BPT – I Solution

Subject: Maths

Topic: Mathematical Logic, Matrices, Trigonometric Functions, Pair of Straight lines, Probability Distribution, Binomial Distribution

Marks: 80 marks

Section – I

Q 1.(A)

i. (b) T, F, F [2m]

ii. (c) $\frac{2\pi}{3}$ [2m]

(B)

i. $AX = B$

$$\therefore \quad \begin{bmatrix} 1 & 2 \\ -1 & 3 \end{bmatrix} X = \begin{bmatrix} 0 & 1 \\ 2 & 4 \end{bmatrix}$$

By $R_2 + R_1$, we get

$$\begin{bmatrix} 1 & 2 \\ 0 & 5 \end{bmatrix} X = \begin{bmatrix} 0 & 1 \\ 2 & 5 \end{bmatrix}$$ [1m]

By $\left(\frac{1}{5}\right) R_2$, we get

$$\begin{bmatrix} 1 & 2 \\ 0 & 1 \end{bmatrix} X = \begin{bmatrix} 0 & 1 \\ 2/5 & 1 \end{bmatrix}$$

By $R_1 - 2R_2$, we get

$$\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} X = \begin{bmatrix} -4/5 & -1 \\ 2/5 & 1 \end{bmatrix}$$

$$\therefore \quad X = \begin{bmatrix} -4/5 & -1 \\ 2/5 & 1 \end{bmatrix}$$ [1m]

ii. We know that

$\sin\frac{\pi}{6} = \frac{1}{2}$ and $\sin(\pi + x) = -\sin x, \sin(2\pi - x) = -\sin x$

$$\therefore \quad \sin\left(\pi + \frac{\pi}{6}\right) = -\sin\frac{\pi}{6} = -\frac{1}{2}$$

$$\& \quad \sin\left(2\pi - \frac{\pi}{6}\right) = -\sin\frac{\pi}{6} = \frac{-1}{2}$$ [1m]

$\therefore \quad \sin\frac{7\pi}{6} = \sin\frac{11\pi}{6} = -\sin\frac{\pi}{6} = \frac{-1}{2}$, where

$0 < \frac{7\pi}{6} < 2\pi$ and $0 < \frac{11\pi}{6} < 2\pi$

$\therefore \quad \sin x = \frac{-1}{2}$ gives

$$\sin x = \sin \frac{7\pi}{6} = \sin \frac{11\pi}{6}$$

$\therefore \quad x = \frac{7\pi}{6}$ and $x = \frac{11\pi}{6}$ [1m]

Hence, the required principal solutions are

$$x = \frac{7\pi}{6} \text{ and } x = \frac{11\pi}{6}$$

iii. Comparing the equation $kx^2 + 4xy - y^2 = 0$

with $ax^2 + 2hxy + by^2 = 0$, we get,

$a = k$, $2h = 4$, $b = -1$

Let m_1 and m_2 be the slopes at the lines represented by $kx^2 + 4xy - y^2 = 0$

$\therefore \quad m_1 + m_2 = \frac{-2h}{b} = \frac{-4}{-1} = 4$

and $m_1 m_2 = \frac{a}{b} = \frac{k}{-1} = -k$ [1m]

we are given that $m_2 = m_1 + 8$

$\therefore \quad m_1 + m_1 + 8 = 4$

$\therefore \quad 2m_1 = 4 - 8$

$\therefore \quad 2m_1 = -4$

$\therefore \quad m_1 = -2$

Also $m_1 \quad m_1 + 8 \quad = -k$ (*)

$-2 \quad -2 + 8 \quad = -k$ (from *)

$\therefore \quad -2 \quad 6 \quad = -k$

$\therefore \quad -12 = -k$

$\therefore \quad k = 12$ [1m]

Q2(A)

(i)

(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)
p	q	$p \leftrightarrow q$	$\sim q$	$\sim q$	$p \wedge \sim q$	$\sim \ p \wedge \sim q$ (I)	$q \wedge \sim p$	$\sim \ q \wedge \sim p$ (II)	(I) $\wedge$ (II)
T	T	T	F	F	F	T	F	T	T
T	F	F	F	T	T	F	F	T	F
F	T	F	T	F	F	T	T	F	F
F	F	T	T	T	F	T	F	T	T

The entries in columns (3) and (10) are identical.

$\therefore \quad p \leftrightarrow q \equiv \sim \ p \wedge \sim q \ \wedge \sim \ q \wedge \sim p$

[1 mark for column (3), 1 mark for column (10) and 1 mark for conclusion.]

ii. The given equation is $2x^2 + 8xy + py^2 + qx + 2y - 15 = 0$

comparing it with $ax^2 + 2hxy + by^2 + 2gx + 2fy + c = 0$

we get,

$a = 2, h = 4, b = p, g = \frac{q}{2}, f = 1, c = -15$

Since the lines are parallel, $h^2 = ab$

$\therefore \quad 4^{2} = 2p \quad \therefore \; p = 8$ [1m]

Since the given equation represents a pair of lines

$\therefore \quad D = \begin{vmatrix} a & h & g \\ h & b & f \\ g & f & c \end{vmatrix} = 0$, where $b = p = 8$

i.e., $\begin{vmatrix} 2 & 4 & \frac{q}{2} \\ 4 & 8 & 1 \\ \frac{q}{2} & 1 & -15 \end{vmatrix} = 0$ [1m]

i.e. $2\;\; -120-1\;\; -4\left(-60-\frac{q}{2}\right)+\frac{q}{2}\;\; 4-4q\;\; = 0$

i.e. $\quad -242+240+2q+2q-2q^2 = 0$

i.e. $\quad -2q^2+4q-2 = 0$

$\therefore \quad q^2-2q+1 = 0$

$\therefore \quad q-1^{\;2} = 0$

$\therefore \quad q-1 = 0$

$\therefore \quad q = 1$ [1m]

Hence, $p = 8$ and $q = 1$

(B)

i. Let $A \begin{bmatrix} 7 & -6 & -2 \\ -18 & 16 & 5 \\ -10 & 9 & 3 \end{bmatrix}$

$\therefore \quad |A| = \begin{vmatrix} 7 & -6 & -2 \\ -18 & 16 & 5 \\ -10 & 9 & 3 \end{vmatrix}$

$= 7\;\; 48-45\;\; +6\;\; -54+50\;\; -2\;\; -162+160$

$= 21-24+4 = 1 \neq 0$

$\therefore \quad A^{-1}$ exists

we have to find the cofactor matrix $= \left[A_{ij}\right]_{3\times 3}$,

Where $A_{ij} = \;-1^{\;i+j}.M_{ij}$

Now, $A_{11} = \;-1^{\;1+1} M_{11} = \begin{vmatrix} 16 & 5 \\ 9 & 3 \end{vmatrix} = 48-45 = 3$

$A_{12} = \;-1^{\;1+2} M_{12} = -\begin{vmatrix} -18 & 5 \\ -10 & 3 \end{vmatrix} = -\;\; -54+50\;\; = 4$

$$A_{13} = \ -1\ ^{1+3} M_{13} = \begin{vmatrix} -18 & 16 \\ -10 & 9 \end{vmatrix} = -162 + 160 = -2$$

$$A_{21} = \ -1\ ^{2+1} M_{21} = -\begin{vmatrix} -6 & -2 \\ 9 & 3 \end{vmatrix} = -\ \ -18 + 18\ \ = 0$$

$$A_{22} = \ -1\ ^{2+2} M_{22} = \begin{vmatrix} 7 & -2 \\ -10 & 3 \end{vmatrix} = 21 - 20 = 1$$ [1m]

$$A_{23} = \ -1\ ^{2+3} M_{23} = -\begin{vmatrix} 7 & -6 \\ -10 & 9 \end{vmatrix} = -\ \ 63 - 60\ \ = -3$$

$$A_{31} = \ -1\ ^{3+1} M_{31} = \begin{vmatrix} 6 & -2 \\ 16 & 5 \end{vmatrix} = -30 + 32 = 2$$

$$A_{32} = \ -1\ ^{3+2} M_{32} = -\begin{vmatrix} 7 & -2 \\ -18 & 5 \end{vmatrix} = -\ \ 35 - 36\ \ = 1$$

$$A_{33} = \ -1\ ^{3+3} m_{33} = \begin{vmatrix} 7 & -6 \\ -18 & 16 \end{vmatrix}$$

$$= 112 - 108 = 4$$ [1m]

$\therefore$ the cofactor matrix $= \begin{bmatrix} A_{11} & A_{12} & A_{13} \\ A_{21} & A_{22} & A_{23} \\ A_{31} & A_{32} & A_{33} \end{bmatrix} = \begin{bmatrix} 3 & 4 & -2 \\ 0 & 1 & -3 \\ 2 & 1 & 4 \end{bmatrix}$

$\therefore$ $\operatorname{adj} A = \begin{bmatrix} 3 & 0 & 2 \\ 4 & 1 & 1 \\ -2 & -3 & 4 \end{bmatrix}$ [1m]

$\therefore$ $A^{-1} = \dfrac{1}{|A|}\ \operatorname{adj} A\ \ = \dfrac{1}{1}\begin{bmatrix} 3 & 0 & 2 \\ 4 & 1 & 1 \\ -2 & -3 & 4 \end{bmatrix}$

$\therefore$ $A^{-1} = \begin{bmatrix} 3 & 0 & 2 \\ 4 & 1 & 1 \\ -2 & -3 & 4 \end{bmatrix}$ [1m]

ii. Let m_1 and m_2 be the slopes of the lines represented by the equation $ax^2 + 2hxy + by^2 = 0$ (1)
then their separate equations are
$y = m_1x$ and $y = m_2x$

$\therefore$ their combined equation is $m_{1x-y} \quad m_2x - y \ \ = 0$

i.e. $m_1m_2x^2 - \ \ m_1 + m_2 \ \ xy + y^2 = 0$ (2) [1m]

since (1) and (2) represent the same two lines, capering the coefficients, we have.

$$\frac{m_1m_2}{a} = \frac{-\ \ m_1 + m_2}{2h} = \frac{1}{b}$$

$\therefore$ $m_1 + m_2 = \dfrac{-2h}{b}$ and $m_1m_2 = \dfrac{a}{b}$

$\therefore$ $\ m_1 - m_2\ ^2 = \ m_1 + m_2\ ^2 - 4m_1m_2$

$$= \frac{4h^2}{b^2} - \frac{4a}{b} = \frac{4(h^2 - ab)}{b^2}$$

$\therefore \quad |m_1 - m_2| = \left|\frac{2\sqrt{h^2 - ab}}{b}\right|$

If θ is the acute angle between the lines, then

$$\tan\theta = \left|\frac{m_1 - m_2}{1 + m_1 m_2}\right|, \text{ if } m_1 m_2 \neq -1$$

$$= \left|\frac{2\sqrt{h^2 - ab}/b}{1 + (a/b)}\right|, \text{ if } \frac{a}{b} \neq -1$$

$$= \left|\frac{2\sqrt{h^2 - ab}}{a + b}\right|, \text{ if } a + b \neq 0$$ [1m]

i. If the lines are perpendicular to each other,

then $m_1 m_2 = -1$, $\quad \therefore \frac{a}{b} = -1$, $\quad \therefore a = -b$

$\therefore \quad a + b = 0$

i.e., (coeff. Of x^2) + (coeff. of y^2) = 0

this is the condition for the lines to be perpendicular to each other. [1m]

ii. If the lines are parallel (coincident), then the angle θ between them is 0.

$\therefore \quad \tan\theta = 0$

$\therefore \quad \frac{2\sqrt{h^2 - ab}}{a + b} = 0$

$\therefore \quad h^2 - ab = 0$

This is the condition for the lines to be parallel (coincident) [1m]

3(A) i. Let p: the switch s_1 is closed.

q: the switch s_2 is closed

$\sim p$: the switch s_1' is closed

$\sim q$: the switch s_2' is closed

We observe from the given circuit the lamp L is 'on' if (s_1 is closed) and (s_1' is closed or s_2' is closed) and (s_2 is closed)

Hence, the logical expression of the given circuit is $p \wedge (\sim p \vee \sim q) \wedge q$ [1m]

Using the laws of logic, we have

$p \wedge (\sim p \vee \sim q) \wedge q$ [By Associative law]

$\equiv [p \wedge (\sim p \vee \sim q)] \wedge q$ [By Distributive law]

$\equiv [(p \wedge \sim p) \vee (p \wedge \sim q)] \wedge q$ [By complement law]

$\equiv [F \vee (p \wedge \sim q)] \wedge q$ [By Identity law]

$\equiv (p \wedge \sim q) \wedge q$ [By Associative law]

$\equiv p \wedge F$ [By Complement law]

$\equiv F$ [By Identify law] [1m]

Hence, the circuit will here work and if will be always off irrespective of the status of switches [1m]

ii. Show that $\sin^{-1}\left(\frac{8}{17}\right)+\sin^{-1}\left(\frac{3}{5}\right)=\sin^{-1}\left(\frac{77}{85}\right)$

Sol. Let $\sin^{-1}\left(\frac{8}{17}\right)=x, \sin^{-1}\left(\frac{3}{5}\right)=y$

and $\sin^{-1}\left(\frac{77}{85}\right)=z$

then $\sin x=\frac{8}{17}$, where $0<x<\frac{\pi}{2}$

$\sin y=\frac{3}{5}$, where $0<y<\frac{\pi}{2}$

and $\sin z=\frac{77}{85}$, where $0<z<\frac{\pi}{2}$ [1m]

$\therefore$ $\cos x>0, \cos y>0$

Now, $\cos x=\sqrt{1-\sin^2 x}=\sqrt{1-\frac{64}{289}}=\sqrt{\frac{225}{289}}=\frac{15}{17}$

$\cos y=\sqrt{1-\sin^2 y}=\sqrt{1-\frac{9}{25}}=\sqrt{\frac{16}{25}}=\frac{4}{5}$

We have to show that, $x+y=z$ [1m]

Now, $\sin\ x+y\ =\sin x.\cos y+\cos x.\sin y$

$=\left(\frac{8}{17}\right)\left(\frac{4}{5}\right)+\left(\frac{15}{17}\right)\left(\frac{3}{5}\right)$

$=\frac{32}{85}+\frac{45}{85}=\frac{77}{85}$

Q3(B) i. The given equations can be written in the matrix from as:

$$\begin{bmatrix}2 & 3 & 4\\ 3 & 1 & 2\\ 4 & 2 & 2\end{bmatrix}\begin{bmatrix}x\\ y\\ z\end{bmatrix}=\begin{bmatrix}-9\\ -12\\ -12\end{bmatrix}$$ [1m]

By $R_1 \leftrightarrow R_2$

$$\begin{bmatrix}3 & 1 & 2\\ 2 & 3 & 4\\ 4 & 2 & 2\end{bmatrix}\begin{bmatrix}x\\ y\\ z\end{bmatrix}=\begin{bmatrix}-12\\ -9\\ -12\end{bmatrix}$$

By R_2-R_2

$$\begin{bmatrix}1 & -2 & -2\\ 2 & 3 & 4\\ 4 & 2 & 2\end{bmatrix}\begin{bmatrix}x\\ y\\ z\end{bmatrix}=\begin{bmatrix}-3\\ -9\\ -12\end{bmatrix}$$

By R_2-2R_1 & R_3-4R_1

$$\begin{bmatrix} 1 & -2 & -2 \\ 0 & 7 & 8 \\ 0 & 10 & 10 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} -3 \\ -3 \\ 0 \end{bmatrix}$$

By $10R_2$,

$$\begin{bmatrix} 1 & -2 & -2 \\ 0 & 70 & 80 \\ 0 & 10 & 10 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} -3 \\ -30 \\ 0 \end{bmatrix}$$

By $R_2 - 7R_3$

$$\begin{bmatrix} 1 & -2 & -2 \\ 0 & 0 & 10 \\ 0 & 10 & 10 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} -3 \\ -30 \\ 0 \end{bmatrix}$$ [1m]

$\therefore$ $$\begin{bmatrix} x-2y-2z \\ 0+0+10z \\ 0+10y+10z \end{bmatrix} = \begin{bmatrix} -3 \\ -30 \\ 0 \end{bmatrix}$$

By equality of matrices,

$x - 2y - 2z = -3$ (1)

$10z = -30$ (2)

$10y + 10z = 0$ (3) [1m]

From (2) $z = -3$

Substituting $z = -3$ in (3), we get,

$10y - 30 = 0 \quad \therefore \quad y = 3$

Substituting $y = 3, z = -3$ in (1), we get,

$x - 6 + 6 = -3 \therefore x = -3$

Hence, $x = -3, y = 3, z = -3$ is the required solution. [1m]

ii. Let $A = \left[\begin{bmatrix} 2 & 0 & -1 \\ 5 & 1 & 0 \\ 0 & 1 & 3 \end{bmatrix}\right]$

$\therefore$ $|A| = \begin{vmatrix} 2 & 0 & -1 \\ 5 & 1 & 0 \\ 0 & 1 & 3 \end{vmatrix} = 1 \neq 0$

$\therefore$ A^{-1} exists

Consider $AA^{-1} = I$

$\therefore$ $$\begin{bmatrix} 2 & 0 & -1 \\ 5 & 1 & 0 \\ 0 & 1 & 3 \end{bmatrix} A^{-1} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

By $3R_1$,

$$\begin{bmatrix} 6 & 0 & -3 \\ 5 & 1 & 0 \\ 0 & 1 & 3 \end{bmatrix} A^{-1} = \begin{bmatrix} 3 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

By $R_1 - R_2$,

$$\begin{bmatrix} 1 & -1 & -3 \\ 5 & 1 & 0 \\ 0 & 1 & 3 \end{bmatrix} A^{-1} = \begin{bmatrix} 3 & -1 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$ [1m]

By $R_2 - 5R_3$

$$\begin{bmatrix} 1 & -1 & -3 \\ 0 & 1 & 0 \\ 0 & 1 & 3 \end{bmatrix} A^{-1} = \begin{bmatrix} 3 & -1 & 0 \\ -15 & 6 & -5 \\ 0 & 0 & 1 \end{bmatrix}$$ [1m]

By $R_1 + R_2$ and $R_3 - R_2$

$$\begin{bmatrix} 1 & 0 & -3 \\ 0 & 1 & 0 \\ 0 & 0 & 3 \end{bmatrix} A^{-1} = \begin{bmatrix} -12 & 5 & -5 \\ -15 & 6 & -5 \\ 15 & -6 & 6 \end{bmatrix}$$ [1m]

By $\left(\frac{1}{3}\right)R_3$,

$$\begin{bmatrix} 1 & 0 & -3 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} A^{-1} = \begin{bmatrix} -12 & 5 & -5 \\ -15 & 6 & -5 \\ 5 & -2 & 2 \end{bmatrix}$$

By $R_1 + 3R_3$,

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} A^{-1} = \begin{bmatrix} 3 & -1 & 1 \\ -15 & 6 & -5 \\ 5 & -2 & 2 \end{bmatrix}$$

$$\therefore \quad A^{-1} = \begin{bmatrix} 3 & -1 & 1 \\ -15 & 6 & -5 \\ 5 & -2 & 2 \end{bmatrix}$$ [1m]

Q4 (A)

1. (a) $k = 2$
2. (d) 5
3. (b) 0.55

(B) i. we have

$$p(x) = \frac{x-1}{3} \quad , \ x = 1, 2, 3$$

$$= 0 \qquad , \text{otherwise}$$

This can be written as

$X = x$	1	2	3
$p(X = x)$	0	1/3	2/3

[1m]

Here, $0 \le pi \le 1$

$$\sum pi = 0 + \frac{1}{3} + \frac{2}{3} = 1$$

Hence $p(X = x)$ can be regarded as the p.m.f. of the random variable X. [1m]

ii. $p(x<0) = \int_{-5}^{0} f(x)\,dx$

$= \int_{-\infty}^{-5} f(x)\,dx + \int_{-5}^{0} f(x)$ [1m]

$= 0 + \int_{-5}^{0} f(x)\,dx$ $\quad [\because f(x) = 0 \text{ for } x < -5]$

$= \int_{-5}^{0} \frac{1}{10dx}$

$= \frac{1}{10}[x]_{-5}^{0}$

$= \frac{1}{10}[0-(-5)]$

$= \frac{5}{10} = \frac{1}{2}$ [1m]

iii. The p.m.f. of r.v.x is

$p(x) = \binom{4}{x}\left(\frac{5}{9}\right)^{x}\left(\frac{4}{9}\right)^{4-x}$

Comparing this with $p(x) = {}^{n}C_{x}p^{x}q^{n-x}$

We get,

$n = 4, p = \frac{5}{9}, q = \frac{4}{9}$

$\therefore \quad E(x) = np = 4\left(\frac{5}{9}\right) = \frac{20}{9} = 2.22$ [1m]

and $\operatorname{var}(x) - npq = 4\times\frac{5}{9}\times\frac{4}{9} = \frac{80}{81} = 0.9876$

hence $E(x) = 2.22, \operatorname{var}(x) = 0.9876$ [1m]

Q5(A)i. Two cards can be drawn from a pack of 52 cards in ${}^{52}C_2$ ways

$\therefore \quad n(s) = {}^{52}C_2$

Since $X =$ number of red cards.

$\therefore$ X takes the value 0, 1, 2. [1m]

$p(x=0) = p(0) = p$ (no red card) $= p$ (both black cards)

$\frac{{}^{26}C_2}{{}^{52}C_2} = \frac{26\times 25}{52\times 51} = \frac{25}{102}$

$p(X=1) = p(1) = p$ (red card 1 black card)

$= \frac{{}^{26}C_1 \times {}^{26}C_1}{{}^{52}C_2} = \frac{26\times 26\times 1\times 2}{52\times 51} = \frac{26}{51}$

$p(X=2) = p(2) = p$ (both red cards)

$= \frac{{}^{26}C_2}{{}^{52}C_2} = \frac{26\times 25}{52\times 51} = \frac{25}{102}$

$\therefore$ Probability mars function is given by

$$p(x) = \frac{25}{102} \quad , \quad x = 0$$
$$= \frac{26}{51} \quad , \quad x = 1$$
$$= \frac{25}{102} \quad , \quad x = 2$$ [1m]

ii. Let $X =$ number of heads

$P =$ probability of getting head

$\therefore$ $P = \frac{1}{2}$

$\therefore$ $q = 1 - p = 1 - \frac{1}{2} = \frac{1}{2}$

Given $n = 8$

$\therefore$ $X \sim B(n, p)$

$\therefore$ $X \sim B\left(8, \frac{1}{2}\right)$

The p.m.f. of X is given as

$$p(X = x) = p(x) = {}^nC_x p^x q^{n-x}$$

i.e. $p(x) = {}^8C_x \left(\frac{1}{2}\right)^{xx} \left(\frac{1}{2}\right)^{8-x}, \ x = 0, 1, 2, \ldots, 8$ [1m]

$\therefore$ p (getting heads larger number of times than tails)

$$= p(x \geq 5) = p(5) + p(6) + p(7) + p(8)$$

$$= {}^8C_5 \left(\frac{1}{2}\right)^5 \left(\frac{1}{2}\right)^{8-5} + {}^8C_6 \left(\frac{1}{2}\right)^6 \left(\frac{1}{2}\right)^{8-6} + {}^8C_7 \left(\frac{1}{2}\right)^7 \left(\frac{1}{2}\right)^{8-7} + {}^8C_8 \left(\frac{1}{2}\right)^{8-8}$$ [1m]

$$= {}^8C_3 \left(\frac{1}{2}\right)^5 \left(\frac{1}{2}\right)^3 + {}^8C_2 \left(\frac{1}{2}\right)^6 \left(\frac{1}{2}\right)^2 + {}^8C_1 \left(\frac{1}{2}\right)^7 \left(\frac{1}{2}\right) + {}^8C_8 \left(\frac{1}{2}\right)^8 \ldots. \quad \left[\because {}^nC_x = {}^nC_{n-x}\right]$$

$$= \left(\frac{1}{2}\right)^8 \left[\frac{8 \times 7 \times 6}{1 \times 2 \times 3} + \frac{8 \times 7}{1 \times 2} + 8 + 1\right]$$

$$= \frac{1}{256}(56 + 28 + 8 + 1)$$

$$= \frac{93}{256}$$

$\therefore$ $p(x \geq 5) = 0.36328$

Hence, the probability of getting heads larger number of times than tails $= 0.36328$ [1m]

(B) i. $x =$ number of right turns

$p =$ probability that the rat takes the right turn

Then $q =$ probability that the rat takes the left turn

$\therefore$ $p = \frac{1}{2}$ and $q = \frac{1}{2}$

Given: $n = 10$

$\therefore$ $X \sim B(n, p)$

$\therefore \quad X \sim B\left(10, \frac{1}{2}\right)$

The p.m.f. of X is given as:

$$p(X = x) = {}^{n}C_x \left(\frac{1}{2}\right)^x \left(\frac{1}{2}\right)^{10-x} = {}^{10}C_x \left(\frac{1}{2}\right)^{10},$$

$$x = 0, 1, 2, \ldots, 10$$

$\therefore \quad p(x=0) = p(0) = {}^{10}C_0 \left(\frac{1}{2}\right)^{10} = \frac{1}{2^{10}}$ [1m]

$p(x=1) = p(1) = {}^{10}C_1 \left(\frac{1}{2}\right)^{10} = \frac{10}{2^{10}}$

$p(x=2) = p(2) = {}^{10}C_2 \left(\frac{1}{2}\right)^{10} = \frac{10 \times 9}{1 \times 2}\left(\frac{1}{2}\right)^{10} = \frac{45}{2^{10}}$

$p(x=3) = p(3) = {}^{10}C_3 \left(\frac{1}{2}\right)^{10} = \frac{10 \times 9 \times 8}{1 \times 2 \times 3}\left(\frac{1}{2}\right)^{10} = \frac{120}{2^{10}}$

$p(x=4) = p(4) = {}^{10}C_4 \left(\frac{1}{2}\right)^{10} = \frac{10 \times 9 \times 8 \times 7}{1 \times 2 \times 3 \times 4}\left(\frac{1}{2}\right)^{10} = \frac{210}{2^{10}}$

$p(x=5) = p(5) = {}^{10}C_5 \left(\frac{1}{2}\right)^{10} = \frac{10 \times 9 \times 8 \times 7 \times 6}{1 \times 2 \times 3 \times 4 \times 5}\left(\frac{1}{2}\right)^{10} = \frac{252}{2^{10}}$

$p(x=6) = p(6) = {}^{10}C_6 \left(\frac{1}{2}\right)^{10} = {}^{10}C_4 \left(\frac{1}{2}\right)^{10} = \frac{210}{2^{10}}$ $\left[\because {}^{n}C_r = {}^{n}C_{n-r}\right]$

$p(x=7) = p(7) = {}^{10}C_7 \left(\frac{1}{2}\right)^{10} = {}^{10}C_3 \left(\frac{1}{2}\right)^{10} = \frac{120}{2^{10}}$

$p(p=8) = p(8) = {}^{10}C_8 \left(\frac{1}{2}\right)^{10} = {}^{10}C_2 \left(\frac{1}{2}\right)^{10} = \frac{45}{2^{10}}$

$p(x=9) = p(9) = {}^{10}C_9 \left(\frac{1}{2}\right)^{10} = {}^{10}C_1 \left(\frac{1}{2}\right)^{10} = \frac{10}{2^{10}}$

$p(x=10) = p(10) = {}^{10}C_{10} \left(\frac{1}{2}\right)^{10} = \frac{1}{2^{10}}$ [1m]

$\therefore$ the probability distribution of X is as below:

x	0	1	2	3	4	5	6	7	8	9	10
p(x)	$\frac{1}{2^{10}}$	$\frac{10}{2^{10}}$	$\frac{45}{2^{10}}$	$\frac{120}{2^{10}}$	$\frac{210}{2^{10}}$	$\frac{252}{2^{10}}$	$\frac{210}{2^{10}}$	$\frac{120}{2^{10}}$	$\frac{45}{2^{10}}$	$\frac{10}{2^{10}}$	$\frac{1}{2^{10}}$

$\therefore$ p (at least 9 rats will take the same turn)

= p (at least 9 rats take either right or left turn)

= p (9 right turn or 10 right turn or 9 left turn or 10 left turn)

= p (9 right turn or 10 right turn or 1 right turn or 0 right turn)

= p (9 right turn or 10 right turn or 1 right turn or 0 right turn) [1m]

$= p(x=9) + p(x=10) + p(x=1) + p(x=0)$

$$= p(9) + p(10) + p(1) + p(0)$$

$$= \frac{10}{2^{10}} + \frac{1}{2^{10}} + \frac{10}{2^{10}} + \frac{1}{2^{10}}$$

$$= \frac{10+1+10+1}{2^{10}} = \frac{22}{1024} = 0.0215$$

Hence, the probability that at least 9 rats will turn the same way is 0.0215 [1m]

ii. $f(x) = \int_0^x f(x)\,dx$

$$= \int_0^x 3(1-2x^2)\,dx = 3\int_0^x (1-2x^2)\,dx\sqrt{a^2+b^2}$$

$$= 3\left[x - \frac{2x^3}{2}\right]_0^x = 3\left[x - \frac{2x^3}{3} - 0\right]$$

$\therefore \quad f(x) = 3\left(x - \frac{2x^3}{3}\right)$ [1m]

$$\frac{p\left(\frac{1}{4}<x<\frac{1}{3}\right)\text{by using p.d.f}}{p\left(\frac{1}{4}<x<\frac{1}{3}\right) = \int_{1/4}^{1/3} f(x)\,dx}$$

$$= \int_{1/4}^{1/3} 3(1-2x^2)\,dx = \int_{1/4}^{1/3} (1-2x^2)\,dx$$

$$= 3\left[x - 2\frac{x^3}{3}\right]_{1/4}^{1/3}$$

$$= 3\left(\frac{25}{81} - \frac{23}{96}\right) = 3\left(\frac{2400-1863}{7776}\right)$$

$$= 3\left(\frac{537}{7776}\right) = \frac{537}{2592} = \frac{179}{864}$$ [1m]

$$\frac{p\left(\frac{1}{4}<x<\frac{1}{3}\right)\text{by using c.d.f}}{p\left(\frac{1}{4}<x<\frac{1}{3}\right) = f\left(\frac{1}{3}\right) - f\left(\frac{1}{4}\right)}$$ [1m]

Where $f(x) = 3\left(x - \frac{2x^3}{3}\right)$

$$= 3\left(\frac{1}{3} - \frac{2}{81}\right) - 3\left(\frac{1}{4} - \frac{1}{96}\right)$$

$$= 3\left(\frac{25}{81}\right) - 3\left(\frac{23}{96}\right)$$

$$= \frac{25}{27} - \frac{23}{32}$$

$$= \frac{800-621}{864} = \frac{179}{864}$$ [1m]

Q6.(A) i. Let p: A sequence is bounded

q: It is convergent

then the symbolic form of the given statement is $p \to q$

Converser: $q \to p$ is the converse of $p \to q$

i.e. If a sequence is convergent then it is bounded. [1m]

Inverse: $\sim p \to \sim q$ is the inverse of $p \to q$

i.e. If a sequence is not bounded then it is not convergent. [1m]

Contraposisitve: $\sim q \to \sim p$ is the contrapositive of $p \to 1$

i.e. If a sequence is not convergent then it is not bounded. [1m]

ii. Let m_1 and m_2 be the slopes of the lines represented by $2x^2 + 7xy + 3y^2 = 0$ which is of the form $ax^2 + 2hxy + by^2 = 0$

$\therefore \quad a = 2, 2h = 7, b = 3$

$\therefore \quad m_1 + m_2 = \frac{-2h}{b} = \frac{-7}{3}$

and $\quad m_1.m_2 = \frac{a}{b} = \frac{2}{3}$ [1m]

Now required lines are perpendicular to these lines.

$\therefore$ Their slopes are $-1/m_1$ and $-1/m_2$

Since these lines are passing through the origin.

$\therefore$ Their separate equations are $y = \frac{-1}{m_1}x$ and $y = \frac{-1}{m_2}x$

i.e. $\quad m_1 y = -x$ and $m_2 y = -x$ [1m]

i.e. $\quad x + m_1 y = 0$ and $x + m_2 y = 0$

$\therefore$ Their joint equation is $(x + m_1 y)(x + m_2 y) = 0$

$\therefore \quad x^2 + \left(\frac{-7}{3}\right)xy + \frac{2}{3}y^2 = 0$ [By(1)]

$\therefore \quad 3x^2 - 7xy + 2y^2 = 0$ [1m]

Q6(B)i $\quad 2\tan x - \cot x + 1 = 0$

$\therefore \quad 2\tan x - \frac{1}{\tan x} + 1 = 0$

$\therefore \quad 2\tan^2 x - 1 + \tan x = 0$

$\therefore \quad 2\tan^2 x + 2\tan x - \tan x - 1 = 0$

$\therefore \quad (\tan x + 1)(2\tan x - 1) = 0$

$\therefore$ either $\tan x + 1 = 0$ or $2\tan x - 1 = 0$

$\therefore$ either $\tan x = -1$ or $\tan x = \frac{1}{2}$ [1m]

$\therefore$ either $\tan x = -\tan\frac{\pi}{4}$ or $\tan x = \frac{1}{2}$ $\left[\because \tan\frac{\pi}{4} = 1\right]$

$\therefore$ either $\tan x = \tan\left(\pi - \frac{\pi}{4}\right)$ or $\tan x = \frac{1}{2}$ $\left[\because \tan(\pi - \theta) = -\tan\theta\right]$ [1m]

$\therefore$ either $\tan x = \tan\frac{3\pi}{4}$ or $\tan x = \tan\left(\tan^{-1}\frac{1}{2}\right)$ $\left[\because \tan\ \pi-\theta\ = -\tan\theta\right]$ [1m]

The general solution of

$\tan\theta = \tan\alpha$ is $\theta = n\pi + \alpha,\ n \in Z$ [1m]

$\therefore$ the required general solution is given by

$x = n\pi + \frac{3\pi}{4}$ or $x = n\pi + \tan^{-1}\frac{1}{2}, n \in Z$ [1m]

ii. By sine rule,

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C} = k$$

$\therefore$ $a = k\sin A, b = k\sin B, c = k\sin C$ [1m]

R.H.S. $b^2 - c^2\ \ \sin A$

$= \ k^2\sin^2 B - k^2\sin^2 C\ \ \sin A$

$= k^2\ \ \sin^2 B - \sin^2 C\ \ \sin A$

$= k^2 \times 2\sin\left(\frac{B+C}{2}\right).\cos\left(\frac{B-C}{2}\right) \times 2\sin\left(\frac{B-C}{2}\right).\cos\left(\frac{B-C}{2}\right) \times \sin A$

$= k^2 \times 2\sin\left(\frac{B+C}{2}\right).\cos\left(\frac{B+C}{2}\right) \times 2\sin\left(\frac{B-C}{2}\right).\cos\left(\frac{B.C}{2}\right) \times \sin A$

$= k^2 \times \sin\ \ B+C\ \times \sin\ \ B-C\ \times \sin A$

$= k^2\sin\ \ \pi - A\ \ \sin\ \ B-C\ \ .\sin A$ $\because A+B+B=\pi$ [1m]

$= k^2\sin A.\sin\ \ B-C\ \ .\sin A$

$= \ k\sin A\ \ ^2.\sin\ \ B-C$

$a^2\sin\ \ B-C\ = L.H.S$ [1m]