**Answers Key (Main)**

**DATE: 16/03/19**

**IIT-JEE: 2019 MAJOR TEST - 3**

**CODE: 2**

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**Note:** Detailed solution to this test is available on Monday after 02.00 pm on our website: [www.iitianspace.com](http://www.iitianspace.com)
PART (A) : PHYSICS

1. (1)
Given equation is dimensionally correct because both sides are dimensionless but numerically wrong because the correct equation is $\tan \theta = \frac{v^2}{rg}$.

2. (2)
Percentage error in $g = (\% \text{ error in } l) + 2 (\% \text{ error in } T) = 1\% + 2(3\%) = 7\%$

3. (3)
As $v^2 = u^2 + 2as \Rightarrow (2u)^2 = u^2 + 2as \Rightarrow 2as = 3u^2$
Now, after covering an additional distance $s$, if velocity becomes $v$, then,
$v^2 = u^2 + 2a(2s) = u^2 + 4as = u^2 + 6u^2 = 7u^2$
$\therefore \quad v = \sqrt{7} u$

4. (4)
The velocity time graph for given problem is shown in the figure.

5. (1)
$\frac{mv^2}{r} = 10 \Rightarrow \frac{1}{2}mv^2 = 10 \times \frac{r}{2} = 1J$

6. (2)
Equation of trajectory for oblique projectile motion
$y = x \tan \theta - \frac{gx^2}{2u^2 \cos^2 \theta}$
Substituting $x = D$ and $u = v_0$
$h = D \tan \theta - \frac{gD^2}{2u_0^2 \cos^2 \theta}$.

7. (3)
The coin falls behind him it means the velocity of train was increasing otherwise the coin would fall directly into the hands of thrower.

8. (2)
$v = \sqrt{\mu \ g \ r} = \sqrt{0.8 \times 9.8 \times 15} = 10.84 \ m/s$
9. (4)

By the conservation of momentum, \( m_A v_A = m_B v_B \Rightarrow m \times 16 = 2m \times v_B \Rightarrow v_B = 8 \text{ m/s} \)

Kinetic energy of system = \( \frac{1}{2} m_A v_A^2 + \frac{1}{2} m_B v_B^2 = \frac{1}{2} m \times (16)^2 + \frac{1}{2} \times (2m) \times 8^2 = 192 \text{ m J} \)

10. (2)

Let speed of the bullet = \( v \)

Speed of the system after the collision = \( V \)

By conservation of momentum \( m v = (m + M) V \)

\[ \Rightarrow V = \frac{m v}{M + m} \]

So the initial K.E. acquired by the system

\[ \frac{1}{2} (M + m) V^2 = \frac{1}{2} (m + M) \left( \frac{m v}{M + m} \right)^2 = \frac{1}{2} \frac{m^2 v^2}{(m + M)} \]

This kinetic energy goes against work done by friction = \( \mu R \times x = \mu (m + M) g \times x \)

By the law of conservation of energy

\[ \frac{1}{2} \frac{m^2 v^2}{(m + M)} = \mu (m + M) g \times x \Rightarrow v^2 = 2 \mu g x \left( \frac{m + M}{m} \right)^2 \]

\[ \therefore v = \sqrt{2 \mu g x \left( \frac{m + M}{m} \right)} \]

11. (3)

Heat is lost by steam in two stages

(i) for change of state from steam at 100ºC to water at 100ºC is \( m \times 540 \)

(ii) to change water at 100ºC to water at 80ºC is \( m \times 1 \times (100 - 80) \), where \( m \) is the mass of steam condensed.

Total heat lost by steam is \( m \times 540 + m \times 20 = 560 \text{ m (cals)} \) Heat gained by calorimeter and its contents is \( (1.1 + 0.02) \times (80 - 15) = 1.12 \times 65 \text{ cals.} \)

using Principle of calorimetry, Heat gained = heat lost

\[ \therefore 560m = 1.12 \times 65, m = 0.130 \text{ gm} \]

12. (1)

Initial diameter of tyre = \((1000 - 6) \text{ mm} = 994 \text{ mm}\), so initial radius of tyre \( R = \frac{994}{2} = 497 \text{ mm} \)

and change in diameter \( \Delta D = 6 \text{ mm} \) so \( \Delta R = \frac{6}{2} = 3 \text{ mm} \)

After increasing temperature by \( \Delta \theta \) tyre will fit onto wheel

Increment in the length (circumference) of the iron tyre

\( \Delta L = L \times \alpha \times \Delta \theta = L \times \frac{\gamma}{3} \times \Delta \theta \) [As \( \alpha = \frac{\gamma}{3} \)]
\[ 2\pi \Delta R = 2\pi R \left( \frac{\gamma}{3} \right) \Delta \theta \Rightarrow \Delta \theta = \frac{3}{\gamma} \frac{\Delta R}{R} = \frac{3 \times 3}{3.6 \times 10^{-5} \times 497} \]
\[ \Rightarrow \Delta \theta = 500^\circ C \]

13. (2)

**Hint:** Let \( l' \) be the length of rope hanging from the table and \( l \) be the length resting on the table.
If \( l' \) is maximum, then the rope is just about to slip.
Thus, tension due to part \( l' = \) friction on part \( l \).
Where, tension due to part \( l' = \) weight of part \( l' \) and
friction on part \( l \) = limiting friction = \( \mu \times \) weight of part \( l \).

14. (4)

\[ W_{BCOB} = - \text{Area of triangle } BCO = - \frac{P_0V_0}{2} \]
\[ W_{AODA} = + \text{Area of triangle } AOD = + \frac{P_0V_0}{2} \]

15. (1)

Suppose thickness of each wall is \( x \) then
\[ \left( \frac{Q}{t} \right)_{\text{combination}} = \left( \frac{Q}{t} \right)_A \]
\[ \Rightarrow K_s A(\theta_1 - \theta_2) = \frac{2KA(\theta_1 - \theta)}{x} \]
\[ \therefore K_s = \frac{2 \times 2K \times K}{(2K + K)} = \frac{4}{3} K \text{ and } (\theta_1 - \theta_2) = 36^\circ \]
\[ \Rightarrow \frac{4}{3} KA \times 36 \]
\[ \Rightarrow \frac{2KA(\theta_1 - \theta)}{x} \]

Hence temperature difference across wall \( A \) is
\[ (\theta_1 - \theta) = 12^\circ C \]

16. (4)

According to Newton law of cooling
\[ \frac{\theta_1 - \theta_2}{t} = K \left[ \frac{\theta_1 + \theta_2}{2} - \theta_0 \right] \]

\[ 80^\circ C \rightarrow \text{1 min} \rightarrow 64^\circ C \]
\[ 80^\circ C \rightarrow \text{10 min} \rightarrow 52^\circ C \]
\[ 80^\circ C \rightarrow \text{15 min} \rightarrow \theta = ? \]

For first process: \[ \frac{(80 - 64)}{5} = K \left[ \frac{80 + 64}{2} - \theta_0 \right] \]
\[ \text{(i)} \]

For second process: \[ \frac{(80 - 52)}{10} = K \left[ \frac{80 + 52}{2} - \theta_0 \right] \]
\[ \text{(ii)} \]

For third process: \[ \frac{(80 - \theta)}{15} = K \left[ \frac{80 + \theta}{2} - \theta_0 \right] \]
\[ \text{…(iii)} \]

On solving equation (i) and (ii) we get \( K = \frac{1}{15} \) and \( \theta_0 = 24^\circ C \)
Putting these values in equation (iii) we get $\theta = 42.7^\circ C$

17. (3) 
\[ Q \propto T^4 \Rightarrow \frac{Q_1}{Q_2} = \left(\frac{T_1}{T_2}\right)^4 \]
\[ \Rightarrow \frac{Q_1}{Q_2} = \left(\frac{T}{T+T/2}\right)^4 = \frac{16}{81} \Rightarrow Q_2 = \frac{81}{16} Q_1 \]

% increase in energy = \[\frac{Q_2 - Q_1}{Q_1} \times 100 = 400\%\]

18. (4) 
Highly polished mirror like surfaces are good reflectors, but not good radiators.

19. (3) 
\[ x_{CM} = \frac{\int x \, dm}{\int dm} \text{ if } n = 0, \text{ then } x_{CM} = \frac{L}{2} \]

As \( n \) increases, the centre of mass shift away from \( x = \frac{L}{2} \), thus option (1) is satisfying.

Alternatively:
\[ x_{CM} = \frac{\int_0^L k \left(\frac{x}{L}\right)^n \, dx}{\int_0^L k \left(\frac{x}{L}\right)^n \, dx} \]

Solving, \( x_{CM} = L \left[\frac{n+1}{n+2}\right] \)

20. (2) 
M.I of the system about an axis through \( A \) in the plane of the figure-2 parallel to \( BC \) is
\[ I_1 = m_b (4cm)^2 + m_c (4cm)^2 \]
\[ = 2 \times 16 \times 10^{-4} + 5 \times 16 \times 10^{-4} \]
\[ = 7 \times 16 \times 10^{-4} = 112 \times 10^{-4} kgm^2 \]

M.I of the system about an axis passing through \( A \) and perpendicular to the plane of the figure
\[ I_2 = m_b (4cm)^2 + m_c (5cm)^2 \]
\[ = 2 \times 16 \times 10^{-4} m + 5 \times 25 \times 10^{-4} m \]
\[ = (32 + 125) \times 10^{-4} = 147 \times 10^{-4} kgm^2 \]

\[ \text{ratio} \frac{I_1}{I_2} = \frac{112}{147} = \frac{16}{21} \]
21. (2)
The friction provides the opposing torque.
\[
\alpha = \frac{fR}{I}
\]
It will decrease the angular velocity.
\[
\omega = \omega_0 - \alpha t = \omega_0 - \frac{\mu mg R}{2I} t = \omega_0 - \frac{2\mu gt}{R}
\]
Due to the friction, \( f = \mu mg \) linear velocity will increase.
\[\nu = 0 + at = 0 + \mu gt.\]
For pure rolling, \( v = R\omega \)
\[
\therefore \mu gt = \omega_0 R - 2\mu gt \Rightarrow 3\mu gt = \omega_0 R \Rightarrow t = \frac{\omega_0 R}{3\mu g}
\]

22. (3)
For the translational equilibrium
\[
R_f + R_r = W
\]
\( R_f = \) Reaction on front wheels
\( R_r = \) Reaction on rear wheels
\( R_f + R_r = 2000 \times 10 = 20000 \text{ N} \)
For the rotational equilibrium of the car (about the C.M.)
\[1.5 \times R_f = 0.5 R_r\]
Also \( R_f = \frac{R_r}{3} \) or \( R_f = 3R_f \)
\[R_f + R_r = 20000\]
And \( R_f + 3R_f = 20000 \Rightarrow R_f = 5000 \text{ N} \)
\[R_r - 3R_f = 15000 \text{ N}\]
Reaction on each front wheel = \( \frac{5000}{2} = 2500 \text{ N} \)
Reaction on each rear wheel = \( \frac{15000}{2} = 7500 \text{ N} \)
Correct option is (3).

23. (1)
Initial angular momentum of the system = Angular momentum of bullet before collision
\[= Mv \left( \frac{L}{2} \right) \quad \text{..... (i)}\]
Assume the rod rotates with angular velocity \( \omega \).
Final angular momentum of the system = \( \left( \frac{ML^2}{12} \right) \omega + M \left( \frac{L}{2} \right)^2 \omega \quad \text{.....(ii)}\)
By equation (i) and (ii) \( Mv \frac{L}{2} = \left( \frac{ML^2}{12} + \frac{ML^2}{4} \right) \omega \) or \( \omega = \frac{3v}{2L} \)
24.  

\[ T = 2\pi \sqrt{\frac{2R}{g}} = 2\pi \sqrt{\frac{1}{g}} = 2 \text{ sec} \]  

[As diameter \( 2R = 1 \) \text{ meter given}]

25.  

\[ \frac{1}{2} m (2\pi f)^2 A^2 = K.E. + P.E. \]

26.  

\[ y = 0.2 \sin(10\pi t + 1.5\pi) \cos(10\pi t + 1.5\pi) \]

\[ = 0.1 \sin 2(10\pi t + 1.5\pi) \quad \text{[:: \sin 2A = 2 \sin A \cos A]} \]

\[ = 0.1 \sin(20\pi t + 3.0\pi) \]

\[ \therefore \text{ Time period, } T = \frac{2\pi}{\omega} = \frac{2\pi}{20\pi} = \frac{1}{10} = 0.1 \text{ sec} \]

27.  

Let the piston be displaced through distance \( x \) towards left, then volume decreases, pressure increases. If \( \Delta P \) is increase in pressure and \( \Delta V \) is decrease in volume, then considering the process to take place gradually (i.e. isothermal)

\[ PV_1 = PV_2 \Rightarrow PV = (P + \Delta P)(V - \Delta V) \]

\[ \Rightarrow PV = PV + \Delta PV - P\Delta V - \Delta P V \]

\[ \Rightarrow \Delta P V - P \Delta V = 0 \quad \text{(neglecting } \Delta P. \Delta V) \]

\[ \Delta P(Ah) = P(Ax) \Rightarrow \Delta P = \frac{P.Ax}{h} \]

This excess pressure is responsible for providing the restoring force \( (F) \) to the piston of mass \( M \).

Hence \( F = \Delta P.A = \frac{PAx}{h} \)

Comparing it with \( |F| = kx \Rightarrow k = M \omega^2 = \frac{PA}{h} \)

\[ \Rightarrow \omega = \sqrt{\frac{PA}{Mh}} \Rightarrow T = 2\pi \sqrt{\frac{Mh}{PA}} \]

28.  

Let \( T_1 \) and \( T_2 \) are the time period of the two pendulums \( T_1 = 2\pi \sqrt{\frac{100}{g}} \) and \( T_2 = 2\pi \sqrt{\frac{121}{g}} \) \( (T_1 < T_2) \) because \( l_1 < l_2 \).

Let at \( t = 0 \), they start swinging together. Since their time periods are different, the swinging will not be in unison always. Only when number of completed oscillation differs by an integer, the two pendulum will again begin to swing together.

Let longer length pendulum complete \( n \) oscillation and shorter length pendulum complete \( (n + 1) \) oscillation, for the unison swinging, then \( (n + 1)T_1 = nT_2 \)

\[ (n+1) \times 2\pi \sqrt{\frac{100}{g}} = n \times 2\pi \sqrt{\frac{121}{g}} \Rightarrow n = 10 \]
29. (4)
Velocity component along common tangent does not change while velocity component along common normal will interchange in case of elastic collision between two identical masses.

![Diagram showing velocities before and after collision]

30. (4)
In this case, spring force is zero initially.
Hence, tension in the string must also be zero initially.

![FBD of A and B]

$FBD$ of $A$ and $B$ will be as shown.
PART (B) : CHEMISTRY

31. (3)
Suppose the mol.wt. of enzyme = \( x \)
Given 100 g of enzyme wt of Se =0.5 gm
\( \therefore \) In x of enzyme wt. of Se = \( \frac{0.5 \times x}{100} \)
Hence, \( 78.4 = \frac{0.5 \times x}{100} \)
\( \therefore x = 15680 = 1.568 \times 10^4 \)

32. (2)
No. of radial nodes in 3p- orbital
\( = (n - \ell - 1) \)
[For p orbital \( \ell = 1 \)]
\( = 3 - 1 - 1 = 1 \)

33. (2)
Suppose the nucleus of hydrogen atom have charge of one proton i.e. The electron revolves in a radius of \( r \) around it. Therefore the centripetal force is supplied by electrostatic force of attraction i.e.
\[ \frac{mv^2}{r} = \frac{ze^2}{r^2} \]
or, \[ mv^2 = \frac{ze^2}{r} \]
or, \[ \frac{1}{2}mv^2 = \frac{1}{2} \frac{ze^2}{r} = \text{K.E.} \]
Now, total energy \( (E_n) = \text{K.E.} + \text{P.E.} \) in first excited state
\[ E = \frac{1}{2}mv^2 + \left( -\frac{ze^2}{r} \right) \]
\[ = + \frac{1}{2} \frac{ze^2}{r} - \frac{ze^2}{r} \]
\[ -3.4\text{eV} = - \frac{1}{2} \frac{ze^2}{r} \]
\( \therefore \) K.E. = \( \frac{1}{2} \frac{ze^2}{r} = +3.4\text{eV} \)

34. (1)
\( \text{SO}_2 > \text{P}_2\text{O}_5 > \text{SiO}_2 > \text{Al}_2\text{O}_3 \)
Acidic Weak acidic Amphoteric

35. (4)
As difference of electro negativity increases % ionic character increases and covalent character decreases i.e, electro negativity difference decreases and covalent character increases. Further greater the charge on the cation more will be its covalent character. Be has maximum (+2) charge.
36. (2)  
\[
\frac{\text{Moles of } H_2}{\text{Moles of } O_2} = \frac{8}{1} \text{(Given)}
\]
\[
\left(\frac{\text{M.W. of } O_2}{\text{M.W. of } H_2}\right) \left(\frac{\text{weight of } H_2}{\text{weight of } O_2}\right) = \frac{8}{1}
\]
\[
\text{weight of } H_2 = \frac{8 \times 2}{32} = 1
\]
\[
\text{weight of } O_2 = \frac{32 \times 1}{2} = 2
\]

37. (3)  
\[V_1 = 300 \text{ c.c.}, T_1 = 273 + 27K = 300K,\]
\[P_1 = 620 \text{ mm of } Hg\]
\[T_2 = 273 + 47K = 320K, P_2 = 640 \text{ mm of } Hg\]
By gas equation,
\[
\frac{P_1 V_1}{T_1} = \frac{P_2 V_2}{T_2}
\]
\[
\frac{300 \times 620}{300} = \frac{640 \times V_2}{320}
\]
\[\Rightarrow V_2 = \frac{300 \times 620 \times 320}{300 \times 640} = 310 \text{ cc}
\]

38. (1)  
\[\Delta H = \Delta E + \Delta nRT\]
\[\Delta n = 3 - (1 + 5) = 3 - 6 = -3\]
\[\Delta H - \Delta E = (-3RT)\]

39. (4)  
(i) Vander Waal’s radius > covalent radius  
(iii) Nitrogen has half filled orbitals  
(iv) Halogen 1\textsuperscript{st} electron gain enthalpy negative while 2\textsuperscript{nd} positive.

40. (3)  
\[\Delta S = \frac{\text{Latent heat of fusion}}{\text{Melting point}} = \frac{\Delta H_f}{T}\]
\[= \frac{2930}{300} \text{ JK}^{-1}\text{mol}^{-1}\]
\[= 9.77 \text{ JK}^{-1}\text{mol}^{-1}\]

41. (1)  
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<th>Hybridisation</th>
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<td>sp\textsuperscript{3}</td>
</tr>
<tr>
<td>ClO\textsubscript{3}\textsuperscript{+}</td>
<td>3</td>
<td>sp\textsuperscript{2}</td>
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42. (3) 
\[ H_2C_2O_4 \rightarrow 2H^+ + C_2O_4^{2-} \quad \Delta H_1 \]
\[ 2H^+ + 2OH^- \rightarrow 2H_2O \quad -57.32 \times 2 \]
\[ -53.35 = -114.64 + \Delta H_1 \]
\[ \Delta H_1 = 61.29 \]

43. (1) 
A salt of strong base with weak acid undergoes anionic hydrolysis to give basic solution.

44. (2) 
\[ K_p = \frac{P^2_{\text{CO}_2}}{P_{\text{CO}_2}}; K_p = \frac{4 \times 4}{2} = 8; C(s) = 1; \]
The concentration of solids and liquids are taken as unity.

45. (4) 
\[ \left[ H^+ \right] = \frac{K_a[\text{ACID}]}{[\text{SALT}]}; \left[ H^+ \right] = \frac{1.8 \times 10^{-6} \times 0.1}{0.5} \]
\[ = 0.36 \times 10^{-6}; \log \left[ H^+ \right] = -\log \left[ 0.36 \times 10^{-6} \right] \]
\[ \therefore \quad \text{pH} = 6.44 \]

46. (1) 
NaCl is a salt of strong acid and strong base hence its aqueous solution will be neutral i.e. pH = 7.
NaHCO₃ is an acidic salt hence pH < 7.
Na₂CO₃ is a salt of weak acid and strong base.
Hence its aqueous solution will be strongly basic i.e. pH > 7.
NH₄Cl is salt of weak base and strong acid, hence its aqueous will be strongly acid i.e. pH < 7.

47. (4) 
\[ 2\text{SO}_2 + \text{O}_2 \overset{\text{Heat}}{\longrightarrow} 2\text{SO}_3 \]
Exothermic reaction with a reduction in number of moles is favoured by low temperature and high pressure.

48. (1) 
Carbon has the maximum oxidation state of +4, therefore carbon dioxide (CO₂) cannot acts as a reducing agent.

49. (4) 
In KI it shows −1 O.S & in IF₇ & in IE₇ it shows +7 O.S hence O.S of I varies from −1 to +7.

50. (2) 
In KMnO₄ the O.N. of Mn is +7, in MnO₄²⁻ +6, in MnO₂ +4, in Mn₂O₃ +3 and in Mn²⁺ is +2.
The difference being 1, 3, 4 and 5 respectively.
51.  (3)
Hint: Et$_2$NH do not show resonance.
Sol: Due to conjugation

52.  (4)
Hint: Check the stability of conjugate base.
Sol: H$_p$ is most acidic as conjugate base is aromatic

53.  (3)
Hint: More stable resonating structure contribute more towards resonance hybrid.
Sol: Order of stability: I = II > III, i.e., structures I and II will contribute equally towards resonance hybrid.

54.  (1)
Hint: Effect of back bonding will dominate over inductive effect.
Sol: ‘S’ has vacant 3$d$ orbitals in which delocalization of negative charge from the adjacent carbon atom occurs.
55. (4)
Hint: least delocalised lone pair has maximum basic strength.

Sol:

56. (2)
Hint: Consider only the alkenes.
Sol: C_4H_6 → \[\text{CH}_2=\text{C} \equiv \text{CH} - \text{CH}_3\]

57. (4)
Hint: Empirical formula calculation.
Sol: H_2C = COOH
CH_2COOH

58. (2)
Hint: Isomers should have same molar formula.
Sol: Given compound

Option-(3)

59. (1)
Hint: Due to resonance, \(\ell_3\) decreases

Sol: \(\ell_1 - sp^3 - sp^3\)
\(\ell_2 - sp^3 - sp^3\)
\(\ell_3 - \) due to resonance double bond character
\[\therefore \ell_1 > \ell_2 > \ell_3\]
60. (2)
Hint: Ketone has the maximum priority among all.
Sol:

```
1 2 3 4 5 6 7
Cl
F
```

```text
OF
CH₃
O
CH₃
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CH₃
1
2
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```
61. (2)
Since, \(0 < \sin \frac{\pi}{3} < 1\), the inequality \(\log_{\sin(\pi/3)} \left( x^2 - 3x + 2 \right) \geq 2 \)
\[ x^2 - 3x + 2 \leq \left( \sin \frac{\pi}{3} \right)^2 \text{ and } x^2 - 3x + 2 > 0 \]
\[ 4x^2 - 12x + 5 \leq 0 \text{ and } x^2 - 3x + 2 > 0 \]
\[ (2x-1)(2x-5) \leq 0 \text{ and } (x-1)(x-2) > 0 \]
\[ \frac{1}{2} \leq x \leq \frac{5}{2} \text{ and } (-\infty < x < 1 \text{ or } 2 < x < \infty) \]
\[ \frac{1}{2} \leq x \leq \frac{5}{2} \text{ or } 2 < x \leq \frac{5}{2} \]
\[ x \in \left[ \frac{1}{2}, 1 \right] \cup \left( 2, \frac{5}{2} \right] \]

62. (4)
Given, \(2 \sin^2 \theta + \sqrt{3} \cos \theta + 1 = 0\)
\[ 2(1 - \cos^2 \theta) + \sqrt{3} \cos \theta + 1 = 0 \]
\[ -2 \cos^2 \theta + \sqrt{3} \cos \theta + 3 = 0 \]
\[ 2 \cos^2 \theta - \sqrt{3} \cos \theta - 3 = 0 \]
\[ 2 \cos^2 \theta - 2\sqrt{3} \cos \theta + \sqrt{3} \cos \theta - 3 = 0 \]
\[ (\cos \theta - \sqrt{3})(2 \cos \theta + \sqrt{3}) = 0 \]
\[ \cos \theta = -\frac{\sqrt{3}}{2} \]
\[ \theta = \frac{5\pi}{6} \quad (\because \cos \theta \neq \sqrt{3}) \]

63. (2)
For the equation \(2x^2 + 3x + 4 = 0\)
\[ \text{Discriminant } = (3)^2 - 4 \cdot 2 \cdot 4 < 0 \]
\[ \therefore \text{ Roots of } 2x^2 + 3x + 4 = 0 \text{ are imaginary since, the equations } x^2 + bx + c = 0 \text{ and } 2x^2 + 3x + 4 = 0 \text{ are given to have a common root, therefore both roots will be common.} \]
Hence, both the equations are identical.
\[ a:b:c = 2:3:4 \]
\[ \Rightarrow \text{ Least value of } a + b + c = 9 \]

64. (1)
Given, \(|x-2|^2 + |x-2| - 2 = 0\)
\[ y^2 + y - 2 = 0, \text{ where } y = |x-2| \]
\[ (y+2)(y-1) = 0 \]
\[ y = 1 \] :: \[ y = |x - 2| = -2 \text{ not possible} \]
\[ |x - 2| = 1 \]
\[ x - 2 = \pm 1 \]
\[ x = 3, 1 \]
\[ \therefore \text{Sum of roots is 4.} \]

65. (2)
Discriminant \[ 4a^2 - 4(a^2 + a - 3) > 0 \]
\[ \Rightarrow a < 3 \] .....(i)
Since, the roots are less than 3.
\[ \therefore \text{Sum of the roots is less than 6} \]
i.e., \[ \frac{2a}{1} < 6 \]
\[ \Rightarrow a < 3 \] .....(ii)
Now, \[ f(3) > 0 \]
\[ \Rightarrow (a - 2)(a - 3) > 0 \]
\[ \Rightarrow a < 2 \text{ or } a > 3 \] .....(iii)
From equations (i), (ii) and (iii), we get \[ a < 2. \]

66. (4)
\[ S_1 = \frac{a}{1 - r^2} \] (i.e. Sum of odd terms)
Given, \[ S = 5S_1 \Rightarrow a = \frac{1}{5 \times 1 - r^2} \]
\[ \Rightarrow 1 + r = \frac{1}{5} \]
\[ \Rightarrow r = -\frac{4}{5} \]

67. (3)
Given the sum of first \( n \) natural numbers \( = \frac{1}{78} \) (the sum of their cubes)
\[ \Rightarrow \frac{n(n+1)}{2} = \frac{1}{78} \times \frac{n^2(n+1)^2}{4} \]
\[ \Rightarrow 156 = n(n+1) \]
\[ \Rightarrow n^2 + n - 156 = 0 \]
\[ \Rightarrow (n + 13)(n - 12) = 0 \]
\[ \Rightarrow n = 12 \] (:: \( n \neq -13 \))

68. (2)
Total number of ways in which all letters of the word ‘GARDEN’ can be arranged = 6!
There are two vowels in the word ‘GARDEN’.
Total number of ways in which these two vowels can be arranged = 2!
∴ Total number of required ways = \( \frac{6!}{2!} = 360 \)

69. (1)
Total number of numbers without restriction = \( 2^5 \)
Two numbers have all the digits equal.
So, the required number of numbers = \( 2^5 - 2 = 30 \)

70. (3)
∴ Sum of coefficient of the expression
\[
\left( \frac{1}{x} + 2x \right)^n = 6561
\]
∴ \( (1 + 2)^n = 3^8 \)
⇒ \( 3^n = 3^8 \)
⇒ \( n = 8 \)
Let \( (r+1) \) th term is independent of \( x \).
∴ \( T_{r+1} = \binom{8}{r} \left( \frac{1}{x} \right)^r (2x)^{8-r} = \binom{8}{r} 2^{8-r} x^{8-2r} \)
Since, this term is independent of \( x \), then
\( 8 - 2r = 0 \)
⇒ \( r = 4 \)
∴ Coefficient of \( T_5 = \binom{8}{4} 2^4 \)
\[ = 16 \cdot \binom{8}{4} \]

71. (4)
\( T_{r+1} \) in \( \left( 2 + \frac{x}{3} \right)^n = \binom{n}{r} 2^{n-r} \left( \frac{x}{3} \right)^r = \binom{n}{r} \frac{2^{n-r}}{3^r} x^r \)
∴ Coefficient of \( x^r = \binom{n}{r} \frac{2^{n-r}}{3^r} \)
∴ \[ \binom{8}{r} \frac{2^{n-r}}{3^r} = \binom{8}{r} \frac{2^{n-8}}{3^8} \]
⇒ \[ \frac{2}{n-7} = \frac{1}{8 \times 3} \]
⇒ \( n - 7 = 48 \)
⇒ \( n = 55 \)

72. (1)
Let \( A \) and \( B \) are two given events.
∴ \( P(A) = \frac{2}{7}, \quad P(B) = \frac{6}{11} \)
∴ Required probability = \( 1 - P(\bar{A})P(\bar{B}) \)
73. (3) 
\[ P(A \cap B) \leq \min \{P(A), P(B)\} = \min \{0.65, 0.80\} = 0.65 \]
\[ \therefore P(A \cap B) \leq 0.65 \]
Also, 
\[ P(A \cap B) = P(A) + P(B) - P(A \cup B) \geq 0.65 + 0.80 - 1 = 0.45 \]
\[ \therefore 0.45 \leq P(A \cap B) \leq 0.65 \]

74. (4) 
Given equation is \((a + 2b)x + (a - 3b)y = a - b\)
It can be rewritten as 
\[ a(x + y - 1) + b(2x - 3y + 1) = 0 \]
This is the form of intersection of two lines.
\[ x + y - 1 = 0 \quad \text{and} \quad 2x - 3y + 1 = 0 \]
On solving, we get 
\[ x = \frac{2}{5} \quad \text{and} \quad y = \frac{3}{5} \]

75. (2) 
Let equation of line parallel to 
\[ 3x - y = 7 \] be \(3x - y = \lambda\)
It passes through \((1, 2)\).
\[ 3 - 2 = \lambda \quad \Rightarrow \lambda = 1 \]
\[ \therefore \] Line is \(3x - y = 1\).
The point of intersection of \(x + y + 5 = 0\) and \(3x - y = 1\) is \((-1, -4)\).
\[ \therefore \] Distance between \((1, 2)\) and \((-1, -4)\)
\[ = \sqrt{2^2 + 6^2} = \sqrt{40} \]

76. (4) 
Since, the lines \(x + 3y - 9 = 0\), \(4x + by - 2 = 0\) and \(2x - y - 4 = 0\) are concurrent, then
\[
\begin{vmatrix}
1 & 3 & -9 \\
4 & b & -2 \\
2 & -1 & -4 \\
\end{vmatrix} = 0
\]
\[ \Rightarrow 1(-4b - 2) - 3(-16 + 4) - 9(-4 - 2b) = 0 \]
\[ \Rightarrow -4b - 2 + 36 + 36 + 18b = 0 \]
\[ \Rightarrow 14b + 70 = 0 \]
\[ \Rightarrow b = -5 \]

77. (2) 
Equation \(x^2 + pxy + y^2 - 5x - 7y + 6 = 0\) will represent a pair of straight lines, if
\[
1 \cdot 1.6 + 2 \times \left( \frac{-7}{2} \right) \left( \frac{-5}{2} \right) \left( \frac{p}{2} \right) - 1 \cdot \left( \frac{-7}{2} \right)^2 - 1 \cdot \left( \frac{-5}{2} \right)^2 - 6 \left( \frac{p}{2} \right)^2 = 0
\]

\[
\Rightarrow 6 + \frac{35p}{4} - 49 - \frac{25}{4} - \frac{6p^2}{4} = 0
\]

\[
\Rightarrow 24 + 35p - 74 - 6p^2 = 0
\]

\[
\Rightarrow 6p^2 - 35p + 50 = 0
\]

\[
\Rightarrow (2p - 5)(3p - 10) = 0
\]

\[
\Rightarrow p = \frac{5}{2} \text{ or } \frac{10}{3}
\]

78. (3)

Since, \(D\) is the mid point of \(BC\).

So, coordinates of \(D\) are \(\left( \frac{x_1 + x_2}{2}, \frac{y_2 + y_3}{2} \right)\).

Given, \(G(7, 5)\) is the centroid of \(\Delta ABC\).

\[
\Rightarrow 7 = \frac{2 + x_1 + x_2}{3} \quad \text{and} \quad 5 = \frac{3 + y_2 + y_3}{3}
\]

\[
\Rightarrow x_2 + x_3 = 19 \quad \text{and} \quad y_2 + y_3 = 12
\]

\[
\Rightarrow \frac{x_2 + x_3}{2} = \frac{19}{2} \quad \text{and} \quad \frac{y_2 + y_3}{2} = 6
\]

\[
\therefore \text{ Coordinate of } D = \left( \frac{19}{2}, 6 \right).
\]

79. (2)

Let \(y = 4x - x^2\)

\[
\Rightarrow x^2 - 4x + y = 0
\]

\[
\Rightarrow x = \frac{4 \pm \sqrt{16 - 4y}}{2}
\]

\[
\Rightarrow x = 2 \pm \sqrt{4 - y}
\]

But, \(x \leq 2\), so \(x = 2 - \sqrt{4 - y}\)

\[
\therefore f^{-1}(x) = 2 - \sqrt{4 - x}
\]
80. (3) Let the point of the line divides the line in ratio \( m : 1 \).

\[ \therefore \text{Coordinate of point is } \frac{5m+1}{m+1}, \frac{7m-1}{m+1} \text{ lies on } y + x = 4 \]

\[ \Rightarrow \frac{7m-1+5m+1}{m+1} = 4 \]

\[ \Rightarrow \frac{12m}{m+1} = 4 \]

\[ \Rightarrow 12m = 4m + 4 \]

\[ \Rightarrow 8m = 4 \]

\[ \Rightarrow m = \frac{1}{2} \]

\[ \therefore \text{Required ratio is } 1 : 2. \]

81. (1) \[ (fog)(x) = f\left[ g(x) \right] = f(3x + 4) \]

Since, the domain of \( f \) is \([-3, 5]\)

\[ -3 \leq 3x + 4 \leq 5 \]

\[ \Rightarrow -5 \leq 3x + 4 \leq 5 \]

\[ \Rightarrow -9 \leq 3x \leq 1 \]

\[ \Rightarrow -3 \leq x \leq \frac{1}{3} \]

\[ \therefore \text{Domain of } fog \text{ is } [-3, \frac{1}{3}] \]

82. (4) The given function is \( f(x) = \sqrt{1-2x} + 2\sin^{-1}\left(\frac{3x-1}{2}\right) \)

For domain of \( \sqrt{1-2x}, 1-2x \geq 0 \)

\[ \Rightarrow 1 \geq 2x \Rightarrow x \leq \frac{1}{2} \]

\[ \Rightarrow x \in \left(-\infty, \frac{1}{2}\right) \] and for domain of \( 2\sin^{-1}\left(\frac{3x-1}{2}\right) \)

\[ -1 \leq \frac{3x-1}{2} \leq 1 \]

\[ \Rightarrow -2 \leq 3x-1 \leq 2 \]

\[ -2+1 \leq 3x \leq 2+1 \]

\[ \Rightarrow -1 \leq 3x \leq 3 \]

\[ \Rightarrow \frac{1}{3} \leq x \leq 1 \]

\[ \therefore \text{Domain of } f(x) = \left[-\frac{1}{3}, \frac{1}{2}\right] \]
83. (3)

We have, \( \left[ \frac{1}{2} + \frac{x}{100} \right] = \begin{cases} 0, & \text{if } 0 < x < 49 \\ 1, & \text{if } 50 \leq x \leq 99 \end{cases} \)

Thus, \( \left[ \frac{1}{2} \right] + \left[ \frac{1}{2} + \frac{1}{100} \right] + \left[ \frac{1}{2} + \frac{2}{100} \right] + \ldots + \left[ \frac{1}{2} + \frac{99}{100} \right] = 50 \)

84. (2)

We have, \( 2f(x) + f(1-x) = x^2 \) .... (i)

In Eq. (i) \( x \) is replaced by \( (1-x) \), we get \( 2f(1-x) + f(x) = (1-x)^2 \) .... (ii)

On solving Eqs. (i) and (iii) we get
\[
\Rightarrow 2f(1-x) \pm f(x) = -(1-x)^2
\]

\[
\Rightarrow 3f(x) = 2x^2 - (1-x)^2
\]

\[
\Rightarrow f(x) = \frac{1}{3} \{x^2 + 2x - 1\}
\]

\[
\Rightarrow f(4) = \frac{1}{3} \{16 + 8 - 1\} = \frac{23}{3}
\]

85. (4)

\[
y = \frac{x^2 + x + 2}{x^2 + x + 1}
\]

\[
yx^2 + yx + y = x^2 + x + 2
\]

\[
(y-1)x^2 + (y-1)x + y - 2 = 0
\]

Clearly, \( y \neq 1 \)

Since, \( x \) is real
\[
D > 0
\]

\[
(y-1)^2 - 4(y-1)(y-2) \geq 0
\]

\[
(y-1)[y-1 - 4y + 8] \geq 0
\]

\[
(y-1)(3y-7) \leq 0
\]

\[
y \in \left[ 1, \frac{7}{3} \right] \quad (\because y \neq 1)
\]

86. (3)

Let \( 2x + 3y = X \) and \( 2x - 7y = Y \)
\[
\Rightarrow x = \frac{7X + 3Y}{20}
\]

\[
\therefore f(X, Y) = \frac{20(7X + 3Y)}{20}
\]

\[
\Rightarrow f(X, Y) = 7X + 3Y
\]
87. (2) 
Here, \( f(x) = |\sin 2x| + |\cos 8x| \)
Now, \( |\sin 2x| = |\sin (2x + \pi)| \) \[ \because \sin (2x + \pi) = -\sin (2x) \]
But then \( |\sin (2x + \pi)| = |\sin 2x| = \left| \sin \left( 2 \left( x + \frac{\pi}{2} \right) \right) \right| \)
So, \( |\sin 2x| \) is periodic with fundamental period of \( \frac{\pi}{2} \).
Also, \( |\cos 8x| = |\cos (8x + \pi)| \) \[ \because \cos (8x + \pi) = -\cos (8x) \]
But then \( |\cos (8x + \pi)| = |\cos 8x| = \left| \cos \left( 8 \left( x + \frac{\pi}{8} \right) \right) \right| \)
So, \( |\cos 8x| \) is periodic with period \( \frac{\pi}{8} \).
So, the sum of \( |\sin 2x| \) and \( |\cos 8x| \) is periodic with fundamental period,
\[
\text{L.C.M.} \left( \frac{\pi}{2}, \frac{\pi}{8} \right) = \frac{\text{L.C.M.} \left( \pi, \pi \right) \times \text{H.C.F.} (2, 8)}{2} = \frac{\pi}{2}.
\]

88. (3) 
\[
\lim_{x \to 0} \sqrt{x^2 + 1} - \frac{1}{x^2 + 9} - 3 = \left[ \frac{0}{0} \right. \text{ Form} \]
Applying L’ Hospital rule
\[
\lim_{x \to 0} \frac{2x}{\sqrt{x^2 + 1} - 3} = \lim_{x \to 0} \frac{2x}{\sqrt{x^2 + 1}} = \frac{3}{3} = 1.
\]

89. (3) 
\[
\lim_{x \to 3} \frac{x^2 + 3x^2 - x - 3}{x^2 + x - 6} = \lim_{x \to 3} \frac{(x^2 - 1)(x + 3)}{(x + 3)(x - 2)} = \lim_{x \to 3} \frac{x^2 - 1}{x - 2} = \frac{8}{5}
\]

90. (3) 
\[
\text{LHL} = \lim_{x \to 0^-} \left[ \frac{x^2}{x^2} \right] = 0
\]
\[
\text{RHL} = \lim_{x \to 0^+} \left[ \frac{x^2}{x^2} \right] = 0
\]
\[
\therefore \lim_{x \to 0} \left[ \frac{x^2}{x} \right] = 0
\]