \mu C \) (4) \(-18 \mu C\)

20. (3)

Let \( q_1 \) and \( q_2 \) be charge on 6 \( \mu F \) and 4 \( \mu F \) respectively.

\[ q_1 + q_2 = q \quad \text{ .......... (i)} \]

Also, \[ \frac{q_1}{C_1} = \frac{q_2}{C_2} \quad \text{ .......... (ii)} \]

\[ \therefore C_1 \text{ and } C_2 \text{ are in parallel combination} \]

\[ q_2 = \frac{C_2}{C_1} q_1 \]

\[ = \frac{4}{6} q_1 \quad \text{ .......... (iii)} \]

Using (i) and (iii),

\[ \frac{10}{6} q_1 = q \]

\[ \therefore q = \frac{5}{3} q_1 \]

\[ q_1 = \frac{3}{5} \times 30 = 18 \mu C \]

\[ \text{SECTION-II : (NUMERICAL VALUE TYPE)} \]

This section contains 05 questions. The answer to each question is a NUMERICAL VALUE. For each question, enter the correct numerical value (e.g. 6.25, 7, 0.33, 30.275, 127.30, -127)
21. A block of mass 15 kg is placed at a distance of 10 m from the rear end of a long trolley as shown in figure. The coefficient of friction between the block and the surface below is 0.4. Starting from rest, the trolley is given a uniform acceleration of 5 m/s². At what distance (in metres) from the starting point will the block fall off the trolley? (Take g = 10 m/s²)

![Diagram of trolley with block](image)

Acceleration of the block = acceleration of the trolley = \( a = 5 \text{ m/s}^2 \)

\[ F = ma = 15 \times 5 = 75 \text{ N} \] in the backward direction.

The force of limiting friction \( f \) is given by,

\[ f = \mu mg = 0.4 \times 15 \times 10 = 60 \text{ N} \] in forward direction.

Hence, the net force on the block towards the right i.e., towards the rear and of the trolley is,

\[ F' = F - f = 75 - 60 = 15 \text{ N} \]

Acceleration due to this force is,

\[ a' = \frac{F'}{m} = \frac{15}{15} = 1 \text{ m/s}^2 \]

Let \( t \) be the time taken for the block to fall from the rear end of the trolley travelling a distance

\[ s = 10 \text{ m} \]

\[ s = ut + \frac{1}{2}at^2 \]

\[ 10 = 0 + \frac{1}{2} \times 1 \times t^2 \]

\[ \therefore t = \sqrt{20} \text{ s} \]

The distance covered by the trolley in time

\[ t = \sqrt{20} \text{ s} \] is given by,

\[ s' = ut + \frac{1}{2}at^2 = 0 + \frac{1}{2} \times 5 \times 20 \]

\[ \therefore s' = 50 \text{ m} \]

22. A steel rod of cross-sectional area 1 m² is acted upon by forces shown in the figure. Determine the total elongation of the bar in \( \mu \text{m} \).

[Take \( Y = 2.0 \times 10^{11} \text{ N/m}^2 \) ]

![Diagram of steel rod with forces](image)

22. (1.3)
The action of forces on each part of rod is shown below.

As, \( \ell = \frac{FL}{AY} \)

\[ \therefore \ell_{PQ} = \frac{(60 \times 10^3) \times 1.5}{1 \times 2 \times 10^{11}} = 4.5 \times 10^{-7} \text{ m} \]

\[ \ell_{QR} = \frac{(70 \times 10^3) \times 1}{1 \times 2 \times 10^{11}} = 3.5 \times 10^{-7} \text{ m} \]

And \( \ell_{RS} = \frac{(50 \times 10^3) \times 2}{1 \times 2 \times 10^{11}} = 5.0 \times 10^{-7} \text{ m} \)

The total extension

\[ \ell = 4.5 \times 10^{-7} + 3.5 \times 10^{-7} + 5.0 \times 10^{-7} \]

\[ = 13 \times 10^{-7} \text{ m} \]

\[ = 1.3 \times 10^{-6} \text{ m} \]

\[ \therefore \ell = 1.3 \mu \text{m} \]

23. In the given diagram, CP represents a wavefront and AO and BP, the corresponding two rays. The condition of \( \theta \) for constructive interference at P between the rays BP and reflected ray AOP is found to be \( \cos \theta = \frac{Q \lambda}{d} \). The value of Q is \_________. \
23. (0.25)
Path difference between the two rays is given by
\[ \Delta = CO + PO + \frac{\lambda}{2} \]

So, \[ PO = d \sec \theta \]
And \[ CO = PO \cos 2\theta = d \sec \theta \cos 2\theta \]

So, \[ \Delta = \left( d \sec \theta + d \sec \theta \cos 2\theta \right) + \frac{\lambda}{2} \]
Phase difference between two rays is \( \phi = \pi \) (as one ray is reflected one and another is direct).
Now, for constructive interference, path difference should be even multiple of half wavelength i.e., \( \lambda, 2\lambda, 3\lambda \).

\[ \therefore d \sec \theta + d \sec \theta \cos 2\theta + \frac{\lambda}{2} = \lambda \]

\[ \therefore d \sec \theta (1 + \cos 2\theta) = \frac{\lambda}{2} \]

\[ \therefore d \sec \theta (2 \cos^2 \theta) = \frac{\lambda}{2} \]

\[ \therefore \cos \theta = \frac{\lambda}{4d} \]

Comparing with \( \frac{\lambda}{d} \)

\[ \Rightarrow Q = \frac{1}{4} = 0.25 \]
24. Each diode shown in the figure has a forward bias resistance of 40 Ω and reverse bias resistance of 1.5Ω. The current through the resistance 100Ω is _______ mA.

\[
\begin{align*}
48 \Omega & \quad 86.5 \Omega \\
12 \text{V} & \quad 100 \Omega
\end{align*}
\]

24. (83.33)

According to given values of diode resistances, the equivalent circuit becomes

\[
\begin{align*}
R_{\text{net}} &= \frac{88 \times 88}{88 + 88} + 100 = 144 \Omega \\
I &= \frac{12}{144} = \frac{1}{12} = 0.08333 \text{A} = 83.33 \text{mA}
\end{align*}
\]

25. A parallel plate capacitor of area A, plate separation d and capacitance C is filled with three different dielectric materials \(K_1 = 4, K_2 = 5\) and \(K_3 = 3\) (as shown in figure). If a single dielectric material is to be used to have the same capacitance C in this capacitor, then what is the value of its dielectric constant K?

\[
\begin{align*}
C_1 &= \frac{\varepsilon_0 \left(\frac{A}{2}\right) K_1}{d} = \frac{\varepsilon_0 AK_1}{d} \\
C_2 &= \frac{\varepsilon_0 K_2}{d} = \frac{\varepsilon_0 AK_2}{d} \\
C_3 &= \frac{\varepsilon_0 AK_3}{d} = \frac{2\varepsilon_0 AK_3}{d}
\end{align*}
\]

The capacitors \(C_1\) and \(C_2\) are in parallel and their equivalent capacitance,
C' = C_1 + C_2 = \frac{\varepsilon_0 A}{d} (K_1 + K_2)

This combination is in series with C_3. Hence the net capacitance

\[
\frac{1}{C''} = \frac{1}{C'} + \frac{1}{C_3} = \frac{d}{\varepsilon_0 A (K_1 + K_2)} + \frac{d}{2\varepsilon_0 A K_3}
\]

\[
\therefore \frac{1}{C''} = \frac{d}{\varepsilon_0 A} \left( \frac{1}{K_1 + K_2} + \frac{1}{2K_3} \right)
\]

\[
\therefore \frac{d}{\varepsilon_0 A K} = \frac{d}{\varepsilon_0 A} \left( \frac{1}{K_1 + K_2} + \frac{1}{2K_3} \right)
\]

\[
\therefore \frac{1}{K} = \frac{1}{K_1 + K_2} + \frac{1}{2K_3} = \frac{1}{4 + 5} + \frac{1}{2 \times 3} = \frac{1}{9} + \frac{1}{6} = \frac{1 + 1}{9 + 6} = \frac{2 + 3}{18} = \frac{5}{18}
\]

\[
\therefore K = \frac{18}{5} = 3.6
\]
PART (B) : CHEMISTRY

SECTION-I : (SINGLE ANSWER CORRECT TYPE)

This section contains 20 multiple choice questions. Each question has four choices (1), (2), (3) and (4) out of which ONLY ONE is correct.

26. The molecular formula of a commercial resin used for exchanging ions in water softening is \( C_8H_7SO_3Na \) (Mol. wt. 206). What would be the maximum uptake of \( Ca^{2+} \) ions by the resin when expressed in mole per gram resin?
   (1) \( \frac{1}{103} \)  
   (2) \( \frac{1}{206} \)  
   (3) \( \frac{2}{309} \)  
   (4) \( \frac{1}{412} \)

26. (4)
\[
2C_8H_7SO_3Na + Ca^{2+} \rightarrow (C_8H_7SO_3)_2Ca
\]
2 mole \( \rightarrow \) 1 mole
2 \times 206 gm take 1 mole of \( Ca^{2+} \)
\[\therefore 1 \text{ gm takes} \quad \frac{1}{412} \text{ mole of } Ca^{2+}\]

27. Which of the following is the energy of a possible excited state of hydrogen?
   (1) \(+ 13.6 \text{ eV}\)  
   (2) \(- 6.8 \text{ eV}\)  
   (3) \(- 3.4 \text{ eV}\)  
   (4) \(+ 6.8 \text{ eV}\)

27. (3)
\[
\frac{13.6z^2}{n^2} \Rightarrow \text{for hydrogen; } z = 1
\]
\[
= \frac{13.6}{n^2}
\]
Possible is \(- 13.6, -3.4, -1.5\) etc.

28. The following reaction is performed at 298 K?
\[
2NO(g) + O_2(g) \rightleftharpoons 2NO_2(g)
\]
The standard free energy of formation of NO(g) is 86.6 kJ/mol at 298 K. What is the standard free energy of formation of \( NO_2(g) \) at 298 K? \( (K_p = 1.6 \times 10^{12}) \)
   (1) \( R(298) \ln \left(1.6 \times 10^{12} \right) - 86600 \)  
   (2) \( 86600 + R(298) \ln \left(1.6 \times 10^{12} \right) \)  
   (3) \( 86600 - \frac{\ln \left(1.6 \times 10^{12} \right)}{R(298)} \)  
   (4) \( 0.5[2 \times 86,600 - R(298) \ln \left(1.6 \times 10^{12} \right)] \)

28. (4)
\[
\frac{R \times 298 eV \times 1.6 \times 10^{12}}{2} = \Delta G^0_r = 2\Delta G^0_{NO_2} - 2\Delta G^0_{NO}
\]
\[
\Delta G^0_{NO_2} = 86.6 \times 10^3 - \frac{298 K eV \times 1.6 \times 10^{12}}{2}
\]
29. The standard Gibbs energy change at 300K for the reaction $2A \rightarrow B + C$ is 2494.2 J. At a given time, the composition of the reaction mixture is $[A] = \frac{1}{2}$, $[B] = 2$ and $[C] = \frac{1}{2}$. The reaction proceeds in the : $[R = 8.314 \text{ J/K/mol}, e = 2.718]$

1) forward direction because $Q > K_C$  
2) reverse direction because $Q > K_C$

3) forward direction because $Q < K_C$  
4) reverse direction because $Q < K_C$

29. (2)
$\Delta G^\circ$ at 300 K = 2494.2 J
$2A \rightarrow B + C$
$\Delta G^\circ = -RT\ln K$
$- 2494.2 = -8.314 \times 300 \ln K$
$K = 2.718$

$Q = \frac{[B][C]}{[A]^2} = \frac{2 \times \frac{1}{2}}{\left(\frac{1}{2}\right)^2} = 4$

$Q > K_C \Rightarrow$ reverse direction

30. The ionic radii (in $\text{Å}$) of $\text{N}^{3-}$, $\text{O}^{2-}$ and $\text{F}^-$ are respectively:

1) 1.36, 1.40 and 1.71  
2) 1.36, 1.71 and 1.40

3) 1.71, 1.40 and 1.36  
4) 1.71, 1.36 and 1.40

30. (3)
Isoelectronic species. If number of protons are more size will be less.

31. From the following statements regarding $\text{H}_2\text{O}_2$, choose the incorrect statement?

1) It can act only as an oxidizing agent  
2) It decomposes on exposure to light

3) It has to be stored in plastic or wax lined glass bottles in dark  
4) It has to be kept away from dust

31. (1)
It acts as oxidizing as well as reducing agent.

32. Which one of the following alkaline earth metal sulphates has its hydration enthalpy greater than its lattice enthalpy?

1) $\text{CaSO}_4$  
2) $\text{BeSO}_4$

3) $\text{BaSO}_4$  
4) $\text{SrSO}_4$

32. (2)
$\text{BeSO}_4$ is only the soluble sulphate because its hydration energy more than its lattice energy, rest of all are ppt.

33. Which among the following is the most reactive?

1) $\text{Cl}_2$  
2) $\text{Br}$

3) $\text{I}_2$  
4) $\text{ICl}$
33. (4) It has some dipole moment value and it is polar, rest of all are nonpolar and \( \mu = 0 \).

34. Which one has the highest boiling point?
   (1) He  (2) Ne  (3) Kr  (4) Xe

34. (4) More is the atomic weight more will be boiling point.

35. In Carius method of estimation of halogens, 250 mg of an organic compound gave 141 mg of AgBr. The percentage of bromine in the compound is : (at. mass Ag = 108, Br = 80)
   (1) 24  (B) 36  (3) 48  (4) 60

35. (1) Moles of Br = \( 1 \times \text{moles of AgBr} \)
       \[ = 1 \times \frac{141 \times 10^{-3}}{188} \times 80 \]
       Mass of Br = \( \frac{141 \times 10^{-3}}{188} \times 80 \)
       \[ \therefore \% \text{ of Br} = \frac{141 \times 10^{-3}}{250 \times 10^{-3}} \times 100 = 24\% \]

36. Which of the following compounds will exhibit geometrical isomerism?
   (1) 1-phenyl-2-butene  (2) 3-phenyl-1-butene  (3) 2-phenyl-1-butene  (4) 1, 1-Diphenyl-1-propane

36. (1) \( \text{H}_2\text{C} - \text{HC} = \text{CH} - \text{CH}_2 - \text{Ph} \)
   Both double bonded carbon are differently disubstituted.

37. Which compound would give 5 – keto – 2 - methyl hexanal upon ozonolysis?
   (1)  (2)  (3)  (4)

37. (2)
38. The synthesis of alkyl fluorides is best accomplished by:
   (1) Free radical fluorination  (2) Sandmeyer's reaction
   (3) Finkelstein reaction  (4) Swarts reaction

38. (4)
   \[ R - Cl \xrightarrow{AgF/dmF} R - F + AgCl/AgBr \]
   \[ R - Br \]
   Swarts reaction

39. In the reaction
   \[ \begin{array}{c}
   \textrm{NH}_2 \\
   \textrm{NaNO}_2/\text{HCl} \\
   0 - 5^\circ\text{C} \\
   \textrm{CuCN/KCN} \\
   \Delta \\
   \end{array} \xrightarrow{} \begin{array}{c}
   \text{D} \\
   \text{E + N}_2 \\
   \end{array} \]
   the product E is:
   (1) \[ \begin{array}{c}
   \text{COOH} \\
   \end{array} \]
   (2) \[ \begin{array}{c}
   \text{H}_3\text{C} - \text{C} - \text{C} - \text{H}_3 \\
   \end{array} \]
   (3) \[ \begin{array}{c}
   \text{CN} \\
   \end{array} \]
   (4) \[ \begin{array}{c}
   \text{CH}_3 \\
   \end{array} \]

39. (3)

40. Which polymer is used in the manufacture of paints and lacquers?
   (1) Bakelite  (2) Glyptal
   (3) Polypropene  (4) Poly vinyl chloride

40. (2)
   Glyptal polymer is used in the manufacture of paints and lacquers.
41. Which of the following compounds is not an antacid?
   (1) Aluminium hydroxide  (2) Cimetidine
   (3) Phenelzine  (4) Ranitidine

41. Phenelzine is not an antacid.

42. Which of the following compounds is not colored yellow?
   (1) $\text{Zn}_2[\text{Fe} (\text{CN})_6]$  (2) $\text{K}_3[\text{Co} (\text{NO}_2)_6]$
   (3) $(\text{NH}_4)_3[\text{As} (\text{Mo}_3 \text{O}_{10})_4]$  (4) $\text{BaCrO}_4$

42. (1)
   (1) $(\text{NH}_4)_3[\text{As} (\text{Mo}_3 \text{O}_{10})_4]$ = Yellow
   (2) $\text{BaCrO}_4$ = Yellow
   (3) $\text{Zn}_2[\text{Fe} (\text{CN})_6]$ = White
   (4) $\text{K}_3[\text{Co} (\text{NO}_2)_6]$ = Yellow

43. The vapour pressure of acetone at $20^\circ\text{C}$ is 185 torr. When 1.2 g of a non-volatile substance was dissolved in 100 g of acetone at $20^\circ\text{C}$, its vapour pressure was 183 torr. The molar mass (g mol$^{-1}$) of the substance is :
   (1) 32  (2) 64  (3) 128  (4) 488

43. (2)
   \[p^0 = 185\]
   \[\frac{p^0 - p}{p} = \frac{n}{N}\]
   \[\frac{185 - 183}{183} = \frac{1.2}{100/58}\]
   \[M = 64\]

44. The number of geometric isomers that can exist for square planar $[\text{Pt} (\text{Cl}) (\text{py}) (\text{NH}_3) \text{NH}_2\text{OH}]^+$ is (py = pyridine) :
   (1) 2  (2) 3  (3) 4  (4) 6

44. (2)
   dsp$^2$Mabcdef and hence
   Its number of geometrical isomers = 3

45. 1 g of an ideal gas X is introduced into an evacuated flask kept at 295 K. The pressure is found to be 1 atm. If 2 g of another ideal gas Y is added to the same flask, the total pressure becomes 1.5 atm. The molecular mass of Y is ________ times greater than the molecular mass of X.
   (1) 2  (2) 3  (3) 5  (4) 4
45. \[(4)\]
Pressure of Y = 1.5 – 1 = 0.5 atm
Now,
\[PV = nRT = \frac{wRT}{M_x}\]
\[P_xV = \frac{1 \times RT}{M_x}\]
And \[P_yV = \frac{2 \times RT}{M_y}\]
\[\therefore \frac{P_x}{P_y} = \frac{M_y}{2M_x}\]
\[\frac{M_y}{M_x} = \frac{2P_x}{P_y}\]
\[\therefore M_y = \frac{2 \times 1}{0.5} = 4\]
\[\therefore M_y = 4M_x\]

$SECTION-II : (NUMERICAL VALUE TYPE)$

This section contains 05 questions. The answer to each question is a NUMERICAL VALUE. For each question, enter the correct numerical value (e.g. 6.25, 7, 0.33, 30.275, 127.30, -127)

46. \[(4)\]
100 mL of a water sample contains 0.81 g of calcium bicarbonate and 0.73 g of magnesium bicarbonate. The hardness of this water sample expressed in terms of equivalents of CaCO$_3$ is $1 \times 10^3$ ppm. The value of x is __________.
[Given: molar mass of calcium bicarbonate is 162 g mol$^{-1}$ and magnesium bicarbonate is 146 g mol$^{-1}$]

46. \[(4)\]
Total hardness = \[\frac{\text{Molarity } \left( \text{Ca}^{2+} \text{ and Mg}^{2+} \right) \times \text{Molar mass of CaCO}_3 \times 10^6}{10^3}\]

No. of moles of Ca(HCO$_3$)$_2$ = \[\frac{0.81}{162}\]
= 0.005 mol
\[\therefore \text{Molarity of Ca}^{2+}\text{ ions} = \frac{0.005}{0.1} = 0.05 \text{ M}\]

No. of moles of Mg(HCO$_3$)$_2$ = \[\frac{0.73}{146}\]
= 0.005 mol
\[\therefore \text{Molarity of Mg}^{2+}\text{ ions} = \frac{0.005}{0.1} = 0.05 \text{ M}\]

Hardness in terms of CaCO$_3$
\[= \frac{(0.05+0.05) \times 100}{10^3} \times 10^6\]
\[ 10,000 \text{ ppm} = 1 \times 10^4 \text{ ppm} \]
\[ \therefore \text{ Value of } x = 4 \]

**Alternate method**

\[ 162 \text{ g of } \text{Ca(HCO}_3\text{)}_2 = 100 \text{ g of } \text{CaCO}_3 \]

100 mL water sample contain 0.81 g \( \text{Ca(HCO}_3\text{)}_2 \)

\[ \text{i.e., } \frac{0.81 \times 100}{162} = 0.5 \text{ g } \text{CaCO}_3 \text{ in } 100 \text{ mL} \]

\[ \text{i.e., } 5 \text{ g } \text{CaCO}_3 \text{ in } 1 \text{ L} = 5000 \text{ mg } \text{CaCO}_3 \text{ in } 1 \text{ L} \]

\[ = 5000 \text{ ppm } \text{CaCO}_3 \]

Similarly, 146 g of \( \text{Mg(HCO}_3\text{)}_2 = 100 \text{ g } \text{CaCO}_3 \)

100 mL water sample contain 0.73 g \( \text{Mg(HCO}_3\text{)}_2 \)

\[ \text{i.e., } \frac{0.73 \times 100}{146} = 0.5 \text{ g } \text{CaCO}_3 \text{ in } 100 \text{ mL} \]

\[ \text{i.e., } 5000 \text{ ppm } \text{CaCO}_3 \]

Total hardness in terms of \( \text{CaCO}_3 \)

\[ = 5000 + 5000 = 10,000 = 1 \times 10^4 \text{ ppm} \]

\[ \therefore \text{ Value of } x = 4 \]

47. 3, 4–Dimethylhex–3–ene undergoes addition reaction with HCl to give a product ‘X’. The total number of stereoisomers possible for the product ‘X’ is _____.

47. (4)

Product, 3–chloro–3, 4–dimethylhexane has 2–chiral centers. Hence, four stereoisomers are possible.

\[
\begin{align*}
\text{H}_3\text{CCH}_2 \text{C} = \text{C} \text{–CH}_2\text{CH}_3 & \xrightarrow{\text{HCl}} \text{H}_3\text{CCH}_2 \text{C}^* \text{–CH}_2\text{CH}_3 \\
\text{CH}_3\text{ CH}_3 & \text{ CH}_3\text{ CH}_3
\end{align*}
\]

3, 4-Dimethylhex-3-ene

3-Chloro-3,4-dimethylhexane

48. In a face centred cubic arrangement of A and B atoms in which ‘A’ atoms are the corners of the unit cell and ‘B’ atoms are at the face centers. One of the ‘A’ atom is missing from one corner in unit cell. The simplest formula of compound is \( A_x B_y \). The value of \( x \) is _____.
48. (7)
Number of atoms of A at corners = 7 (one ‘A’ is missing)
∴ Contribution of atoms of ‘A’ to the unit cell
= \(7 \times \frac{1}{8} = \frac{7}{8}\)
Total number of atoms ‘B’ at face = 6
∴ Contribution of atom ‘B’ to the unit cell
= \(6 \times \frac{1}{2} = 3\)
A : B = \(\frac{7}{8} : 3 = 7 : 24\)
Hence, formula = \(A_xB_{24}\)
∴ The value of \(x\) is 7.

49. The number of diamagnetic species among the following is ________.
\[
\left[\text{Ni(CO)}_4\right], \left[\text{NiCl}_4\right]^{2-}, \left[\text{CoF}_6\right]^{3-}, \left[\text{Cu(NH}_3\right)_4]^{2+}, \left[\text{Ni(CN)}_4\right]^{2-}
\]

49. (2)
i. \(\left[\text{Ni(CO)}_4\right]\):
\[
\text{Ni (ground state):} \quad \uparrow \downarrow \uparrow \downarrow \uparrow \uparrow \quad 3d^8 \quad 4s^2 \quad 4p
\]
\[
\text{Ni(CO)}_4 \quad \uparrow \downarrow \uparrow \downarrow \uparrow \downarrow \downarrow \quad \text{sp}^3 \text{ hybrid orbitals}
\]
No. of unpaired electrons = 0
⇒ Diamagnetic
(ii) \(\left[\text{NiCl}_4\right]^{2-}\):
\[
\text{Ni}^{2+} \quad \uparrow \downarrow \uparrow \downarrow \uparrow \quad 3d^8 \quad 4s \quad 4p
\]
\[
\left[\text{NiCl}_4\right]^{2-} \quad \uparrow \downarrow \uparrow \downarrow \uparrow \quad \text{sp}^3 \text{ hybrid orbitals}
\]
No. of unpaired electrons = 2
⇒ Paramagnetic
(iii) \(\left[\text{CoF}_6\right]^{3-}\):
\[
\text{Co}^{3+} \quad \uparrow \downarrow \uparrow \downarrow \uparrow \quad 3d^6 \quad 4s \quad 4p \quad 4d
\]
\[
\left[\text{CoF}_6\right]^{3-} \quad \uparrow \downarrow \uparrow \downarrow \uparrow \quad \text{sp}^3 \text{d}^2 \text{ hybrid orbitals}
\]
No. of unpaired electrons = 4
⇒ Paramagnetic
(iv) \(\left[\text{Cu(NH}_3\right)_4]^{2+}\):
No. of unpaired electrons = 1
⇒ Paramagnetic

(v) \([\text{Ni(CN)}]^{-2}\):

No. of unpaired electrons = 0
⇒ Diamagnetic

50. How many compounds amongst the following can be categorized as vicinal dihalides?

\[
\begin{align*}
\text{(I)} & \quad \text{Br} & \quad \text{Br} \\
\text{(II)} & \quad \text{Cl} & \quad \text{Cl} \\
\text{(III)} & \quad \text{Cl} & \quad \text{Cl} \\
\text{(IV)} & \quad \text{Br} & \quad \text{Br} \\
\text{(V)} & \quad \text{Cl} & \quad \text{Cl}
\end{align*}
\]

50. (2)
Vicinal dihalides are compounds in which both the halogen atoms are attached to adjacent C–atoms. Compound (I) and (V) are vicinal dihalides.
PART (C) : MATHEMATICS

SECTION-I : (SINGLE ANSWER CORRECT TYPE)

This section contains 20 multiple choice questions. Each question has four choices (1), (2), (3) and (4) out of which ONLY ONE is correct.

51. A complex number $z$ is the said to be unimodular if $|z| = 1$. Suppose $z_1$ and $z_2$ are complex number such that $\frac{z_1 - 2z_2}{2 - z_1 z_2}$ is unimodular and $z_2$ is not unimodular. Then the point $z_1$ lies on a :

1. straight line parallel to $x$-axis
2. straight line parallel to $y$-axis
3. circle of radius 2
4. circle of radius $\sqrt{2}$

52. If $A = \begin{bmatrix} 1 & 2 & 2 \\ 2 & 1 & -2 \\ a & 2 & b \end{bmatrix}$ is a matrix satisfying the equation $AA^T = 9I$, where $I$ is $3 \times 3$ identity matrix, then the ordered pair $(a, b)$ is equal to

1. $(2, -1)$
2. $(-2, 1)$
3. $(2, 1)$
4. $(-2, -1)$

53. The number of integers greater than 6000 that can be formed, using the digits 3, 5, 6, 7 and 8 without repetition is :

1. 216
2. 192
3. 120
4. 72

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- - - - (Four digit numbers)

\[ 3 \times 4 \binom{C}{1} \times 3! = 72 \]

- - - - - (Five digit numbers)

Total = 192

54. The sum of first 9 terms of the series \( \frac{1^3}{1} + \frac{1^3 + 2^3}{1 + 3} + \frac{1^3 + 2^3 + 3^3}{1 + 3 + 5} + \ldots \) is:

(1) 71  (2) 96  (3) 142  (4) 192

54. (2)

\[ T_n = \frac{\Sigma n^3}{\Sigma (2n-1)} = \frac{n^2(n+1)^2}{4 \times n^2} \]

\[ \Sigma T_n = \frac{1}{4} \left( \Sigma n^2 + 2 \Sigma n + \Sigma 1 \right) \]

\[ = \frac{1}{4} \left( \frac{n(n+1)(2n+1)}{6} + 2 \cdot \frac{n(n+1)}{2} + n \right) \]

\[ = \frac{1}{4} \left( \frac{9 \times 10 \times 19}{6} + 90 + 9 \right) \]

\[ = \frac{1}{4} (285 + 99) = 96 \]

55. \( \lim_{x \to 0} \frac{(1-\cos 2x)(3+\cos x)}{x \tan 4x} \) is equal to

(1) 4  (2) 3  (3) 2  (4) \( \frac{1}{2} \)

55. (3)

\[ \lim_{x \to 0} \frac{(1-\cos 2x)(3+\cos x)}{x \tan 4x} \]

\[ \lim_{x \to 0} \frac{(1-\cos 2x)(3+\cos x)}{x \tan 4x} \]

\[ \lim_{x \to 0} \left[ \frac{(1-\cos 2x)(3+\cos x)}{(2x)^2} \right] \cdot \frac{\tan 4x}{4x} \]

\[ \Rightarrow \frac{1}{2} (3+1) = 2 \]

56. If the function \( g(x) = \begin{cases} k \sqrt{x+1}, & 0 \leq x \leq 3 \\ mx + 2, & 3 < x \leq 5 \end{cases} \) is differentiable, then the value of \( k + m \) is:

(1) 2  (2) \( \frac{16}{5} \)  (3) \( \frac{10}{3} \)  (4) 4

56. (1)
g(x) = \begin{cases} 
\sqrt[3]{kx + 1} & \text{if } x \in [0,3] \\
mx + 2 & \text{if } x \in (3,5]
\end{cases}

g(x) \text{ is diff } \Rightarrow g(x) \text{ continuous}
\therefore g(3^-) = g(3^+)
\Rightarrow k\sqrt[3]{4} = 3m + 2
\Rightarrow 2k = 3m + 2 \quad \ldots (1)

Again
\begin{align*}
g'(3^+) &= g'(3^-) \\
\Rightarrow m &= \frac{k}{2\sqrt[3]{x + 1}} \bigg|_{x = 3} = \frac{k}{4}
\Rightarrow 4m = k \quad \ldots (2)
\end{align*}

From (1) & (2)
2k = 3m + 2 \Rightarrow 8m = 3m + 2
5m = 2
m = \frac{2}{5}
\& k = 4m = \frac{8}{5}
\Rightarrow k + m = \frac{10}{5} = 2

57. Let \( f(x) \) be a polynomial of degree four having extreme values at \( x = 1 \) and \( x = 2 \).
If \( \lim_{x \to 0} \left[ 1 + \frac{f(x)}{x^2} \right] = 3 \), then \( f(2) \) is equal to :
(1) \ -8 \quad (2) \ -4 \quad (3) \ 0 \quad (4) \ 4

57. (3)
\begin{align*}
f(x) &= \lim_{x \to 0} \left[ 1 + \frac{f(x)}{x^2} \right] = 3 \\
\Rightarrow f(x) \text{ must not contain degree 0 & degree 1 term} \\
\Rightarrow f(x) &= ax^4 + bx^3 + cx^2
\end{align*}

Now \( f'(x) = 4ax^3 + 3bx^2 + 2cx \)
\begin{align*}
f'(1) &= 4a + 3b + 2c = 0 \quad \ldots (1) \\
f'(2) &= 32a + 12b + 4c = 0 \quad \ldots (2)
\end{align*}

and \( \lim_{x \to 0} \left[ 1 + \frac{f(x)}{x^2} \right] = 1 + c = 3 \quad \ldots (3) \\
\Rightarrow c &= 2
\begin{align*}
(1) \quad &\Rightarrow 4a + 3b = -4 \quad -12b = 24 \\
(2) \quad &\Rightarrow 32a + 12b = -8 \quad b = -2 \\
8\times(1) \quad &\Rightarrow 32a + 24b = -32 \quad a = \frac{1}{2}
\end{align*}
58. The integral \( \int \frac{dx}{x^2(x^4 + 1)^{3/4}} \) equals:

\[
\begin{align*}
(1) & \quad \left( \frac{x^4 + 1}{x^4} \right)^{1/4} + c \\
(2) & \quad (x^4 + 1)^{1/4} + c \\
(3) & \quad -(x^4 + 1)^{1/4} + c \\
(4) & \quad -\left( \frac{x^4 + 1}{x^4} \right)^{1/4} + c
\end{align*}
\]

58. (4)

\[
\int \frac{dx}{x^2(x^4 + 1)^{3/4}}
= \int \frac{dx}{x^5(1+x^{-4})^{3/4}} = \int \frac{x^{-5}}{(1+x^{-4})^{3/4}} dx \quad \ldots (1)
\]

Put \( 1+x^{-4} = T^4 \)

\[
-4x^{-3}dx = 4T^3 dT
\]

\( \Rightarrow \) (1) becomes

\[
-\int \frac{T^3 dT}{T^4} = -T + c
\]

\[
= -(1+x^{-4})^{1/4} + c
\]

\[
= -\left( \frac{1+x^4}{x^4} \right)^{1/4} + c
\]

59. The integral \( \int_\frac{4}{2} \frac{\log x^2}{\log x^2 + \log(36-12x+x^2)} dx \) is equal to

\[
\begin{align*}
(1) & \quad 2 \\
(2) & \quad 4 \\
(3) & \quad 1 \\
(4) & \quad 6
\end{align*}
\]

59. (3)

\[
I = \int_\frac{4}{2} \frac{\log x^2}{\log x^2 + \log(x-6)^2} \ dx \quad \ldots (1)
\]

Using \( = \int_a^b f(x)dx = \int_a^b f(a + b - x)dx \)

\[
I = \int_\frac{4}{2} \frac{\log(6-x)^3}{\log(6-x)^2 + \log x^2} \ dx \quad \ldots (2)
\]

(1) + (2) gives

\[
2I = \int_2^\frac{4}{2} dx = 2
\]

\[
I = 1
\]

60. The area (in sq. units) of the quadrilateral formed by the tangents at the end points of the latera recta to the ellipse \( \frac{x^2}{9} + \frac{y^2}{5} = 1 \) is

\[
\begin{align*}
(1) & \quad \frac{27}{4} \\
(2) & \quad 18 \\
(3) & \quad \frac{27}{2} \\
(4) & \quad 27
\end{align*}
\]

60. (4)
\[ \frac{x^2}{9} + \frac{y^2}{5} = 1 \]
\[ a = 3b - \sqrt{5} \]
\[ e^2 = 1 - \frac{b^2}{a^2} \]
\[ = 1 - \frac{5}{9} - \frac{4}{9} \]
\[ e = \frac{2}{3} \]

Now the quadrilateral formed will be a rhombus with area \( \frac{2a^2}{e} \)
\[ = \frac{2\cdot 9}{2} \times 3 = 27 \]

61. Locus of the image of the point (2, 3) in the line \((2x - 3y + 4) + k(x - 2y + 3) = 0, k \in \mathbb{R},\) is a:
   (1) straight line parallel to x-axis
   (2) straight line parallel to y-axis
   (3) circle of radius \(\sqrt{2}\)
   (4) circle of radius \(\sqrt{3}\)

61. (3)

\[ RQ = RP \]
\[ (x - 1)^2 + (y - 2)^2 = (2 - 1)^2 + (3 - 2)^2 \]
\[ (x - 1)^2 + (y - 2)^2 = 2 \]

62. Let O be the vertex and Q be any point on the parabola, \(x^2 = 8y\). If the point P divides the line segment OQ internally in the ratio 1 : 3, then the locus of P is
   (1) \(x^2 = y\)
   (2) \(y^2 = x\)
   (3) \(y^2 = 2x\)
   (4) \(x^2 = 2y\)
62. (4)

Let \( P : (h, k) \)

\[
\begin{align*}
\alpha &= \frac{1.\alpha + 3.0}{4} \\
\beta &= \frac{1.\beta + 3.0}{4}
\end{align*}
\]

\( \Rightarrow \alpha = 4h \)

\( \Rightarrow \beta = 4k \)

\( \therefore (\alpha, \beta) \) on parabola

\( \Rightarrow \alpha^2 = 8\beta \Rightarrow (4h)^2 = 8.4k \)

\[16h^2 = 32k \]

\[ x^2 = 2y \]

63. The equation of the plane containing the lines \( 2x - 5y + z = 3; x + y + 4z = 5 \), and parallel to the plane, \( x + 3y + 6z = 1 \), is

(1) \( 2x + 6y + 12z = 13 \)

(2) \( x + 3y + 6z = -7 \)

(3) \( x + 3y + 6z = 7 \)

(4) \( 2x + 6y + 12z = -13 \)

64. Let \( \vec{a}, \vec{b} \) and \( \vec{c} \) be three non-zero vectors such that no two of them are collinear and

\[
(\vec{a} \times \vec{b}) \times \vec{c} = \frac{1}{3} \| \vec{b} \| \| \vec{c} \| \vec{a} \]

If \( \theta \) is the angle between vectors \( \vec{b} \) and \( \vec{c} \), then value of \( \sin \theta \) is:

(1) \( \frac{2\sqrt{2}}{3} \)

(2) \( -\frac{\sqrt{2}}{3} \)

(3) \( \frac{2}{3} \)

(4) \( -\frac{2\sqrt{3}}{3} \)
64.  
(1) \[(\vec{a} \times \vec{b}) \cdot \vec{c} = \frac{1}{3} bc \cdot \vec{a}\]
(2) \[(\vec{a} \cdot \vec{c}) \cdot \vec{b} - (\vec{b} \cdot \vec{c}) \cdot \vec{a} = \frac{1}{3} bc \cdot \vec{a} + 0.\vec{b}\]
(3) \[-\vec{b} \cdot \vec{c} = \frac{1}{3} bc; \vec{a} \cdot \vec{c} = 0\]
(4) \[-bc \cos \theta = \frac{1}{3} bc\]
\[
\cos \theta = \frac{1}{3} \Rightarrow \sin \theta = \frac{2\sqrt{2}}{3}
\]

65. If 12 identical balls are to be placed in 3 identical boxes, then the probability that one of the boxes contains exactly 3 balls is

(1) \[\frac{55}{3} \left(\frac{2}{3}\right)^{11}\]
(2) \[\frac{55}{3} \left(\frac{2}{3}\right)^{10}\]
(3) \[\frac{220}{3} \left(\frac{1}{3}\right)^{12}\]
(4) \[\frac{22}{3} \left(\frac{1}{3}\right)^{11}\]

66. Let \(\tan^{-1} y = \tan^{-1} x + \tan^{-1} \left(\frac{2x}{1-x^2}\right),\) where \(|x| < \frac{1}{\sqrt{3}}.\) Then a value of \(y\) is:

(1) \[\frac{3x-x^3}{1-3x^2}\]
(2) \[\frac{3x+x^3}{1-3x^2}\]
(3) \[\frac{3x-x^3}{1+3x^2}\]
(4) \[\frac{3x+x^3}{1+3x^2}\]

66. (1)
\[
\tan^{-1} y = \tan^{-1} x + \tan^{-1} \left(\frac{2x}{1-x^2}\right)
\]
\[|x| < \frac{1}{\sqrt{3}}\]
\[
\Rightarrow \tan^{-1} \frac{2x}{1-x^2} = 2 \tan^{-1} x
\]
\[
\Rightarrow \tan^{-1} y = \tan^{-1} x + 2 \tan^{-1} x
\]
\[= 3 \tan^{-1} x
\]
\[= \tan^{-1} \left(\frac{3x-x^3}{1-3x^2}\right)
\]
67. The negation of $\sim s \lor (\sim r \land s)$ is equivalent to:

(1) $s \land \sim r$
(2) $s \land (\sim r \land s)$
(3) $s \lor (\sim r \lor \sim s)$
(4) $s \lor r$

67. (4) $\sim s \lor (\sim r \land s)$

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68. Let $A$ and $B$ be two sets containing four and two elements respectively. Then the number of subsets of the set $A \times B$, each having at least three elements is:

(1) 219
(2) 256
(3) 275
(4) 510

68. (1)

$n(A \times B) = 8$

Total subsets $= 2^8$

$8C_0 + 8C_1 + 8C_2 = 37$

No. of Req. Subsets $= 256 - 37 = 219$

69. The number of points, having both co-ordinates as integers, that lie in the interior of the triangle with vertices $(0, 0)$, $(0, 41)$ and $(41, 0)$ is:

(1) 901
(2) 861
(3) 820
(4) 780

69. (4)

$x + y < 41$

$x + y < 39 \Rightarrow x + y \leq 38 \Rightarrow x + y + z = 38$

$38C_{3-1} = 40C_2 = \frac{40 \times 39}{2} = 780$

70. The number of common tangents to the circles $x^2 + y^2 - 4x - 6y - 12 = 0$ and $x^2 + y^2 + 6x + 18y + 26 = 0$, is:

(1) 1
(2) 2
(3) 3
(4) 4

70. (3)
\[ x^2 + y^2 - 4x - 6y - 12 = 0 \]

\[ C_1(2,3) \quad r_1 = \sqrt{2^2 + 3^2 + 12} = 5 \]

\[ x^2 + y^2 + 6x + 18y + 26 = 0 \]

\[ C_2(-3,-9), r_2 = \sqrt{3^2 + 9^2 - 26} = 8 \]

\[ C_1C_2 = \sqrt{25 + 144} = 13 \]

**SECTION-II : (NUMERICAL VALUE TYPE)**

This section contains 05 questions. The answer to each question is a NUMERICAL VALUE. For each question, enter the correct numerical value (e.g. 6.25, 7, 0.33, 30.275, 127.30, –127)

71. A circle touching both the coordinates axes, having centre in the first quadrant and also touching the line \( 4x + 3y = 12 \), is given by \( x^2 + y^2 - 2cx - 2cy + c^2 \). Find the sum of all possible values of \( c \).

71. (7) A circle that touches the coordinate axes is given by

\[(x - c)^2 + (y - c)^2 = c^2\]

This circle touches the line \( 4x + 3y = 12 \) i.e.

\( 4x + 3y - 12 = 0 \)

\[ \Rightarrow \frac{4c + 3c - 12}{\sqrt{(4)^2 + (3)^2}} = c \]

\[ \Rightarrow 7c - 12 = 5c \]

\[ \Rightarrow c = 1, 6 \]

\[ \Rightarrow \text{The sum of all possible values of} \ c = 7 \]

72. Consider the words formed from the letters of word CONSTANT. Let A be the set containing words having 2N together and 2T together. Let B be the set containing words having order of vowels unchanged. Find \( \frac{n(B)}{n(A)} \)

72. (7) Number of words having 2N together and 2T together (i.e. C, O, A, S, (NN), (TT))

\[ = 6! = 720 \]

\[ \Rightarrow n(A) = 720 \]

Number of words having order of vowels unchanged is

\[ = 8C_2 \times 1 \times 6! \]

\[ = 2! \]
73. If the coefficients of the \((r-1)\)th, \(r\)th and \((r+1)\)th terms in the expansion of \((x+1)^n\) are in the ratio 2 : 15 : 70, then find the value of \(n\).

\[(16)\]

\[\begin{align*}
\binom{n}{r-2} : \binom{n}{r-1} : \binom{n}{r} &= 2 : 15 : 70 \\
\frac{\binom{n}{r-2}}{\binom{n}{r-1}} &= \frac{2}{15} \\
\frac{n!}{(r-2)!(n-r+2)!} &= \frac{2}{15} \\
\frac{n!}{(r-1)!(n-r+1)!} &= \frac{2}{15} \\
\Rightarrow \quad \frac{r-1}{n-r+2} &= \frac{2}{15} \\
\Rightarrow \quad 2n - 17r &= -19 \\
\text{........... (i)} \\
\binom{n}{r-1} &= \frac{3}{14} \\
\binom{n}{r} &= \frac{3}{14} \\
\Rightarrow \quad \frac{n!}{r!(n-r)!} &= \frac{3}{14} \\
\Rightarrow \quad \frac{r}{n-r+1} &= \frac{3}{14} \\
\Rightarrow \quad 3n - 17r &= -3 \\
\text{........... (ii)} \\
\text{Solving (i) and (ii), we get} \\
\Rightarrow \quad n &= 16
\end{align*}\]

74. Two integers \(x\) and \(y\) are chosen with replacement out of the set \(\{0, 1, 2, \ldots, 10\}\). The probabilities that \(|x - y| > 4\) and \(|x - y| > 6\) are \(p_1\) and \(p_2\). If \(p_1 - p_2 = \frac{m}{n}\) (\(m, n\) are co-prime), find \(n - m\).

\[(9)\]

\(x\) and \(y\) take values from 0 to 10.

So, the total number of ways of selecting \(x\) and \(y\) is \(11 \times 11 = 121\).

\(|x - y| > 4\) \(\Leftrightarrow x - y < -4\) or \(x - y > 4\)

There are 42 pairs \((x, y)\) that satisfy \(|x - y| > 4\).

\(|x - y| > 6\) \(\Leftrightarrow x - y < -6\) or \(x - y > 6\)

There are 20 pairs \((x, y)\) satisfying \(|x - y| > 6\).

\(\Rightarrow \quad p_1 = \frac{42}{121}, \quad p_2 = \frac{20}{121}\)

\(\Rightarrow \quad p_1 - p_2 = \frac{22}{121} = \frac{2}{11}\)

\(\Rightarrow \quad n = 11, \quad m = 2\)
75. Let \( f(x) = \sin^{-1}\left(\frac{2x}{1+x^2}\right) \) and \( f \) is differentiable everywhere on \( R \) except at two isolated points, say \( x_1 \) and \( x_2 \). Find \( x_1^2 + x_2^2 \).

\[
\begin{align*}
\Rightarrow n - m &= 9 \\
75. \quad &f(x) = \sin^{-1}\left(\frac{2x}{1+x^2}\right) \\
f'(x) &= -\frac{1}{\sqrt{1 - \left(\frac{2x}{1+x^2}\right)^2}} \times \frac{d}{dx} \left(\frac{2x}{1+x^2}\right) \\
&= -\frac{1+x^2}{\sqrt{(1-x^2)^2}} \times \frac{2(1+x^2) - 2x(2x)}{(1+x^2)^2} \\
&= \left(\frac{2}{1+x^2}\right) \left(\frac{1-x^2}{1-x^2}\right) \\
f' \text{ is not defined for } x^2 = 1 \\
i.e. \text{ for } x = 1, -1 \\
\Rightarrow x_1 = 1, x_2 = -1 \quad \text{(or } x_1 = -1, x_2 = 1) \\
\Rightarrow x_1^2 + x_2^2 = 1 + 1 = 2
\end{align*}
\]