IIT – JEE: 2022 **PRACTICE TEST** **DATE: 23/08/20**

TOPIC: TRIGONOMETRY - I

SOLUTION

1. (C)

$2\sin^2\theta + 3\cos^2\theta$

Minimum value is 2,

[If $x\sin^2\theta + y\cos^2\theta$, If $x > y$, then x will be always maximum value and y is minimum if $y > x$, vice versa will happen]

2. (C)

Put, $\theta = 60^o$

$\Rightarrow \cos\theta > \cos^2\theta$

$\Rightarrow \cos 60^o > \cos^2 60^o$

$\Rightarrow \frac{1}{2} > \frac{1}{4}$

$\cos\theta > \cos^2\theta$

3. (A)

If $\tan\theta = 1$

It means $\theta = 45^o$

$$= \frac{8\sin\theta + 5\cos\theta}{\sin^3\theta - 2\cos^3\theta + 7\cos\theta}$$

$$= \frac{8\sin 45^o + 5\cos 45^o}{\sin^3 45^o - 2\cos^3 45^o + 7\cos 45^o}$$

$$= \frac{8\times\frac{1}{\sqrt{2}} + 5\times\frac{1}{\sqrt{2}}}{\left(\frac{1}{\sqrt{2}}\right)^3 - 2\left(\frac{1}{\sqrt{2}}\right)^3 + 7\left(\frac{1}{\sqrt{2}}\right)} = 2$$

4. (B)

$\cos^2\theta + \cos^4\theta = 1$

$\Rightarrow \cos^4\theta = 1 - \cos^2\theta$

$\Rightarrow \cos^4\theta = \sin^2\theta$

$\Rightarrow \cos^2\theta.\cos^2\theta = \sin^2\theta$

$\Rightarrow \cos^2\theta = \frac{\sin^2\theta}{\cos^2\theta}$

$\Rightarrow \cos^2\theta = \tan^2\theta$

$\Rightarrow \tan^2\theta + \tan^4\theta$
$\Rightarrow \cos^2\theta + \cos^4\theta = 1$

5. (C)

$$\frac{3\sin\theta + 2\cos\theta}{3\sin\theta - 2\cos\theta}$$

Divide numerator & denominator by $\cos\theta$

$$= \frac{\frac{3\sin\theta}{\cos\theta} + \frac{2\cos\theta}{\cos\theta}}{\frac{3\sin\theta}{\cos\theta} - \frac{2\cos\theta}{\cos\theta}} \left[\frac{\sin\theta}{\cos\theta} = \tan\theta\right]$$

$$= \frac{3\tan\theta + 2}{3\tan\theta - 2}$$

Put value of $\tan\theta$

$$= \frac{3\times\frac{4}{3} + 2}{3\times\frac{4}{3} - 2}$$

$$= \frac{6}{2}$$

= 3

6. (C)

$$(\sec A - \cos A)^2 + (\operatorname{cosec} A - \sin A)^2 - (\cot A - \tan A)^2$$

$$= (\sec^2 A + \cos^2 A - 2\sec A.\cos A) + (\operatorname{cosec}^2 A + \sin^2 A - 2\operatorname{cosec} A.\sin A)$$
$$-(\cot^2 A + \tan^2 A - 2\cot A.\tan A)$$

$$= \sec^2 A - \tan^2 A + \cos^2 A + \sin^2 A + \operatorname{cosec}^2 A - \cot^2 A - 2$$

= 3 – 2

= 1

Alternate shortcut method :

$$(\sec A - \cos A)^2 + (\operatorname{cosec} A - \sin A)^2 - (\cot A - \tan A)^2$$

Put $\theta = 45^\circ$

$$= (\sec 45^\circ - \cos 45^\circ)^2 + (\operatorname{cosec} 45^\circ - \sin 45^\circ)^2 - (\cot 45^\circ - \tan 45^\circ)^2$$

$$= \left(\sqrt{2} - \frac{1}{\sqrt{2}}\right)^2 + \left(\sqrt{2} - \frac{1}{\sqrt{2}}\right)^2 - (1-1)^2$$

$$= \frac{1}{2} + \frac{1}{2} - 0$$

= 1

7. (D)

$$\frac{\cot 30^\circ - \cot 75^\circ}{\tan 15^\circ - \tan 60^\circ}$$

$$= \frac{\tan 60^\circ - \tan 15^\circ}{\tan 15^\circ - \tan 60^\circ}$$

$= \frac{-(\tan 15^\circ - \tan 60^\circ)}{\tan 15^\circ - \tan 60^\circ}$

$= -1$

8. (A)

$(\sec^4\theta - \tan^4\theta)$

$\Rightarrow (\sec^2\theta - \tan^2\theta)\ (\sec^2\theta + \tan^2\theta)$

$\Rightarrow 1 \times (\sec^2\theta + \tan^2\theta)\ [1 + \tan^2\theta = \sec^2\theta]$

$\Rightarrow 1 \times \frac{7}{12}$

$\Rightarrow \frac{7}{12}$

9. (C)

$\left(\frac{1}{\cos\theta} + \frac{1}{\cot\theta}\right)\left(\frac{1}{\cos\theta} - \frac{1}{\cot\theta}\right)$

$= (\sec\theta + \tan\theta)(\sec\theta - \tan\theta)$

$= \sec^2\theta + \tan^2\theta\ [1 + \tan^2\theta = \sec^2\theta] = 1$

10. (A)

$\cos 1^\circ \cos 2^\circ \cos 3^\circ ....\cos 177^\circ \cos 178^\circ \cos 179^\circ$

$= \cos 90^\circ$

= 0 [0 will make whole series 0] = 0

11. (D)

$\frac{\tan\theta}{1-\cot\theta} + \frac{\cot\theta}{1-\tan\theta}$

$\Rightarrow \frac{\tan\theta}{1-\frac{1}{\tan\theta}} + \frac{\frac{1}{\tan\theta}}{1-\tan\theta}$

$\Rightarrow \frac{\tan^2\theta}{\tan\theta - 1} + \frac{1}{\tan\theta(1-\tan\theta)}$

$\Rightarrow \frac{\tan^2\theta}{\tan\theta - 1} - \frac{1}{\tan\theta(\tan\theta - 1)}$

$\Rightarrow \frac{\tan^3\theta - 1}{\tan\theta(\tan\theta - 1)}$

$\Rightarrow \frac{(\tan\theta - 1)(\tan^2\theta + \tan\theta + 1)}{\tan\theta(\tan\theta - 1)}$

$\Rightarrow \frac{\tan^2\theta + \tan\theta + 1}{\tan\theta}$

$\Rightarrow \tan\theta + \cot\theta + 1$

12. (C)

$$\cos^4\theta-\sin^4\theta=\frac{2}{3}$$

$$\Rightarrow(\cos^2\theta-\sin^2\theta)\ (\cos^2\theta+\sin^2\theta)=\frac{2}{3}$$

$$\Rightarrow\cos^2\theta-\sin^2\theta=\frac{2}{3}$$

$$\Rightarrow1-\sin^2\theta-\sin^2\theta=\frac{2}{3}$$

$$\Rightarrow1-2\sin^2\theta=\frac{2}{3}$$

13. (A)

$$\frac{\sin A}{1+\cos A}+\frac{\sin A}{1-\cos A}$$

$$\Rightarrow\frac{\sin A(1-\cos A)+\sin A(1+\cos A)}{(1+\cos A)(1-\cos A)}$$

$$\Rightarrow\frac{\sin A-\sin A.\cos A+\sin A+\sin A.\cos A}{1-\cos^2 A}$$

$$\Rightarrow\frac{2\sin A}{\sin^2 A}$$

$$\Rightarrow 2\ \text{cosec}\ A$$

14. (B)

$$2\sin\theta+\cos\theta=\frac{7}{3}$$

$$\Rightarrow(\tan^2\theta-\sec^2\theta)$$

$$\Rightarrow(\sec^2\theta-1-\sec^2\theta)\qquad[\because 1+\tan^2\theta=\sec^2\theta]$$

$$\Rightarrow-1$$

15. (A)

$$\sec^2\theta-\frac{\sin^2\theta-2\sin^4\theta}{2\cos^4\theta-\cos^2\theta}$$

$$\Rightarrow\sec^2\theta-\frac{\sin^2\theta(1-2\sin^2\theta)}{\cos^2\theta(2\cos^2\theta-1)}$$

$$[\cos^2\theta-\sin^2\theta=2\cos^2\theta-1=1-2\sin^2\theta]$$

$$\Rightarrow\sec^2\theta-\tan^2\theta$$

$$\Rightarrow 1$$

16. (D)

$$\sqrt{\frac{1+\sin\theta}{1-\sin\theta}}+\sqrt{\frac{1-\sin\theta}{1+\sin\theta}}$$

$= \frac{(\sqrt{1+\sin\theta})^2 + (\sqrt{1-\sin\theta})^2}{\sqrt{1-\sin^2\theta}}$

$= \frac{1+\sin\theta+1-\sin\theta}{\cos\theta}$

$= \frac{2}{\cos\theta}$

$= 2\sec\theta$

Alternate shortcut method :

Put $\theta = 30^o$

$\sqrt{\frac{1+\sin 30^o}{1-\sin 30^o}} + \sqrt{\frac{1-\sin 30^o}{1+\sin 30^o}}$

$\Rightarrow \sqrt{\frac{1+\frac{1}{2}}{1-\frac{1}{2}}} + \sqrt{\frac{1-\frac{1}{2}}{1+\frac{1}{2}}}$

$\Rightarrow \sqrt{\frac{3}{1}} + \sqrt{\frac{1}{3}}$

$\Rightarrow \frac{4}{\sqrt{3}}$

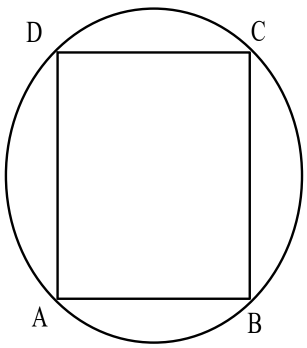
Now check with option by putting $\theta = 30^o$

= 2 sec 30^o

$= \frac{2\times 2}{\sqrt{3}}$

$= \frac{4}{\sqrt{3}}$

17. (A)



$\angle A + \angle C = \angle B + \angle D = 180^o\,1$

$\therefore \angle A = 180^o - \angle C$

$\cos A = \cos\left(180^o\right) - C \Rightarrow \cos C$

Similarly,

$\cos B = -\cos D$

$\Rightarrow\ \cos A + \cos B + \cos C + \cos D$

$\Rightarrow\ \cos A + \cos B + \cos A - \cos B = 0$

Alternate solution :

Put, $A = B = C = D = 90^{o}$

$= \cos A + \cos B + \cos C + \cos D$

$= \cos 90^{o} + \cos 90^{o} + \cos 90^{o} + \cos 90^{o}$

$= 0 + 0 + 0 + 0$

$= 0$

18. (A)

Given,

$\tan\theta + \cot\theta = 5$

$\Rightarrow \tan\theta + \cot\theta = 5$

$\Rightarrow (\tan\theta + \cot\theta)^2 = 5^2$

(Squaring both sides)

$\Rightarrow \tan^2\theta + \cot^2\theta + 2\tan\theta\cot\theta = 25$

$\Rightarrow \tan^2\theta + \cot^2\theta = 25 - 2$

$[\because \tan\theta.\cot\theta = 1]$

$\Rightarrow \tan^2\theta + \cot^2\theta = 23$

19. (C)

$\sin A = m\sin B$

$\sin^2 A = m^2\sin^2 B$ .....(i)

Now, $\tan^2 A = n^2\tan^2 B$

$$\frac{\sin^2 A}{\cos^2 A} = n^2\frac{\sin^2 B}{\cos^2 B}$$

From equation (i)

$$\Rightarrow \frac{1-\cos^2 A}{n^2\cos^2 A} = \frac{\sin^2 B}{\cos^2 B}$$

$$\Rightarrow \frac{1-\cos^2 A}{n^2\cos^2 A} = \frac{\frac{(1-\cos^2 A)}{m^2}}{1-\frac{\sin^2 A}{m^2}}$$

$$\Rightarrow \frac{1-\cos^2 A}{n^2\cos^2 A} = \frac{1-\cos^2 A}{m^2-1+\cos^2 A}$$

$\Rightarrow m^2 - 1 + \cos^2 A = n^2\cos^2 A$

$\Rightarrow m^2 - 1 = \cos^2\theta(n^2 - 1)$

$$\Rightarrow \cos^2 A = \frac{m^2-1}{n^2-1}$$

20. (C)

$\tan 4^{o}.\tan 43^{o}.\tan 47^{o}.\tan 86^{o}$

Here, $\tan 86^{o} = \tan(90^{o} - 4^{o}) = \cot 4^{o}$

$\tan 47^{o} = \tan(90^{o} - 43^{o}) = \cot 43^{o}$

$\tan 4^{o}.\cot 4^{o}.\tan 43^{o}.\cot 43^{o} = 1$

21. (A)

$\Rightarrow \sin 3A = \cos(A - 26°)$

$\Rightarrow 3A + A - 26° = 90°$

[If $\sin A = \cos B$ then $A + B = 90°$]

$\Rightarrow 4A = 116°$

$\Rightarrow A = 29°$

22. (A)

$\sin(\theta + 18°) = \cos 60° (0 < \theta < 90°)$

$\Rightarrow \theta + 18° + 60° = 90°$

$\Rightarrow \theta = 12°$

$\Rightarrow \cos 5\theta$

$\Rightarrow \cos 60°$

$\Rightarrow \frac{1}{2}$

23. (D)

$\sin A - \cos A = \frac{\sqrt{3} - 1}{2}$

Shortcut method:

Put, $\theta = 60°$

$\Rightarrow \sin A - \cos A = \frac{\sqrt{3} - 1}{2}$

$\Rightarrow \sin 60° - \cos 60° = \frac{\sqrt{3} - 1}{2}$

$\Rightarrow \frac{\sqrt{3}}{2} - \frac{1}{2} = \frac{\sqrt{3} - 1}{2}$

$\Rightarrow \frac{\sqrt{3} - 1}{2} = \frac{\sqrt{3} - 1}{2} (Matched)$

Hence,

$\sin A . \cos A$

$\Rightarrow \frac{\sqrt{3}}{2} \times \frac{1}{2}$

$\Rightarrow \frac{\sqrt{3}}{4}$

Alternate:

$\sin A - \cos A = \frac{\sqrt{3} - 1}{2}$

Squaring both side,

$\Rightarrow \sin^2 A + \cos^2 A - 2 \sin A . \cos A = \left(\frac{\sqrt{3} - 1}{2} \right)^2$

$\Rightarrow 1 - 2 \sin A . \cos A = \frac{3 + 1 - 2\sqrt{3}}{4}$

$$\Rightarrow 2\sin A.\cos A = 1 - 2\frac{(2-\sqrt{3})}{4}$$

$$\Rightarrow 2\sin A.\cos A = \frac{2-2+\sqrt{3}}{2}$$

$$\Rightarrow \sin A.\cos A = \frac{\sqrt{3}}{4}$$