1. If the block is released from a position when spring is in its natural length. Then, which of the following is correct. (Use $g = 10 \text{ m/s}^2$ and no air resistance or friction. Pulley is massless)

\[ K = 100 \text{ N/m} \]

\[ m = 1 \text{ kg} \]

(A) At equilibrium, spring is elongated by 0.1 m
(B) Maximum elongation in the spring is 0.2 m
(C) The block will perform SHM with an amplitude of 0.1 m
(D) None of the above

1. (ABC)
Elongation:

\[ KX_{eq} = mg \]

(A) \[ X_{eq} = 0.1 \text{ m} \] \hspace{0.5cm} ....(1)

(B) \[ X_{max} = \frac{2mg}{K} \] \Rightarrow \left[ \frac{KX_{max}^2}{2} = mgX_{max} \right] \text{ mechanical energy conservation.}

\[ X_{eq} + X = \frac{2mg}{K} \]
\[ X = \frac{mg}{K} = 0.1 \text{m} \quad \text{.....(II)} \]

(C) Also block will be performing SHM another amplitude of 0.1 m and time period of \( = \frac{2\pi}{\sqrt{\frac{m}{K}}} \) \( = \frac{\pi}{5} \) seconds.

2. A ring of mass ‘m’ and radius ‘R’ is released on a rough horizontal ground with a linear velocity \( v_0 \). The coefficient of friction between the ring and the ground is \( \mu \). Then, which of the following is correct.

(A) The disc will skid initially on the ground
(B) Kinetic friction will act on the ring in backward direction
(C) Kinetic friction will stop acting after sometime
(D) The ring will develop an anticlockwise angular velocity about its center of mass

2. (ABC)

When pure rolling starts \( v' = \frac{v_0}{2} \) since \( L \) constant about ‘O’

\[ \rightarrow \] The ring will be sliding initially towards right, hence kinetic friction will be acting towards left.

\[ \rightarrow \] Due to the torque of kinetic friction about center, there will be angular accelerator (\( \alpha \)) in clockwise sense, hence \( \omega \) will be clockwise sense.

\[ \rightarrow \] Once the pure rolling starts, kinetic friction will stop acting.

3. Three rods \( AB \), \( BC \) and \( AC \) having thermal resistance of 10 units, 10 units and 20 units respectively are connected as shown in the figure. Ends \( A \) and \( C \) are maintained at constant temperature of \( 100^\circ \text{C} \) and \( 0^\circ \text{C} \), respectively. Which of the following is correct?

(A) Rate at which heat is crossing junction \( B \) is 5 units
(B) Rate of heat flow in rod \( AC \) is same as rate of heat flow through \( B \)
(C) Rate of heat flow through \( B \) is 10 units
(D) None of the above

CENTERS: MUMBAI / DELHI / AKOLA / LUCKNOW / NASHIK / PUNE / NAGPUR / BOKARO / DUBAI #2
3. \[ \frac{1}{R_{eq}} = \frac{1}{R_{AC}} + \frac{1}{R_{(AB+BC)}} \]

\[ R_{eq} = 10 \text{ units} \Rightarrow \Delta i_{\text{total}} = \frac{\Delta T}{R_{eq}} = \frac{100}{10} = 10 \text{ units} \]

\[ i_{\text{Heat through (AB+BC)}} = i_{\text{Heat through AC}} \]

Heat current will flow equally through \((AB + BC)\) & \(AC\).

4. In the given figure a ring of mass \(M\) is kept on a horizontal surface while a body of equal mass \('M'\) attached through an ideal string wound on the ring. When the system is released the ring rolls without slipping. Mark the correct options.

(A) Acceleration of the COM of ring is \(g/3\)
(B) Acceleration of the COM of ring \(2g/3\)
(C) Frictional force (on the ring) acts along the forward direction
(D) Frictional force (on the ring) acts along the backward direction

4. \(T(2R) = 2MR^2\alpha\)
\[ a = R\alpha \]
\[ Mg - T = 2Ma \]
\[ a = \frac{g}{3} \]

Friction force for the ring to pure roll is zero if the external force is applied on the top of the ring tangentially.

5. Which of the following statement is correct for a spherical body rolling without slipping on a rough horizontal ground at rest?

(A) The acceleration of a point in contact with ground is zero
(B) The acceleration of COM of the sphere may or may not be zero
(C) Friction force on sphere may or may not be zero
(D) Work done by friction w.r.t. ground on the body is zero

5. \((BCD)\)
\[ \ddot{a}_{pg} = \ddot{a}_{Pc} + \ddot{a}_{cg} \]
\[ = \left( \ddot{a}_{P_{\text{radial}}} + \ddot{a}_{P_{\text{tangential}}} \right) + \ddot{a}_{cg} \]
\[ = \ddot{a}_{P_{\text{radial}}} \quad \left[ \because a = Ra \right] \]
\[ = \omega^2 R \text{ towards the center} \]

(A) is correct.

6. The end B of the rod AB which makes angle ‘\( \theta \)’ with the smooth floor is being pulled with, a constant velocity \( v_0 \) as shown. The length of the rod is ‘\( \ell \)’. At the given instant when \( \theta = 37^\circ \), then mark the correct statement(s)

(A) Velocity of end A is \( \frac{5v_0}{3} \) downwards.
(B) Angular velocity of rod is \( \frac{5v_0}{3\ell} \)
(C) Angular velocity of rod will vary with time
(D) Velocity of end A will vary with time

6. (BCD)

In a rigid body, \( V_A \sin \theta = V_0 \cos \theta \)
\[ V_A = V_0 \cot \theta \]
\[ V_A = \frac{4V_0}{3} \]
\[ \omega = \frac{V_{AB_{\perp}}}{\ell} = \frac{V_A \cos \theta + V_0 \sin \theta}{\ell} = \frac{5V_0}{3\ell} \]
\[ \cdots \cdots \text{(B)} \]
\[ \rightarrow \text{Angular velocity will change since } \theta \text{ changes with time.} \quad \cdots \cdots \text{(D)} \]

7. The figure shows a process AB undergone by 2 moles of an ideal diatomic gas. The process AB is an such a way. That \( VT = \text{constant} \), \( T_1 = 300 \text{ K} \) and \( T_2 = 500 \text{ K} \). \( (R = \text{gas constant}) \). Mark the correct statements.
The molar heat capacity of gas in the process $A \rightarrow B$ is $\frac{5R}{2}$ J/mol-K

The molar heat capacity of gas in the process $A \rightarrow B$ is $\frac{3R}{2}$ J/mol-K

The work done by the gas is $-400R$ J

The work done by the gas is $-200R$ J

Velocity of a particle moving in a curvilinear path in a horizontal $x$-$y$ plane varies with time as $\vec{v} = (2t\hat{i} + t^2\hat{j})$ m/s. Here $t$ is in second. Which of the following statements are correct?

(A) Acceleration of the particle at $t = 1$ s is $2\sqrt{2}$ m/s$^2$

(B) If particle starts from origin, its equation of trajectory is $y = \frac{x^{3/2}}{3}$

(C) Velocity at $t = 1$ is $\sqrt{3}$ m/s

(D) Velocity at $t = 2$ is 8 m/s

Velocity $\vec{v} = 2t\hat{i} + t^2\hat{j}$

Acceleration $\vec{a} = 2\hat{i} + 2t\hat{j}$

At $t = 1$

$\vec{a} = 2\hat{i} + 2\hat{j}$

$|\vec{a}|_{t=1} = 2\sqrt{2}$ m/s$^2$

$\frac{dx}{dt} = 2t, \quad \frac{dy}{dt} = t^2$

$x = t^2, \quad y = \frac{t^3}{3}$

$y = \frac{x^{3/2}}{3}$
This section contains 08 questions. The answer to each question is a NUMERICAL VALUE. For each question, enter the correct numerical value (in decimal notation, truncated/rounded-off to the second decimal place; e.g. 6.25, 7.00, 0.33, 30.27, 127.30)

9. The time period of small oscillations of the system shown below is given by $\frac{n\pi}{10}$ seconds. Find the value of $n$ given that the mass of the block $(m)$ is 1 kg and the spring constant ‘$k$’ is 100 N/m. The spring, string and the pulley used are ideal.

![Diagram of a spring-mass system]

$$T = 2\pi \sqrt{\frac{m}{K_{eq}}} = 2\pi \sqrt{\frac{1}{25}} = \frac{2\pi}{5} \text{ sec.}$$

10. If $F = 27$ N is applied on a thin plank kept on the top of a solid sphere of same mass as that of plank which is 1 kg. The minimum coefficient of friction between the plank and the sphere such that there is no slipping anywhere is given by 0.35$x$. Find the value of ‘$x$’. Use (g = 10 m/s$^2$)

![Diagram of a plank on a sphere]

10. (2.00)
For plank,
\[ F - f = M(2a) \]

For sphere,
\[ \frac{10f}{7} = Ma \]
\[ \frac{10F}{27M} = a \]
\[ a = 10 \text{ m/s}^2 \]
\[ f = 7 \text{ N} \]
\[ \mu_{\text{min}}Mg = f \]
\[ \mu_{\text{min}} = \frac{7}{10} = 0.7 \]

11. Two particles performing SHM along the same straight line without colliding with same equilibrium position \( x = 0 \), Amplitude \( A \) and time period \( T \), but their initial states of motion are as follows.

One is moving towards left extreme and is at \( x = \frac{+\sqrt{3}A}{2} \), another is at \( x = \frac{-\sqrt{3}A}{2} \) and is moving towards right extreme. Minimum time interval from start when they will cross each other is given by \( T(n) \). Find the value of ‘n’.

11. \( (6.00) \)

Angle rotated by both phasor is \( \frac{\pi}{3} \)
\[ \frac{\pi}{3} = \omega t \]
\[ \frac{\pi}{3} = \frac{2\pi}{T}t \]
\[ t = \frac{T}{6} \]
12. The springs are ideal and identical having spring constant $K = 100 \text{ N/m}$. The block has a mass of ‘$m$’ kg and the system is performing SHM with an angular frequency of 10 rad/s. Find the value of ‘$m$’.

![Diagram of springs and block]

12. **(3.00)**

\[
\omega = \sqrt{\frac{3K}{m}}
\]

\[
10 = \sqrt{\frac{300}{m}}
\]

\[
m = 3 \text{ kg}
\]

13. A sphere performing pure rolling on ground is shown in the figure. $V_{\text{COM}} = 4 \text{ m/s}$ and the radius of the sphere is 1 m. The distance travelled by the centre of mass in the time interval the sphere rotates by $4\pi$ radians is given by $4n$ meters. Find ‘$n$’.

![Diagram of sphere]

13. **(3.14)**

In two full rotations about COM, COM will move $4\pi R$ distance.
14. A rod of mass 1 kg and length 1 m is hinged at one of its end. The acceleration of the point $P$ when the rod becomes vertical is given by $4n$. Find the value of ‘$n$’. (Use: $g = 10 \text{ m/s}^2$)

\[ \begin{align*}
\text{Horizontal} \\
\quad P
\end{align*} \]

14. (7.50)

\[ 
Mg \frac{\ell}{2} = \frac{I \omega^2}{2} \\
\sqrt{\frac{3g}{\ell}} = \omega \\
a_p = \omega^2 \ell \text{ towards hinge.} \\
= \frac{3g}{\ell} \ell \\
= 30
\]

15. A thin uniform rod $AB$ of mass $m = 1$ kg moves translationally with acceleration $a = 2 \text{ m/s}^2$ due to two anti-parallel forces $F_1$ and $F_2$. The distance between the points at which these forces are applied is equal to $\ell = 20 \text{ cm}$ and $F_2 = 5\text{ N}$. The length of the rod is given by $4k$ meters. Find ‘$k$’.

\[ \begin{align*}
\text{F}_1 & \quad A \\
\quad \ell \\
\quad B \\
\text{F}_2
\end{align*} \]

15. (0.25)

\[ 
F_2 - F_1 = Ma \quad \ldots (1) \\
F_1 (\ell + x) = F_2 x \quad \ldots (2) \\
x = 30 \text{ cm} \\
\text{Length of rod} = 100 \text{ cm} \]
16. The moment of inertia of a triangular plate shown about its base is given by $I = \frac{6n}{25}$ Kg-m². Find ‘n’.

[Given: Mass of the plate = 1 kg]

\[ I = \frac{Mh^2}{6} = \frac{(1\text{ Kg})(\frac{12}{5}\text{ m})^2}{6} = \frac{24}{25} \text{ Kg-m}^2 \]

**SECTION-III : (MATRIX-MATCH TYPE)**

This section contains 02 Matrix Match. Each question has contains two columns, **Column - I & Column - II**. Match the entries in **Column - I** with the entries in **Column - II**. One or more entries in **Column - I** may match with one or more entries in **Column - II**.

17. A simple harmonic oscillator consists of a block attached to a spring with $k = 200$ N/m. The block slides on a frictionless horizontal surface, with equilibrium point $x = 0$. A graph of the block’s velocity $v$ as a function of time $t$ is shown. Correctly match the required information in the column-I with the values given in the column-II. (Use $\pi^2 = 10$)

\[ V_{\text{max}} = A\omega \]

\[ A = \frac{V_{\text{max}}}{\omega} = \frac{2\pi}{2\pi} \times (0.2) = 0.20 \text{ m} \]

\[ T = 2\pi\sqrt{\frac{m}{k}} \Rightarrow m = \frac{T^2k}{4\pi^2} = 0.2 \text{ kg} \]

**Column – I**

<table>
<thead>
<tr>
<th>Column – I</th>
<th>Column – II</th>
</tr>
</thead>
<tbody>
<tr>
<td>(A) The block’s mass in kg</td>
<td>(P) – 0.20</td>
</tr>
<tr>
<td>(B) The block’s displacement at t = 0 in metres</td>
<td>(Q) 0.0</td>
</tr>
<tr>
<td>(C) The block’s acceleration at t = 0.10 s in m/s²</td>
<td>(R) 0.20</td>
</tr>
<tr>
<td>(D) The block’s maximum kinetic energy in Joule</td>
<td>(S) 4.0</td>
</tr>
</tbody>
</table>

17. (A) $\rightarrow$ R;  (B) $\rightarrow$ P;  (C) $\rightarrow$ T;  (D) $\rightarrow$ S
At \( t = 0.1 \), acceleration is maximum

\[
\Rightarrow a_{\text{max}} = -\omega^2 A = -\left(\frac{2\pi}{0.2}\right)^2 \times 0.2
\]

\[
= -200 \text{ m/s}^2
\]

Maximum energy \( \frac{1}{2}mV_{\text{max}}^2 = 4 \text{ J} \)

18. In each situation of column-I, the \( x \)-coordinates of a particle moving along \( x \)-axis is given in terms of time \( t \). (\( \omega \) is a positive constant). Match the equation of motion given in column-I with the type of motion given in column-II.

<table>
<thead>
<tr>
<th>Column– I</th>
<th>Column – II</th>
</tr>
</thead>
<tbody>
<tr>
<td>(A) ( \sin \omega t - \cos \omega t )</td>
<td>(P) SHM</td>
</tr>
<tr>
<td>(B) ( \sin^3 \omega t )</td>
<td>(Q) Periodic</td>
</tr>
<tr>
<td>(C) ( \sin \omega t + \sin 3\omega t + \sin 5\omega t )</td>
<td>(R) Periodic but not SHM</td>
</tr>
<tr>
<td>(D) ( \exp(-\omega^2 t^2) )</td>
<td>(S) Non periodic</td>
</tr>
<tr>
<td></td>
<td>(T) SHM but not periodic</td>
</tr>
</tbody>
</table>

18. (A) \( \rightarrow P, Q \); (B) \( \rightarrow Q, R \); (C) \( \rightarrow Q, R \); (D) \( \rightarrow S \)

(A) \( x = \sqrt{2} \left( \frac{1}{\sqrt{2}} \sin \omega t - \frac{1}{\sqrt{2}} \cos \omega t \right) \Rightarrow x = \sqrt{2} \left( \omega t - \frac{\pi}{4} \right) \) is periodic with SHM.

(B) \( x = \sin^3 \omega t \) can not be written

As \( x = A \sin (\omega t + \phi) \) so it is not SHM but periodic motion.

(C) Linear combination of different periodic function is also periodic function.

\[
\frac{d^2x}{dt^2} \text{ is not directly proportional to } x \text{ i.e. this motion is not SHM}
\]

(D) \( x \) continuously decreases with time. So, \( x \) is not periodic function.
PART (B) : CHEMISTRY

SECTION-I : (MULTIPLE CORRECT ANSWER(S) TYPE)

This section contains **08 multiple choice questions.** Each question has four choices (A), (B), (C) and (D) out of which **ONE or MORE than one** is/are correct.

19. Which of the following will change their period if each orbital can contain maximum of 3 electrons?
   (A) Na   (B) C   (C) Cl   (D) K
   (AD)

   Period
   (A) Na
   \[1s^2 2s^2 2p^6 \quad 3s^1\]  \[\text{Change}\]  \[2\]
   \[1s^2 2s^3 2p^5\]  \[\text{No change}\]  \[2\]
   (B) C
   \[1s^2 2s^2 2p^2\]  \[\text{No change}\]  \[2\]
   \[1s^2 2s^3\]  \[\text{No change}\]  \[2\]
   (C) Cl
   \[1s^2 2s^2 2p^6 \quad 3s^2 \quad 3p^5\]  \[\text{No change}\]  \[3\]
   \[1s^2 2s^2 \quad 2p^6 \quad 3p^5\]  \[\text{Change}\]  \[3\]
   (D) K
   \[1s^2 2s^2 2p^6 \quad 3s^2 \quad 3p^6 \quad 4s^1\]  \[\text{Change}\]  \[3\]

20. Which of the following statement is / are correct?
   (A) All amphiprotic species are amphoteric in nature
   (B) Conjugate of weak acid is a strong base
   (C) Formation of H\(_2\)O from its ions is exothermic in nature
   (D) H\(_3\)PO\(_2\) is dibasic acid
   (AC)

   (A) True statement
   (B) Conjugate of weak acid is a weak base
   (C) H\(^+\) + OH\(^-\) \rightarrow H\(_2\)O exothermic
   (D) H\(_3\)PO\(_2\) (monobaric acid)
   H\(_3\)BO\(_3\) (monobaric)

21. Identify the correct statement(s).
   (A) H\(_3\)PO\(_3\) > H\(_3\)PO\(_2\) Acidic strength
   (B) H\(_3\)PO\(_4\) > H\(_3\)PO\(_3\) Oxidation state of P
   (C) H\(_3\)PO\(_2\) > H\(_3\)BO\(_3\) Basicity
   (D) HNO\(_2\) > HNO\(_3\) Oxidation state of N
   (B)

   (B) H\(_3\)PO\(_4\)\(^{1+}\) > H\(_3\)PO\(_2\)\(^{1+}\) oxidation state.
   (C) H\(_3\)PO\(_2\) Basicity = 1
   \[\text{H}_3\text{PO}_3\text{ Basicity} = 2\]
   (D) HNO\(_2\)\(^{1-}\) < HNO\(_3\)\(^{5-}\) oxidation state
22. Which of the following thermodynamic parameter are **INCORRECT** when adiabatic free expansion of an ideal gas occurs?

(A) $q = 0$  
(B) $\Delta S_{\text{sys}} = 0$  
(C) $\Delta G = 0$  
(D) $\Delta H = 0$

22. (BC)

$$\Delta S = nRT \ln \frac{V_2}{V_1} \neq 0$$

$\Delta G < 0$ as the process happens spontaneously.

23. Which of the following have correct IUPAC name?

(A) CH$_3$-CH$_2$-CH$_2$-COOC$_2$H$_5$ (Ethyl butanoate)
(B) CH$_3$-CH(CH$_3$)-CH$_2$-CHO (3-methyl butanal)
(C) CH$_3$-CH(OH)-CH(CH$_3$)$_2$ (2-methyl-3-butanol)
(D) (CH$_3$)$_2$CH-CO-CH$_2$CH$_3$ (2-methyl-3-pentanone)

23. (ABD)

24. Tautomerism is exhibited by

(A) ![Image of compound A](image)
(B) ![Image of compound B](image)
(C) ![Image of compound C](image)
(D) ![Image of compound D](image)

24. (ACD)

25. Identify reaction which not gives aromatic products?

(A) ![Image of reaction A](image)  
(B) ![Image of reaction B](image)  
(C) ![Image of reaction C](image)  
(D) ![Image of reaction D](image)

25. (BC)
26. Which of the following gives effervescence of CO\(_2\) when treated with NaHCO\(_3\)?

(A) \[
\begin{array}{c}
\text{COOH} \\
\text{C} \\
\text{H}_3
\end{array}
\]

(B) \[
\begin{array}{c}
\text{COOH} \\
\text{CH}_3
\end{array}
\]

(C) \[
\begin{array}{c}
\text{SO}_3\text{H} \\
\text{CH}_3
\end{array}
\]

(D) \[
\begin{array}{c}
\text{HO} \\
\text{C} \\
\text{H}_3
\end{array}
\]

26. (ABC)

**SECTION-II : (INTEGER ANSWER TYPE)**

This section contains 08 questions. The answer to each question is a NUMERICAL VALUE. For each question, enter the correct numerical value (in decimal notation, truncated/rounded-off to the second decimal place; e.g. 6.25, 7.00, 0.33, 30.27, 127.30)

27. The % change in degree of dissociation of 0.1 M HCOOH (Ka = 10\(^{-4}\)) when 0.1 M HNO\(_3\) is added into it is \(x\). \(x\) will be:

27. \((96.8)\)

\[
\alpha_1 = \sqrt{\frac{10^{-4}}{0.1}} = \sqrt{10^{-3}} = 10^{-2} \times 3.1
\]

\[
\alpha_2 = \frac{10^{-4}}{0.1} = 10^{-3}
\]

28. 300 ml solution A having pH = 2 is mixed with 200 ml solution B having pH = 3. The final pH of the mixture will be (Assume both solution are of strong acids) [\log 5 = 0.7, \log 2 = 0.3]

28. \((2.20)\)

\[\left[H^+\right] = 10^{-2}, \quad \left[H^+\right] = 10^{-3}\]

\[
\left[H^+\right] = \frac{300 \times 10^{-2} + 200 \times 10^{-3}}{500}
\]

\[
= \frac{3 \times 10^{-2} + 2 \times 10^{-3}}{5}
\]

\[
= \frac{0.03 + 0.002}{5} = \frac{0.032}{5}
\]

\[
pH = -\log \frac{0.032}{5} = -\log \frac{32}{5000}
\]

\[= 2.2\]

29. \(x\) mole of FeSO\(_4\) is required for complete reaction with 1 mole of K\(_2\)Cr\(_2\)O\(_7\) in acidic medium. Find the value of \(x\).

29. \((6.00)\)

Equivalents of FeSO\(_4\) = Equivalents of K\(_2\)Cr\(_2\)O\(_7\)

\[x \times 1 = 1 \times 6\]
30. The equivalent weight of KMnO₄ (GMM = 158) in acidic medium is \( x \). Find \( x \).

30. \((31.6)\)

\[ \text{MnO}_4^- \rightarrow \text{Mn}^{2+} \]

\( n \)-factor = 5

Equivalent weight = \( \frac{158}{5} = 31.6 \)

31. How many carbanions are most stable than \( \text{(CH}_3)_3\overset{\cdot}{\text{C}} \)?

(i) \( \text{CCl}_3 \)

(ii) \( \text{CF}_3 \)

(iii) \( \text{(CH}_3)_3\overset{\cdot}{\text{C}} \)

(iv) \( \text{CH}_3 \)

(v) \( \text{CH}_2\text{NO}_2 \)

(vi) \( \text{PhC} \)

(vii) \( \text{(Et)}_3\overset{\cdot}{\text{C}} \)

31. \((5.00)\)

32. In how many pairs of following compounds I\(^{st}\) is more stable than II\(^{nd}\) compound.

(i)

(ii)

(iii)

(iv)

(v)

(vi)

(vii)

32. \((7.00)\)

33. How many of following compounds are more basic than o-toluidine?

(i)

(ii)

(iii)

(iv)
33. (6.00)

34. How many hyperconjugative structure possible for compound tertiary butyl benzene is?
34. (0.00)

SECTION-III : (MATRIX-MATCH TYPE)

This section contains 02 Matrix Match. Each question has two columns, Column - I & Column - II. Match the entries in Column - I with the entries in Column - II. One or more entries in Column - I may match with one or more entries in Column - II.

35. Match the column:

<table>
<thead>
<tr>
<th>Column – I (Species)</th>
<th>Column – II (Maximum n-factor for redox reaction)</th>
</tr>
</thead>
<tbody>
<tr>
<td>(A) $H_2C_2O_4$</td>
<td>(P) 8</td>
</tr>
<tr>
<td>(B) $H_2C_2O_4 \cdot KHC_2O_4$</td>
<td>(Q) 5</td>
</tr>
<tr>
<td>(C) $KHC_2O_4 \cdot 2H_2C_2O_4$</td>
<td>(R) 6</td>
</tr>
<tr>
<td>(D) $H_2C_2O_4 \cdot 3KHC_2O_4$</td>
<td>(S) 2</td>
</tr>
<tr>
<td></td>
<td>(T) 4</td>
</tr>
</tbody>
</table>

35. (A) $\rightarrow$ S; (B) $\rightarrow$ T; (C) $\rightarrow$ R; (D) $\rightarrow$ P

Redox

(A) $H_2C_2O_4$ 2
(B) $H_2C_2O_4 \cdot KHC_2O_4$ 4
(C) $KHC_2O_4 \cdot 2H_2C_2O_4$ 6
(D) $H_2C_2O_4 \cdot 3KHC_2O_4$ 8
36. Match the column.

<table>
<thead>
<tr>
<th>Column – I (Compounds)</th>
<th>Column – II (Property)</th>
</tr>
</thead>
<tbody>
<tr>
<td>(A) CH≡CH</td>
<td>(P) Compound release CO₂ gas with NaHCO₃</td>
</tr>
<tr>
<td>(B) <img src="https://example.com/structure.png" alt="Structure" /></td>
<td>(Q) Compound release H₂ gas with Na metal</td>
</tr>
<tr>
<td>(C) CH₃-COOH</td>
<td>(R) Compound soluble in dil. NaOH</td>
</tr>
<tr>
<td>(D) CH₃-CH₂-OH</td>
<td>(S) Most acidic compound</td>
</tr>
<tr>
<td></td>
<td>(T) More acidic compound than H₂CO₃</td>
</tr>
</tbody>
</table>

36. (A) → Q; (B) → P, Q, R, S, T; (C) → P, Q, R, T; (D) → Q
This section contains **08 multiple choice questions**. Each question has four choices (A), (B), (C) and (D) out of which **ONE OR MORE than one is/are correct**.

### 37. The number of ways in which 20 identical coins can be distributed in 4 persons if each person receives at least 2 coins and at most 5 coins, is

(A) \(15 \binom{12}{4} \cdot 11 \binom{8}{6} + 6 \cdot 7 \binom{4}{2} - 4\)  
(B) \(15 \binom{3}{3} - 6 \cdot 11 \binom{8}{8} + 7 \binom{4}{4} - 4\)  
(C) \(23 \binom{3}{3}\)  
(D) \(15 \binom{0}{0}\)

Number of ways = coefficient of \(x^{20}\) in 
\[
\left( x^2 + x^3 + x^4 + x^5 \right)^4 = x^8 \left( 1 + x + x^2 + x^3 \right)^4
\]

= coefficient of \(x^{12}\) in \(\left( \frac{1 - x^4}{1 - x} \right)^4 = (1 - x^4)(1 - x)^{-4}\)

= \(15 \binom{12}{4} \cdot 11 \binom{8}{6} + 6 \cdot 7 \binom{4}{2} - 4 = 1 = \binom{12}{0}\)

### 38. The equation \(\left( \frac{\sqrt{2}}{2} \right)^{\frac{4x-3}{3x-4}} = \left( \frac{\sqrt{2}}{2} \right)^{\frac{4x-3}{3x-4}}\) has a solution which is

(A) an even integer  
(B) a prime number  
(C) coprime with 5  
(D) none of these

\(\left( \frac{\sqrt{2}}{2} \right)^{\frac{4x-3}{3x-4}} = \left( \frac{\sqrt{2}}{2} \right)^{\frac{4x-3}{3x-4}}\)

\(\Rightarrow 8x - 6 = 9x - 12\)

\(\Rightarrow x = 6\)

### 39. If distinct positive real number \(a, b, c\) are in H.P., then which of the following is true? (Assume that everything is defined)

(A) \(\frac{a}{1 - 2a}, \frac{b}{1 - 2b}, \frac{c}{1 - 2c}\) are in H.P.  
(B) \(\ln \left( \frac{a - b}{2} \right), \ln \frac{b}{2}, \ln \left( \frac{c - b}{2} \right)\) are in H.P.  
(C) \(\frac{c - b}{2}, \frac{b - a}{2}\) are in A.P.  
(D) \(e^{\frac{a}{2}}, e^{\frac{b}{2}}, e^{\frac{c}{2}}\) are in G.P.

\(\frac{a}{1 - 2a}, \frac{b}{1 - 2b}, \frac{c}{1 - 2c}\) are in H.P.  
\(\frac{1, 1}{a, b, c}\) are in A.P.  
\(\frac{a - b}{2}, \frac{b - a}{2}\) are in A.P.  
\(\ln \left( \frac{a - b}{2} \right), \ln \frac{b}{2}, \ln \left( \frac{c - b}{2} \right)\) are in A.P.  
\(\frac{c - b}{2} \left( \frac{a - b}{2} \right) = \frac{ac - b(a + c) + b^2}{4} = \frac{b^2}{4}\)
\[ c - \frac{b}{2} \cdot \frac{b}{2}, \quad a - \frac{b}{2} \] are in G.P.

\[ \text{(D) } \frac{1}{a}, \frac{1}{b}, \frac{1}{c} \text{ are in A.P.} \quad \Rightarrow \quad e^\frac{1}{a}, e^\frac{1}{b}, e^\frac{1}{c} \text{ are in G.P.} \]

40. Let \((1 + x^2)^2 (1 + x)^n = A_0 + A_1 x + A_2 x^2 + \ldots\). If \(A_1, A_2\) and \(A_3\) are in A.P., then the value of \(n\) is (are)

\[ \text{(A) } 2 \quad \text{(B) } 3 \quad \text{(C) } 5 \quad \text{(D) } 7 \]

40. (AB)

\[ (1 + x^2)^2 (1 + x)^n = (1 + 2x^2 + x^4)(a_{C_0} + a_{C_1}x + a_{C_2}x^2 + \ldots) \]

\[ = a_{C_0} + a_{C_1}x + \left( a_{C_2} + 2 \cdot a_{C_0} \right)x^2 + \ldots \]

\[ \therefore \quad A_0 = a_{C_0}, \quad A_1 = a_{C_1}, \quad A_2 = a_{C_2} + 2 \text{ which are in A.P.} \]

\[ \Rightarrow \quad 2A_1 = A_0 + A_2 \quad \Rightarrow \quad 2a_{C_1} = 1 + a_{C_2} + 2 \]

\[ \Rightarrow \quad 2n = 3 + \frac{n(n-1)}{2} \quad \Rightarrow \quad n^2 - 5n + 6 = 0 \]

\[ \therefore \quad n = 2, 3 \]

41. There is a bag containing three balls. Two balls are drawn from the bag and are found to be white. If probability that bag contains all white balls is rational \( \frac{p}{q} \) in lowest form, then \( p + q \) is not equal

\[ \text{(A) } 7 \quad \text{(B) } 3 \quad \text{(C) } 4 \quad \text{(D) } 5 \]

41. (BCD)

Let us consider one bag out of \( n \) bags

\( E_1 \) : event of two white balls
\( E_2 \) : event of three white balls
\( W \) : event of drawing two balls, found to be white.

\[ P(E_1) = P(E_2) = \frac{1}{2}, \quad P \left( \frac{W}{E_1} \right) = \frac{2C_2}{3C_2} = \frac{1}{3} \]

\[ P \left( \frac{W}{E_2} \right) = 1, \quad P \left( \frac{E_2}{W} \right) = \frac{1}{2} \cdot 1 = \frac{3}{4} \]

\[ \therefore \quad \frac{p}{q} = \frac{3}{4} \]

\[ \Rightarrow \quad p + q = 7 \]

42. The diagonals of the parallelogram whose sides are \( \ell x + my + n = 0, \ell x + my + n' = 0, mx + \ell y + n = 0, mx + \ell y + n' = 0 \) cannot be inclined at an angle equal to (Among the given options)

\[ \text{(A) } \frac{\pi}{3} \quad \text{(B) } \frac{\pi}{2} \quad \text{(C) } \tan^{-1} \left( \frac{\ell^2 - m^2}{\ell^2 + m^2} \right) \quad \text{(D) } \tan^{-1} \left( \frac{2\ell m}{\ell^2 + m^2} \right) \]

42. (ACD)

Since,

\[ \left| \frac{n - n'}{\sqrt{\ell^2 + m^2}} \right| = \left| \frac{n - n'}{\sqrt{m^2 + \ell^2}} \right| \]
The given parallelogram is rhombus

\[ \therefore \text{ Angle between diagonals is always } \frac{\pi}{2} \]

43. Let \( f(x) = [x]^2 + [x+1] - 3 \) (where \([\cdot]\) denotes G.I.F.) then and \( f: R \rightarrow R \)
(A) many-one (B) one-one (C) onto (D) into

43. (AD)
Let \( f(x) = [x]^2 + [x+1] - 3 \)
\[ = [x]^2 + [x] - 2 = ([x] + 2)([x] - 1) \]
Clearly \( f(x) = 0 \)
\[ [x] = -2 \text{ or } [x] = 1 \]
\[ \Rightarrow f \text{ is many-one and range contains only integers } \Rightarrow \text{ range } \neq \text{ co-domain } \Rightarrow f \text{ is into} \]

44. The equations of bisectors of two lines \( L_1 \) and \( L_2 \) are \( 2x - 16y - 5 = 0 \) and \( 64x + 8y + 35 = 0 \). If the line \( L_1 \) passes through the point \((-11, 4)\), then
(A) The equation of acute angle bisector is \( 2x - 16y - 5 = 0 \)
(B) The equation of acute angle bisector is \( 64x + 8y + 35 = 0 \)
(C) The equation of obtuse angle bisector is \( 2x - 16y - 5 = 0 \)
(D) The equation of obtuse angle bisector is \( 64x + 8y + 35 = 0 \)

44. (AD)
\[ P_1 = \frac{-22 - 64 - 5}{\sqrt{256 + 4}} = \frac{91}{\sqrt{260}} \]
\[ P_2 = \frac{-11 \times 64 + 32 + 35}{8\sqrt{8^2 + 1}} = \frac{637}{8\sqrt{65}} \]
\[ P_1 < P_2 \]
SECTION-II : (INTEGER ANSWER TYPE)

This section contains 08 questions. The answer to each question is a NUMERICAL VALUE. For each question, enter the correct numerical value (in decimal notation, truncated/rounded-off to the second decimal place; e.g. 6.25, 7.00, 0.33, 30.27, 127.30)

45. For a game in which every possible pair must play with every other possible pair, \(2m \) men are available. If the number of games that were played is 630, then find the value of \(m\).

45. (5.00)

With each selection of 4 men, say A, B, C & D 3 games can be played (AB, CD; AC, BD; AD, BC)

\[ \therefore \ 2m \cdot 3 = 630 \]

\[ \therefore \ m = 5 \]

46. Sum of integers from 1 to 100 which are neither divisible by 3 nor by 5 is \(1000a + 100b + 10c + d\), where \(a, b, c, d \in \{0, 1, 2, ....., 9\}\), then find the value of \(a + b + c - d\).

46. (9.00)

Sum of numbers divisible by 3 is \(\frac{3 + 6 + 9 + .... + 99}{2} = \frac{33}{2}(3 + 99) = 1683\)

Sum of numbers divisible by 5 is \(\frac{5 + 10 + 15 + .... + 100}{2} = \frac{20}{2}(5 + 100) = 1050\)

Sum of numbers divisible by 3 and 5 is \(\frac{15 + 30 + 45 + .... + 90}{2} = \frac{20}{2}(15 + 90) = 3(105) = 315\)

Sum of numbers divisible by 3 or 5 is 1683 + 1050 – 315 = 2418

Sum of numbers from 1 to 100 is \(\frac{1 + 2 + 3 + .... + 100}{2} = \frac{100}{2}(1 + 100) = 5050\)

Required sum = 5050 – 2418 = 2632.

47. If \((1 + x – 2x^2)^6 = 1 + a_1x + a_2x^2 + .... + a_{12}x^{12}\).

Then find the value of expression \(a_2 + a_4 + a_6 + .... + a_{12} - 24\).

47. (7.00)

\[(1 + x – 2x^2)^6 = 1 + a_1x + a_2x^2 + .... + a_{12}x^{12} \quad \ldots \ldots (1)\]

Put \(x = 1\) in equation (i)

\[0 = 1 + a_1 + a_2 + a_3 + a_4 + .... + a_n + a_{11} + a_{12} \quad \ldots \ldots (2)\]

\[64 = 1 – a_1 + a_2 – a_3 + a_4 – .... – a_{11} + a_{12} \quad \ldots \ldots (3)\]

By adding (2) & (3)

\[64 = 2 + 2(a_2 + a_4 + .... + a_{12}) \]

\[\therefore \ a_2 + a_4 + .... + a_{12} = \frac{64 - 2}{2} = \frac{62}{2} = 31\]

\[\therefore \ a_2 + a_4 + .... + a_{12} - 24 = 7\]
48. A ray of light emanating from the point $A(3, 10)$, incident at $M(1, 4)$, gets reflected from a line $L$ passing through the point $M$. After reflection it passes through the point $B(4, 3)$. If the equation of the line $L$ is $ax + by - 12 = 0$, then find the perpendicular distance of the point $(a, b)$ from the line $3x + 4y + 20 = 0$.

48. (8.00)
Equation of AM is $3x - y + 1 = 0$, Equation of BM is $x + 3y - 13 = 0$

$\therefore$ Angle bisectors are $x - 2y + 7 = 0$ and $2x + y - 6 = 0$

Since, $6 + 10 - 6 = 10 > 0$ and $8 + 3 - 6 = 5 > 0$

$\therefore$ A and B lie on the same side of $2x + y - 6 = 0$

$\therefore$ The required line is $2x + y - 6 = 0$

$\therefore$ The point $(a, b)$ is $(4, 2)$.

Perpendicular distance of $(4, 2)$ from the line $3x + 4y + 20 = 0$ is 8.

49. If distance between the pair of parallel lines $x^2 + 2xy + y^2 - 8ax - 8ay - 9a^2 = 0$ is $25\sqrt{2}$, then find the value of $|a|$.

49. (5.00)
Lines are $x + y + a = 0$ and $x + y - 9a = 0$

Distance $(d) = \frac{|10a|}{\sqrt{2}} = \frac{10|a|}{\sqrt{2}}$

$\Rightarrow 10|a| = 50$

$\therefore |a| = 5$

50. A person flips 4 fair coins and discards those which turn up tails. He again flips the remaining coin and then discards those which turn up tails. If $P = \frac{m}{n}$ (expressed in lowest form) denotes the probability that he discards at least 3 coins, find the value of $\frac{m+n}{100}$.

50. (4.45)
Let $C_1, C_2, C_3, C_4$ are coins.

4 coins tossed twice $\rightarrow$ each coin is tossed twice.

Let $S$: denotes the success that a coin is discarded

$P(S) = 1 - $ coin is not discarded

$= 1 - P(HH) = 1 - \frac{1}{4} = \frac{3}{4}$

Hence $S$ can take value 0, 1, 2, 3, 4

$P(S = 3 \text{ or } 4) = P(S = 3) + P(S = 4)$

$= ^4C_1 \left( \frac{3}{4} \right)^3 \left( \frac{1}{4} \right) + ^4C_4 \left( \frac{3}{4} \right)^4 = \left( \frac{3}{4} \right)^3 \left( 1 + \frac{3}{4} \right) = \frac{27}{256} + \frac{81}{256} = \frac{108}{256} = \frac{27}{64}$

$\therefore m + n = 445$

Alternatively: Difference possibilities are as follows:

1. 0 coin discard in first toss and (3 or 4 coins of the remaining 4 coins discards in the second toss)
2. 1 coin discard in first toss and (2 or 3 coins of the remaining three coins discards in the second toss)
3. 2 coins discard in first toss and (1 or 2 coins of the remaining two coins discards in the second toss)
4. 3 coins discard in first toss and (1 or 0 coin of the remaining one coin discards in the second toss)
5. All 4 coins discard in first toss.
\[ \therefore \text{Required probability} = \frac{1}{16} \left[ \frac{4}{16} + \frac{1}{16} \right] + \frac{4}{16} \left[ \frac{3}{16} + \frac{1}{16} \right] + \frac{6}{16} \left[ \frac{2}{16} + \frac{1}{16} \right] + \frac{4}{16} \left[ \frac{1}{16} + \frac{1}{16} \right] + \frac{1}{16} (1) = \frac{189}{256} \]

51. If \( f(x) = \text{sgn}(x^2 + x + 1) \), then the sum of all distinct values of \( f(x) \) is

51. \( (1.00) \)
\[ x^2 + x + 1 > 0 \quad \forall \ x \in R \]
\[ \Rightarrow \text{sgn}(x^2 + x + 1) = 1 \text{ only} \]
\[ \Rightarrow \text{sum} = 1 \]

52. If \( f \) satisfies the relation \( f(x + y) + f(x - y) = 2 \cdot f(x) \cdot f(y) \) \( \forall \ x, y \in R \) and \( f(0) \neq 0 \); then \( f(10) - f(-10) = \)

52. \( (0.00) \)
If \( y = x \) then \( f(2x) + f(0) = 2 \left[ f(x) \right]^2 \)
If \( y = -x \) then
\[ f(2x) + f(0) = 2f(x) \cdot f(-x) \]
\[ \Rightarrow 2 \left[ f(x) \right]^2 = 2f(x) \cdot f(-x) \Rightarrow f(x) = f(-x) \]
\[ \Rightarrow f(x) - f(-x) = 0 \]
\[ \Rightarrow f(10) - (-10) = 0 \]
This section contains 02 Matrix Match. Each question contains two columns, Column - I & Column - II. Match the entries in Column - I with the entries in Column - II. One or more entries in Column - I may match with one or more entries in Column - II.

53. Consider $f(x) = x^2 - ax + b$, where $a, b \in R$

<table>
<thead>
<tr>
<th>Column – I</th>
<th>Column – II</th>
</tr>
</thead>
<tbody>
<tr>
<td>(A) If the roots of $f(x) = 0$ differ by unity, then $a^2 =$</td>
<td>(P) $b(ab + 2)$</td>
</tr>
<tr>
<td>(B) If the roots of $f(x) = 0$ differ by unity, then $a^2 + 4b^2 =$</td>
<td>(Q) $1 + 4b$</td>
</tr>
<tr>
<td>(C) If one of the root of $f(x) = 0$ be twice the other, then $2a^2 =$</td>
<td>(R) $(1 + 2b)^2$</td>
</tr>
<tr>
<td>(D) If the sum of roots of the equation $f(x) = 0$ is equal to sum of squares of their reciprocal, then $a^2 =$</td>
<td>(S) $9b$</td>
</tr>
</tbody>
</table>

53. (A) $\rightarrow$ Q; (B) $\rightarrow$ R; (C) $\rightarrow$ S; (D) $\rightarrow$ P

(A) Let roots are $\alpha, \alpha + 1$

$\alpha + \alpha + 1 = a$

$\alpha(\alpha + 1) = b$

Eliminating $\alpha \Rightarrow a^2 - 4b = 1$

$\Rightarrow a^2 = 1 + 4b$

(B) $a^2 + 4b^2 = 1 + 4b + 4b^2 = (1 + 2b)^2$

(C) If one root is twice of other, than roots are $\alpha, 2\alpha$

$\Rightarrow \alpha + 2\alpha = 3\alpha = a$

$\alpha \cdot 2\alpha = 2\alpha^2 = b$

$\Rightarrow 2a^2 = 9b$

(D) Let roots are $\alpha, \beta$ sum of root $\alpha + \beta = a$ and

$$\frac{1}{\alpha^2} + \frac{1}{\beta^2} = \frac{\alpha^2 + \beta^2}{\alpha^2\beta^2} = \frac{(\alpha + \beta)^2 - 2\alpha\beta}{(\alpha\beta)^2}$$

$\Rightarrow a = \frac{a^2 - 2b}{b^2}$

$ab^2 + 2b = a^2$
54. Match the column:

<table>
<thead>
<tr>
<th>Column – I</th>
<th>Column – II</th>
</tr>
</thead>
<tbody>
<tr>
<td>(A) sin^2 5° + sin^2 10° + sin^2 15° + .......... + sin^2 90° = (P) 0</td>
<td></td>
</tr>
<tr>
<td>(B) If cosec A + cot A = \frac{11}{2}, then tan A = (Q) \frac{19}{2}</td>
<td></td>
</tr>
<tr>
<td>(C) Maximum value of \sin^4 \theta + \cos^4 \theta is (R) 3</td>
<td></td>
</tr>
<tr>
<td>(D) If the equation 4\cos x + 3\sin x = 2\lambda + 1 has a solution, then \lambda may take value equal to (S) 1</td>
<td></td>
</tr>
<tr>
<td>(T) 44/117</td>
<td></td>
</tr>
</tbody>
</table>

54. (A) \rightarrow Q; (B) \rightarrow T; (C) \rightarrow S; (D) \rightarrow P, S, T

(A) \sin^2 5° + \sin^2 10° + \sin^2 15° + ...... + \sin^2 40° + \sin^2 45° + \sin^2 50° + ...... + \sin^2 85° + \sin^2 90° = 
\left(\sin^2 5° + \sin^2 85°\right) + ...... + \left(\sin^2 40° + \sin^2 50°\right) + \sin^2 45° + \sin^2 90° = 8 + \frac{1}{2} + 1 = \frac{19}{2}

(B) cosec^2 A – cot^2 A = 1 
\Rightarrow cosec A – cot A = \frac{2}{11}
\Rightarrow 2\cot A = \frac{11}{2} - \frac{2}{11}
\Rightarrow 117 = \frac{22}{2} \Rightarrow tan A = \frac{44}{117}

(C) \sin^4 \theta + \cos^4 \theta \leq \sin^2 \theta + \cos^2 \theta = 1 
Maximum value = 1

(D) \ -5 \leq 2\lambda + 1 \leq 5
\ -\frac{6}{2} \leq \lambda \leq \frac{4}{2}
\ -3 \leq \lambda \leq 2
Possible value of \lambda are \frac{44}{117}, 1, 0.