IIT – JEE: 2020 **TW TEST (ADV)** **DATE: 03/05/19**

TOPIC: CONTINUITY

SOLUTION

41. **(B)**

$$\lim_{x\to\frac{\pi^-}{2}} \frac{2\cos x - \sin 2x}{(\pi-2x)^2}$$

$$= \lim_{x\to\frac{\pi^-}{2}} \frac{2\cos x - 2\sin x\cos x}{(\pi-2x)^2}$$

$$= \lim_{x\to\frac{\pi^-}{2}} \frac{2\cos x(1-\sin x)}{(\pi-2x)^2}$$

$$x = \frac{\pi}{2} - h$$

$$= \lim_{h\to 0^+} \frac{2\sin h(1-\cos h)}{h^2}$$

$$= \lim_{h\to 0^+} 2\cdot\frac{\sin h}{h}\cdot\frac{1-\cos h}{h^2}\cdot h$$

$$= 0$$

$$\lim_{x\to\frac{\pi^+}{2}} \frac{e^{-\cos x}-1}{8x-4\pi}$$

$$= \lim_{x\to\frac{\pi}{2}} \left(\frac{e^{-\cos x}-1}{-\cos x}\right)\left(\frac{-\cos x}{8x-4\pi}\right)$$

$$= \lim_{x\to\frac{\pi^+}{2}} \left(\frac{-\cos x}{8x-4\pi}\right)$$

$$x = \frac{\pi}{2} + h$$

$$\lim_{h\to 0^+} \frac{\sin h}{8h} = \frac{1}{8}$$

LHL $\neq$ RHL

So, irremovable discontinuity at $x = \frac{\pi}{2}$.

42. **(C)**

$$\tan\theta - \cot\theta = \tan\theta - \frac{1}{\tan\theta}$$

$$= \frac{\tan^2\theta - 1}{\tan\theta}$$

$$=-\left(\frac{1-\tan^2\theta}{2\tan\theta}\right)$$

$$=\frac{-2}{\tan 2\theta}$$

$$=-2\cot 2\theta$$

$$\tan\theta=\cot\theta-2\cot 2\theta$$

$$\frac{1}{2}\tan\left(\frac{\theta}{2}\right)+\frac{1}{4}\tan\left(\frac{\theta}{4}\right)+\frac{1}{8}\tan\left(\frac{\theta}{8}\right)+.....$$

$$=\frac{1}{2}\left\{\cot\frac{\theta}{2}-2\cot\theta\right\}+\frac{1}{4}\left\{\cot\frac{\theta}{4}-2\cot\frac{\theta}{2}\right\}+\frac{1}{8}\left\{\cot\frac{\theta}{8}-2\cot\frac{\theta}{4}\right\}+.....$$

$$=\frac{1}{2^n}\cot\left(\frac{\theta}{2^n}\right)-\cot\theta$$

$$\lim_{x\to\frac{\pi}{2}}\left(\frac{\frac{\theta}{2^n}}{\theta\cdot\tan\left(\frac{\theta}{2^n}\right)}-\cot\theta\right)$$

$n\to\infty$

$$=\frac{1}{\theta}=\frac{2}{\pi}$$

43. (C)

$$6x^2-x-1=0$$

$$\alpha=\frac{1}{2}\qquad\qquad \beta=-\frac{1}{3}$$

$$g(t)=\frac{1}{(\sin t-\alpha)(\sin t-\beta)(\sin t-(\alpha-\beta))}$$

$$=\frac{1}{\left(\sin t-\frac{1}{2}\right)\left(\sin+\frac{1}{3}\right)\left(\sin t-\frac{5}{6}\right)}$$

In $[0, 4\pi]$, there will be 12 points, where denominator = 0.

In $\left[4\pi, 4\pi+\frac{7\pi}{8}\right]$, there will be 4 points.

Total = 16.

44. (D)

$$f(x)=[x]\lim_{n\to\infty}\sum_{r=1}^{n}\frac{(2r-1)}{r+n^2}$$

$$=[x]\left(\frac{1}{1+n^2}+\frac{3}{2+n^2}+\frac{5}{3+n^2}+\frac{7}{4+n^2}+....+\frac{2n-1}{n+n^2}\right)$$

$$=[x]$$

$$\frac{2r-1}{n+n^2}\le\frac{2x-1}{r+n^2}\le\frac{2r-1}{1+n^2}$$

$$\sum\frac{2r-1}{n+n^2}\le\sum\frac{2r-1}{r+n^2}\le\sum\frac{2r-1}{1+n^2}$$

$$\frac{n^2}{n+n^2} \le \sum \frac{2r-1}{r+n^2} \le \frac{n^2}{1+n^2}$$

$$\frac{n}{1+n} \le \sum \frac{2r-1}{r+n^2} \le \frac{n^2}{2+n^2}$$

$$\lim_{n\to\infty} \sum_{r\to 1}^{n} \frac{2r-1}{r+n^2} = 1$$

$$f(x) = [x] \qquad x \ne \frac{\pi}{2}$$

$$x = \frac{\pi}{2}$$

45. **(B)**

$$x \ne 0$$

$$fog(x) = \frac{1}{(g(x)-1)}(g(x)-2)$$

$$= \frac{x^4}{(1-x^2)(1-2x^2)}$$

$$x \ne \pm 1$$

$$x \ne \pm \frac{1}{\sqrt{2}}$$

46. **(ABC)**

(A) Assume $f(a) > f(b)$

$$g(x) = f(x) - \frac{f(a)+f(b)}{2}$$

$$\left.\begin{aligned} g(a) &= \frac{f(a)-f(b)}{2} \\ g(b) &= \frac{f(b)-f(a)}{2} \end{aligned}\right\} g(x) \text{ is a continuous function}$$

$$\left.\begin{aligned} g(a) &> 0 \\ g(b) &< 0 \end{aligned}\right\} So,\ g(x) = 0 \text{ for some } c \in (a, b)$$

(B) $h(x) = f(x) - \sqrt{f(a)f(b)}$

$$h(a) = f(a) - \sqrt{f(a)f(b)}$$

$$= \sqrt{f(a)}\left(\sqrt{f(a)} - \sqrt{f(b)}\right) > 0$$

$$h(b) = f(b) - \sqrt{f(a)f(b)}$$

$$= \sqrt{f(b)}\left(\sqrt{f(b)} - \sqrt{f(a)}\right) < 0$$

(C) $f(c) = \dfrac{3f(a)+2f(b)}{5}$ (section formula)

(D) $f(c) = \dfrac{3f(a)-f(b)}{3-1}$ (external section formula)

47. **(C)**

$x^2+ax+1=ax^2+2x+b$ has roots 1 and 2.

$(a-1)x^2+(2-a)x+b-a=0$

$-\left(\frac{2-a}{a-1}\right)=3$ and $\frac{b-1}{a-1}=2$

$2-a=-3a+3 \qquad b-1=2a-2$

$2a=2 \qquad =1-2$

$a=\frac{1}{2} \qquad =-1$

$b=0$

$a^{-2}+2b=4$

48. **(BD)**

49. **(AC)**

(A) $\frac{\sin x+x}{x}<f(x)<\frac{x^2+x+1}{x^2+1}$

$\lim_{x^-\to\infty}\frac{1+\sin x}{x}=1$

$\lim_{x\to\infty}\left(1+\frac{x}{x^2+1}\right)$

$=\lim_{x\to\infty}\left(1+\frac{x}{x+\frac{1}{x}}\right)$

$=1$

By sandwich

(B) $p=99^{100} \qquad q=100^{99}$

$\frac{q}{p}=\frac{(100)^{99}}{99^{100}}=\left(\frac{100}{99}\right)^{99}\cdot\frac{1}{99}$

$=\left(1+\frac{1}{99}\right)^{99}\cdot\frac{1}{99}$

$\lim_{x\to\infty}\left(1+\frac{1}{x}\right)^x=e$

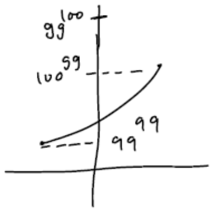
$\left(1+\frac{1}{99}\right)^{99}<e$

$\frac{\left(1+\frac{1}{99}\right)^{99}}{99}<\frac{e}{99}<1$

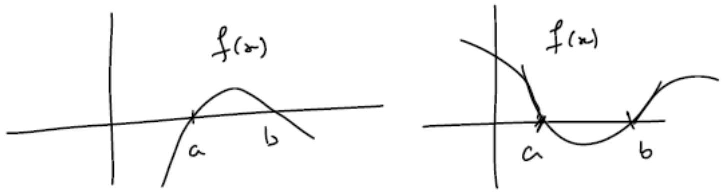
$\frac{p}{q}<1$

$q<p$

One cannot be sure.



(C)



$f\left(a+h\right)>0$ $\qquad$ $f\left(b+h\right)<0$

$f\left(a-h\right)<0$ $\qquad$ $f\left(b-h\right)>0$

$g\left(a+h\right)<0$ $\qquad$ $g\left(b+h\right)<0$

$g\left(a-h\right)<0$ $\qquad$ $g\left(b-h\right)<0$

$g\left(a\right)<0$ $\qquad$ $g\left(b\right)<0$

$g\left(a\right)$ and $g\left(b\right)$ have the same figure.

(D) $f\left(0\right)=163$

$$f\left(1\right)=1+\frac{163}{2+\sin^2 1}$$

$$54\frac{1}{3}<\frac{163}{2+\sin^2 1}<81.5$$

Since $f\left(x\right)$ is a continuous function.

50. (A)

$$f\left(x\right)=\frac{\sqrt{x^2-kx+9}}{x^2-k} \quad R\neq Z$$

$f\left(x\right)$ is continuous $\forall\, n\in R$

Domain:

$x^2-kx+9\geq 0$

$k^2-36\leq 0$

$kt\left\{-6,-5,-4,-3,-2,-1,0,1,2,3,4,5,6\right\}$

$x^2-k\neq 0$

$\therefore \quad k\leq 0$

$\Rightarrow \quad kt\left\{-6,-5,-4,-3,-2,-1\right\}$

51. (BD)

52. (BC)

(A) $g\left(x\right)=\left[x\left[\frac{1}{x}\right]\right]$

Let $x=\frac{1}{2}+h$

$$\lim_{h\to 0^+}\left[\left(\frac{1}{2}+h\right)\left[\frac{1}{\frac{1}{2}+h}\right]\right]=\left[\left(\frac{1}{2}+h\right)\left[\frac{2}{1+2h}\right]\right]=0$$

$\downarrow$

1

$$\lim_{h\to 0^-}\left[\left(\frac{1}{2}+h\right)\left[\frac{2}{1+2h}\right]\right]$$

$\downarrow$

2

$$=\lim_{h\to 0^-}\left[2\left(\frac{1}{2}+h\right)\right]$$

$$=\lim_{h\to 0^-}[1+2h]=0$$

(B) $f(x)=\left[\frac{x}{[x]}\right]$

Let $x=2+h$

$$\lim_{h\to 0^+}=\left[\frac{2+h}{2}\right]$$

$$=\left[1+\frac{h}{2}\right]=1$$

$$\lim_{h\to 0^-}\left[\frac{2-h}{[2-h]}\right]$$

$$\lim_{h\to 0^-}[2-h]$$

$$=1$$

(C) $h(x)=\left[\frac{[x]}{x}\right]$

$h(2)=1$

$$\lim_{p\to 0^+}h(2+p)=\left[\frac{[2+p]}{2+p}\right]=\left[\frac{2}{2+p}\right]=0$$

$$\lim_{p\to 0^+}h(2-p)=\left[\frac{[2-p]}{2-p}\right]=\left[\frac{1}{2-p}\right]=0$$

(D) $k(x)=x\left[\frac{1}{x}\right]+x[x]$

$k(-1)=(-1)(0.1)+(-1)(-1)$

$= 2$

$$\lim_{p\to 0^+}k(-1-p)=(-1-p)\left[\frac{-1}{1+p}\right]+(-1-p)[-1-p]$$

$$=-(1+p)(-1)+(-)(1+p)(-2)$$

$$=1+p+2+2p=3+3p=3$$

$$\lim_{p\to 0^+}k(-1+p)=(-1+p)\left[\frac{1}{-1+p}\right]+(-1+p)[-1+p]$$

$$=(-1+p)(-2)+(-1+p)(-1)$$

$$=2-2p+1-p=3-3p=3$$

53. **(D)**

$$\lim_{x\to 0^+}\frac{e^{0+x}-2}{0+x}$$

$$=\lim_{x\to 0^+}\frac{e^{x}-2}{x}$$

$$f\left(0^+\right)\to -\infty$$

$$\lim_{x\to 0^-}\frac{e^{-1-x}-2}{-1-x}=\frac{e^{-1}-2}{-1}$$

$$=2-\frac{1}{e}$$

$$f\left(x\right)=a\sqrt{x-1}+b\sqrt{2x-1}-\sqrt{2x^2-3x+1}$$

Domain $x\ge 1$ Where $a,b\in R$

$a+2b=2$

(A) $f\left(x\right)=2\left(1-b\right)\sqrt{x-1}+b\sqrt{2x-1}-\sqrt{2x^2-3x+1}$

$$=\left(2\sqrt{x-1}-\sqrt{2x^2+3x+1}\right)+b\left(\sqrt{2x-1}-2\sqrt{x-1}\right)$$

$$=\left(2\sqrt{x-1}-\sqrt{2x-1}\sqrt{x-1}\right)+b\left(\sqrt{2x-1}-2\sqrt{x-1}\right)$$

$$=\sqrt{x-1}\left(2-\sqrt{2x-1}\right)+b\left(\sqrt{2x-1}-2\sqrt{x-1}\right)$$

$f\left(1\right)=b$

$f\left(5\right)=2\left(2-3\right)+b\left(3-4\right)$

$=-2-b=-\left(2+b\right)$

$f\left(1\right)f\left(5\right)<0$

(B) $f\left(1\right)=b$ $f\left(\frac{3}{2}\right)=\sqrt{\frac{1}{2}}\left(2-\sqrt{2}\right)=\sqrt{2}-1$

$f\left(2\right)=\left(2-\sqrt{3}\right)+b\left(\sqrt{3}-2\right)$

$=\left(-\sqrt{3}\right)\left(1-b\right)$

(C) $f\left(1\right)<0$ if $b<-2$

$f\left(5\right)=-\left(2+b\right)>0$ if $b<-2$

$f\left(1\right)f\left(5\right)<0$

(D) $b=0$

$f\left(1\right)=0$ $f\left(2\right)=2-\sqrt{3}$

$f\left(5\right)=2\left(2-3\right)=-2$

$f\left(2\right)\cdot f\left(5\right)<0$

54. **(ABCD)**

55. **(B)**

$f\left(x+2\right)\le f\left(x\right)+2$

$$f(x+4) \le f(x+2)+2 \le f(x)+4$$

$$f(x+6) \le f0x+4+2 \le f(x+6)$$

$$\downarrow$$

$$f(x+10) \le f(x)+10 \qquad .....(1)$$

$$f(x+5) \ge f(x)+5$$

$$f(x)+10 \ge f(x+5)+5 \ge f(x)+10 \qquad .....(2)$$

So looking at (1) and (2) equality roots

$$\left\{\begin{matrix} f(1)=1 \\ f(2)=2 \\ \vdots \\ f(10)=10 \end{matrix}\right\}$$

56. (5)

$$\lim_{x\to 1^-} \frac{\sin\left(ax^2+bx+c\right)}{x^2-1}$$

$$= \lim_{x\to 1^-} \frac{(2ax+b)\cos\left(ax^2+bx+c\right)}{2x}$$

$$= \lim_{x\to 1^-} \frac{(2a+b)\cos\pi}{+2} = \frac{-2a-b}{2}$$

$$= \lim_{x\to 1^+} \text{sgn}(x+1)\cos(2x-2)+bx^2$$

$$= a+b$$

$$\frac{-2a-b}{2}=1 \qquad 2a+b=-2$$

$$a+b=1 \qquad \underline{a+b=1}$$

$$a=-3$$

$$-6+b=-2$$

$$b=4$$

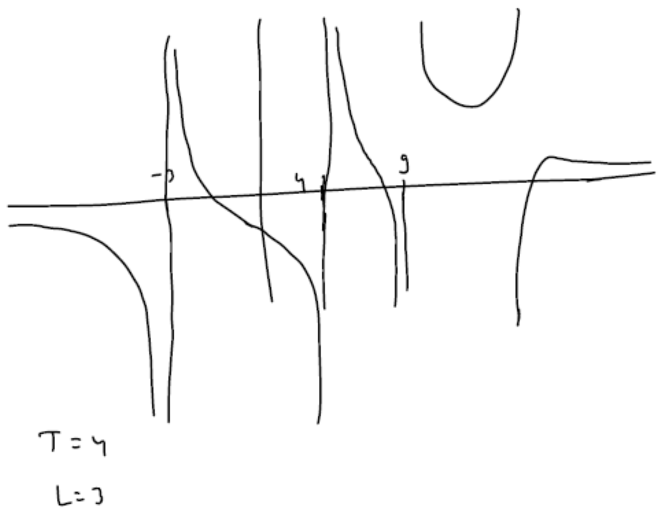
$$\frac{a^2+b^2}{5} = \frac{(-3)^2+4^2}{5} = 5$$

57. (7)

$$f(x) = \frac{2}{x+3} + \frac{5}{x-4} + \frac{7}{x-9} - \frac{1}{x-13} = c$$

$$x \to \infty \quad f(x) \to 0^+$$

$$x \to -\infty \quad f(x) \to 0^-$$



58. **(7)**

$$\lim_{x\to 0^+}\frac{a+\cos(\sin 0)}{x+1}=a+1$$

$$\lim_{x\to 0^-} b+\left[\frac{\sin x - x}{x^3}\right]=b-1$$

$$\lim_{x\to 0}\frac{x-\sin 2x}{x^3}=\frac{16}{}$$

$a+1=2$ $\qquad$ $b-1=2$

$a=1$ $\qquad$ $b=3$

$h2+2b=1+6=7$ a

59. **(0)**

60. **(6)**

$\lim_{x\to 1} f(x)=0$ $\qquad$ function will be discontinuous when $[x^2]$ is an integer.

$\lim_{x\to 2} f(x)=0$

$\lim_{x\to 3} f(x)=0$

$$\left.\begin{array}{l} x=\sqrt{2} \\ x=\sqrt{3} \\ x=\sqrt{5} \\ x=\sqrt{6} \\ x=\sqrt{7} \\ x=\sqrt{8} \end{array}\right\} \text{6 points}$$