Practice Test 4 (advance)Paper # 1 (Phy- Solution)

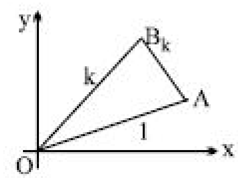
MATHS Paper Solution # 1

41. (A)

[Sol. $OB_k = k$

$\angle AOB_k = \frac{k\pi}{2n}$

$S_k = \frac{1}{2}k\sin\frac{k\pi}{2n}$ (using $\Delta = \frac{1}{2}ab\sin\theta$)



$\therefore \quad L = \frac{k}{2n^2}\sum_{n=1}^{\infty}\sin\frac{k\pi}{2n} = \frac{1}{2n}\sum_{n=1}^{\infty}\frac{k}{n}\sin\frac{k\pi}{2n} = \frac{1}{2}\int_0^1 x\cdot\sin\frac{\pi x}{2}dx$

$= \frac{1}{2}\left[\underbrace{\frac{-2}{\pi}x\cos\frac{\pi x}{2}\Big|_0^1}_{\text{zero}} + \frac{2}{\pi}\int_0^1\cos\frac{\pi x}{2}dx\right] = \frac{1}{2}\left[0+\frac{2}{\pi}\cdot\frac{2}{\pi}\left(\sin\frac{\pi x}{2}\right)_0^1\right] = \frac{2}{\pi^2}$ **Ans.**]

42. (C)

[Sol. For continuity of f at $x = 0$, we have

$k = f(0) = \lim_{x\to 0} f(x) = \lim_{x\to 0}\frac{e^{\tan x} - e^x}{\tan x - x} + \lim_{x\to 0}\frac{ln(\sec x + \tan x) - x}{\left(\frac{\tan x - x}{x^3}\right)x^3}$

$= \lim_{x\to 0}\frac{e^x(e^{\tan x - x} - 1)}{\tan x - x} + 3\lim_{x\to 0}\frac{ln(\sec x - \tan x) - x}{x^3} = 1 + 3\lim_{x\to 0}\frac{\sec x - 1}{3x^2}$ (Using L.H. Rule)

$= 1 + \frac{1}{2} = \frac{3}{2}$ **Ans.**]

43. (C)

Sol. R, R, R ; G, G, G ; B, B,B

$n(S) = \frac{9!}{3!\,3!\,3!}$

$n(A) = \frac{6!}{3!\,3!} \cdot {}^7C_3$ $\quad$ | R | R | R | G | G | G |

$= \frac{6!}{3!\,3!} \cdot \frac{7!}{3!\,4!}$

$P(A) = \frac{6!\,7!}{3!\,3!\,3!\,4!} \cdot \frac{3!\,3!\,3!}{9!} = \frac{6\cdot 5}{9\cdot 8} = \frac{5}{12} = \frac{5}{5+7}$

odds in favour 5 : 7 $\Rightarrow$ odds against 7 : 5 **Ans.**]

44. (B)

[Hint: $(x + y)^2 - 3(x + y) + 2$

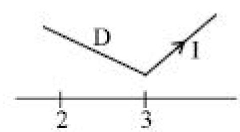
$t^2 - 3t + 2 \Rightarrow (t - 2)(t - 1) \Rightarrow (x + y - 2)(x + y - 1)$

$\Rightarrow$ two parallel lines which are non coincident]

45. (A)

[Sol. $f(x) = x^{3/2} + x^{-3/2} - 4\left(x + \frac{1}{x}\right)$

$f(x) = \left(\sqrt{x} + \frac{1}{\sqrt{x}}\right)^3 - 3\left(\sqrt{x} + \frac{1}{\sqrt{x}}\right) - 4\left[\left(\sqrt{x} + \frac{1}{\sqrt{x}}\right)^2 - 2\right]$



Let $\sqrt{x} + \frac{1}{\sqrt{x}} = t \quad (x > 0)$

let $g(t) = t^3 - 3t - 4t^2 + 8$

now $g(t) = t^3 - 4t^2 - 3t + 8$ where $t \in [2, \infty)$

$g'(t) = 3t^2 - 8t - 3 = (t - 3)(3t + 1)$; $g'(t) = 0 \Rightarrow t = 3 \ (t \neq -1/3)$

$g''(t) = 6t - 8$

$g''(3) = 10 > 0 \Rightarrow g(3)$ is minimum

$g(3) = 27 - 9 - 36 + 8 = -10$ **Ans.**]

46. (A)

[Sol. length of the right angled leg = AO

equation of circle is,

$$(x - r)^2 + (y - r)^2 = r^2$$

put $x = x_1$, $y = 2r$ to get square of the tangent from B

$L^2 = (x_1 - r)^2 + r^2 - r^2$

$L = x_1 - r = BP = BQ$

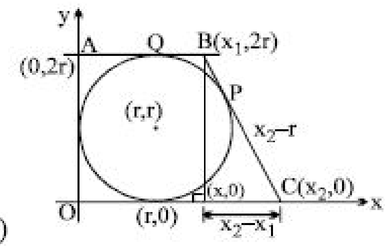
|||ly $\quad x_2 - r = PC \quad$ (Hence $BC = BP + PC = x_1 + x_2 - 2r$)

hence $\quad (x_2 - x_1)^2 + 4r^2 = (x_1 + x_2 - 2r)^2$

$(x_2 + x_1)^2 - 4x_1x_2 + 4r^2 = (x_2 + x_1)^2 + 4r^2 - 4r(x_1 + x_2)$

$x_1x_2 = r(x_1 + x_2)$

$\frac{1}{r} = \frac{1}{x_1} + \frac{1}{x_2} \quad \Rightarrow \quad 2r = \frac{2x_1x_2}{x_1 + x_2} \quad \Rightarrow$ OA is H.M. of the lengths of bases **Ans.**]



47. (A)

[Sol. $-1 \le 3x - 1 \le 1 \quad \Rightarrow \quad 0 \le x \le \frac{2}{3} \quad \Rightarrow$ domain is $\left[0, \frac{2}{3}\right]$

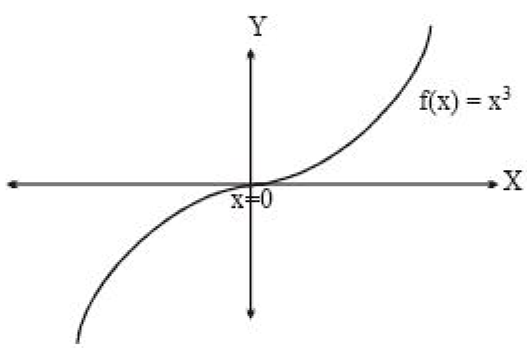
when $x = 0$ then $y = 1$; $x = \frac{2}{3}$, $y = 4$. Hence range is [1, 4] Ans. **Ans.**]

48. (D)

[Sol. Consider the function $f(x) = x^3$.

Its derivative $f'(x) = 3x^2$, vanishes at $x = 0$.

However, as the graph shows that $x = 0$ is not a local extremum of $f(x) = x^3$.



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49. (B,C)

[Sol. We have $f(x) = x + x\int_{-1}^{1} f(t)dt - x^2\int_{-1}^{1} t\,f(t)dt$

$f(x) = (1 + A)\,x - Bx^2$ ... (1) where $A = \int_{-1}^{1} f(t)dt$ and $B = \int_{-1}^{1} t\,f(t)dt$

Now $A = \int_{-1}^{1} f(t)dt = \int_{-1}^{1}\left((1+A)t - Bt^2\right)dt \Rightarrow A = -2B\int_{0}^{1} t^2 dt$; $\therefore A = \frac{-2B}{3}$ ....(2)

|||ly $B = \int_{-1}^{1} t\,f(t)dt = \int_{-1}^{1} t\left((1+A)t - Bt^2\right)dt \Rightarrow B = 2(A+1)\int_{0}^{1} t^2 dt$; $\therefore$ $B = \frac{2}{3}(A+1)$ ....(3)

On solving (2) and (3), we get $A = \frac{-4}{13}$, $B = \frac{6}{13}$

$\therefore$ from equation (1), we get, $f(x) = \frac{9}{13}x - \frac{6}{13}x^2$ $\Rightarrow$ $f'(x) = \frac{9}{13} - \frac{12}{13}x$

$\therefore$ $f'\left(-\frac{1}{3}\right) = \frac{9}{13} - \frac{12}{13}\left(-\frac{1}{3}\right) = \frac{9}{13} + \frac{4}{13} = \frac{13}{13} = 1.$]

50 (A,B,D)

[Sol. **(A)** $r = \frac{\Delta}{s}$; $r_1 = \frac{\Delta}{s-a}$; $r_2 = \frac{\Delta}{s-b}$; $r_3 = \frac{\Delta}{s-c}$ $\Rightarrow$ **(A) is correct**

(B) $\sum \cot\frac{A}{2} = \frac{s}{\Delta}[(s-a)+(s-b)+(s-c)] = \frac{s}{\Delta}[s] = \frac{4s^2}{4\Delta} = \frac{(a+b+c)^2}{4\Delta}$

$\therefore$ $\frac{(a+b+c)^2}{\sum \cot\frac{A}{2}} = 4\Delta$ $\Rightarrow$ **(B) is correct**

(C) Using $a^2 + b^2 - c^2 = 2ab\cos C$

given $(a^2 + b^2 - c^2)\tan B = 2ab\cos C\,\frac{\sin B}{\cos B} \neq 4\Delta$ $\Rightarrow$ **(C) is NOT correct**

Note: C could not be correct if $\tan B \to \tan C$

(D) $b^2 \sin 2C + c^2 \sin 2B$

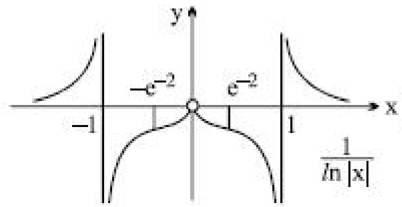
using $b = k\sin B$, $b[\underline{\underline{k}}\sin B \cdot 2\underline{\underline{\sin C}}\cos C] + c\,\overline{\overline{k}}\sin C\; 2\overline{\overline{\sin B}}\cos B$

$2bc\sin B\cos C + 2bc\sin C\cos B$

$2bc(\sin B\cos C + \cos C\sin B)$

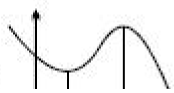
$2bc\sin(B + C) = 2bc\sin A = 4\Delta$ $\Rightarrow$ **(D) is correct**]

51. (A,C)

[Hint: $f(x) = \frac{1}{\ln|x|} = \begin{cases} \frac{1}{\ln x} & \text{if } x > 0 \\ \frac{1}{\ln(-x)} & \text{if } x < 0 \end{cases}$ ]

52. (A,B,C)

[Sol. If $f'(x) = 0$ has n real roots $\Rightarrow$ $f(x) = 0$ has atmost $(n+1)$ roots

(A)  $f(x) = 0$ has 1 real root

(ii) g ($\Rightarrow$ $pt = \ln\frac{p}{(p+1)(e^p-1)}$

on $\therefore$ $t = -\frac{1}{p}\ln\left(\frac{(p+1)(e^p-1)}{p}\right)$ $\Rightarrow$ $t_p = -\frac{1}{p}\ln\left(\frac{(p+1)(e^p-1)}{p}\right)$ **Ans.(ii)**

g (**(iii)** $s_p - t_p = \frac{1}{p}\ln\left(\frac{(p+1)(e^p-1)}{p}\right) - \frac{\ln(p+1)}{p} = \frac{1}{p}\ln\left(\frac{(p+1)(e^p-1)}{p}\cdot\frac{1}{(p+1)}\right) = \frac{1}{p}\ln\left(\frac{e^p-1}{p}\right)$

g (hence $s_p - t_p = \lim_{p\to 0}\left(\frac{1}{p}\ln\frac{(e^p-1)}{p}\right) = \lim_{p\to 0}\ln\left(\frac{e^p-1}{p}\right)^{1/p}$ $\to 1^\infty$ form

g ' $\lambda = \lim_{p\to 0}\frac{1}{p}\left(\frac{e^p-1}{p}-1\right) = \lim_{p\to 0}\left(\frac{e^p-1-p}{p^2}\right) = \frac{1}{2}$ **Ans.(iii)** $\overline{e^p-1)}$

(i **Alternatively for (iii):**

$s_p - t_p = \frac{1}{p}\ln\left(\frac{(p+1)(e^p-1)}{p}\right) - \frac{\ln(p+1)}{p} = \frac{1}{p}\ln\left(\frac{(p+1)(e^p-1)}{p}\cdot\frac{1}{(p+1)}\right) = \frac{1}{p}\ln\left(\frac{e^p-1}{p}\right) - e^x$

$\rightarrow$ $p+1$ $f''(x) = e^x[(p+1)^2 e^{px} - 1]$

$px = -\ln(p+1)$ $f''\left(e^{px} = \frac{1}{p+1}\right) = e^x[(p+1)-1] > 0$

$x = \frac{-\ln(p+1)}{p}$ [verify that $f''(x) > 0$] $\Rightarrow$ *minima*

hence $x = s_p = \frac{-\ln(p+1)}{p}$ **Ans.(i)**

56. (A,D) 57 (A,D) 58. (ACD)

[Sol. Let $A = \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix}$ and $\vec{x} = x_1\hat{i} + x_2\hat{j} + x_3\hat{k}$

$$\therefore \quad A\vec{x} = \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} a_{11}x_1 + a_{12}x_2 + a_{13}x_3 \\ a_{21}x_1 + a_{22}x_2 + a_{23}x_3 \\ a_{31}x_1 + a_{32}x_2 + a_{33}x_3 \end{bmatrix}$$

Since $A\vec{x}$ is orthogonal to $\vec{x}$ for every $\vec{x}$ in R^3, so

$$A\vec{x} \cdot \vec{x} = 0$$

$\Rightarrow (a_{11}x_1 + a_{12}x_2 + a_{13}x_3)\, x_1 + (a_{21}x_1 + a_{22}x_2 + a_{23}x_3)x_2 + (a_{31}x_1 + a_{32}x_2 + a_{33}x_3)\, x_3 = 0$

$\Rightarrow (a_{11}x_1^2 + a_{22}x_2^2 + a_{33}x_3^2) + (a_{12} + a_{21})\, x_1x_2 + (a_{13} + a_{31})\, x_1x_3 + (a_{23} + a_{32})\, x_2x_3 = 0$ .... (1)

$\because$ Above relation (1) hold good for every $\vec{x}$ in R^3 (i.e., $\forall\ x_1, x_2, x_3$)

Hence $a_{ii} = 0\ \forall i$ and $a_{ij} = -a_{ji}\ \forall i \neq j$

$\Rightarrow$ Matrix A must be skew symmetric. Also order of matrix A is 3 and every skew symmetric matrix of odd order is singular.

Hence matrix A is singular also.

(i) Clearly option (A) & (D) are correct

(ii) We have $a_{13} = -2$, $a_{32} = 5$

$\therefore\ a_{31} = 2$, $a_{23} = -5$

Hence option (A) & (D) are correct.

(iii) As matrix A is skew symmetric so sum of all the elements of matrix A is zero, although matrix A cannot be uniquely determined

We know that maximum number of distinct entries in a skew symmetric matrix of order n is $n^2 - n + 1$

$\therefore$ For $n = 3$, maximum number of distinct entries $= 3^2 - 3 + 1 = 7$

As all the diagonal elements of skew symmetric matrix are zero, so trace of matrix A is also zero.

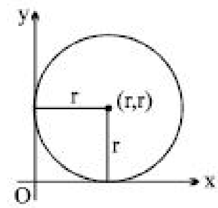
Also pair of conjugate elements of skew-symmetric matrix are additive inverse of each other.

Hence option (A), (C), (D) are true.]

59. (A)-S,(B)-R,(C)-Q,(D)-P

[Sol. Equation of circle touching the coordinates axes and centre (r, r) in the first quadrant is

$$x^2 + y^2 - 2xr - 2yr + r^2 = 0$$



For $r = a$ or b

hence $C_1 : x^2 + y^2 - 2ax - 2ay + a^2$ ....(1)

Centre (a, a), radius = a, a > 0

$C_2 : x^2 + y^2 - 2bx - 2by + b^2$ ....(2)

Centre (b, b), radius b, b > 0

(A) C_1 and C_2 touch each other

radical axis between (1) and (2) is

(1) – (2) = 0

$$2(b - a)x + 2(b - a)y - (b^2 - a^2) = 0$$

$$2x + 2y - (b + a) = 0 \quad ....(3)$$

if it touches both C_1 and C_2 then perpendicular from (a, a) = radius 'a'

$$\left|\frac{2a + 2a - (b + a)}{\sqrt{8}}\right| = a \quad ....(4)$$

$$|3a - b| = 2\sqrt{2}\,a \quad ....(5)$$

now origin and (a, a) must lie on the same side of (3)

but (0, 0) gives – ve sign with (3)

hence (a, a) should also give the same sign i.e. $4a - b - a < 0 \Rightarrow 3a - b < 0$

Hence (5) becomes

$$b - 3a = 2\sqrt{2}\,a \Rightarrow \frac{b}{a} = 3 + 2\sqrt{2} \textbf{ Ans.} \Rightarrow \textbf{(S)}$$

Alternativly: As C_1 and C_2 touch each other externally so,

distance between their centre = sum of their radius

$$\Rightarrow \sqrt{(a - b)^2 + (a - b)^2} = (a + b) \Rightarrow 2(a - b)^2 = (a + b)^2 \Rightarrow a^2 + b^2 - 6ab = 0$$

$$\Rightarrow \frac{b^2}{a^2} - 6\left(\frac{b}{a}\right) + 1 = 0$$

$$\therefore \frac{b}{a} = \frac{6 \pm \sqrt{36 - 4}}{2} = \frac{6 \pm 4\sqrt{2}}{2} = 3 \pm 2\sqrt{2}$$

but $\frac{b}{a} = 3 - 2\sqrt{2}$ (rejected as $\frac{b}{a} > 1$)

Hence $\frac{b}{a} = 3 + 2\sqrt{2}$

hence (a, a) should also give the same sign i.e. $4a - b - a < 0 \quad \Rightarrow \quad 3a - b < 0$
Hence (5) becomes

$$b - 3a = 2\sqrt{2}\,a \quad \Rightarrow \quad \frac{b}{a} = 3 + 2\sqrt{2} \textbf{ Ans.} \quad \Rightarrow \quad \textbf{(S)}$$

Alternativly: As C_1 and C_2 touch each other externally so,
distance between their centre = sum of their radius

$$\Rightarrow \quad \sqrt{(a-b)^2 + (a-b)^2} = (a+b) \Rightarrow \quad 2(a-b)^2 = (a+b)^2 \quad \Rightarrow \quad a^2 + b^2 - 6ab = 0$$

$$\Rightarrow \quad \frac{b^2}{a^2} - 6\left(\frac{b}{a}\right) + 1 = 0$$

$$\therefore \quad \frac{b}{a} = \frac{6 \pm \sqrt{36-4}}{2} = \frac{6 \pm 4\sqrt{2}}{2} = 3 \pm 2\sqrt{2}$$

(C) If common chord is longest then (3) must pass through the centre (a, a) of C_1
i.e. $4a - b - a = 0$

$$3a = b \Rightarrow \quad \frac{b}{a} = 3 \textbf{ Ans.} \quad \Rightarrow \quad \textbf{(Q)}$$

(D) If C_2 passes through the centre of C_1 then (a, a) must satisfy (2)
i.e. $a^2 + a^2 - 2b(2a) + b^2 = 0 \quad \Rightarrow \quad 2a^2 - 4ab + b^2 = 0$

$$\left(\frac{b}{a}\right)^2 - 4\left(\frac{b}{a}\right) + 2 = 0$$

(E

Put $\frac{b}{a} = t$

$$t^2 - 4t + 2 = 0 \quad \Rightarrow \quad (t-2)^2 = 4 - 2 = 2 \quad \Rightarrow \quad t - 2 = \sqrt{2} \text{ or } -\sqrt{2}$$

$$t = 2 + \sqrt{2}, \; t \neq 2 - \sqrt{2} \;\; (\text{as } t > 1) \quad \Rightarrow \quad \textbf{(P)}]$$

$$\left(\frac{}{a}\right) - 4\left(\frac{}{a}\right) + 1 = 0$$

if $\quad \frac{b}{a} = t, \; t^2 - 4t + 1 = 0 \quad \Rightarrow \quad (t-2)^2 = 3 \quad \Rightarrow \quad t - 2 = +\sqrt{3} \text{ or } -\sqrt{3}$

$$t = 2 + \sqrt{3}$$

as $\quad t > 1 \quad \Rightarrow \quad 2 - \sqrt{3}$ is not possible

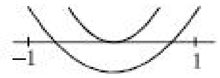
$$\therefore \quad \frac{b}{a} = 2 + \sqrt{3} \textbf{ Ans.} \quad \Rightarrow \quad \textbf{(R)}$$

60. (A)-R,(B)-S,(C)-Q,(D)-Q

[Sol. $f(x) = 2x^2 - 10px + 7p - 1 = 0$

(A) $f(-1) > 0$; $f(1) > 0$

$D \geq 0$ and $-1 < -\dfrac{b}{2a} < 1$



$f(-1) = 2 + 10p + 7p - 1 > 0$ or $17p > -1 \Rightarrow p > -\dfrac{1}{17}$ ....**(i)**

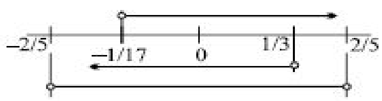
$f(1) = 2 - 10p + 7p - 1 > 0$ or $1 > 3p \Rightarrow p < \dfrac{1}{3}$ ....**(ii)**

$D \geq 0$

$100p^2 - 8(7p - 1) \geq 0 \Rightarrow 100p^2 - 56p + 8 \geq 0 \Rightarrow 25p^2 - 14p + 2 \geq 0$

which is always true $\Rightarrow$ $p \in R$ ....**(iii)**

$-1 < \dfrac{10p}{4} < 1 \Rightarrow -2 < 5p < 2 \Rightarrow -\dfrac{2}{5} < p < \dfrac{2}{5}$ ....**(iv)**



$\Rightarrow \left(-\dfrac{1}{17}, \dfrac{1}{3}\right)$