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<td>36. (0.34)</td>
<td>56. (2.00)</td>
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<td>40. (10.00)</td>
<td>60. (4.00)</td>
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**SOLUTION**

1. **(A)**
   When all the plates are connected together their potential will be same and no electric field will exist between the plates and total charge will be equally distributed on the outer surfaces of the outer plates A and C as shown in figure.

   ![Diagram](image)

   By conservation of charge we have
   
   \[ 2q = Q - Q + Q \]
   
   \[ \Rightarrow q = \frac{Q}{2} = 1 \mu C \]

2. **(A)**
   Over \( Q_1 \), potential is positive so \( Q_1 \) is positive.

   \( V_A = 0 \) and \( A \) point is nearer to \( Q_2 \) so \( Q_2 \) should be negative and \( |Q_1| > |Q_2| \). At \( A \) and \( B \), potential is zero, not the forces so these are not equilibrium points. Equilibrium at \( C \) will depend on the nature of charge which is kept at \( C \). So we cannot comment on the type of equilibrium at \( C \).

3. **(B)**
   Force on dipole \[ P \left| \frac{dE}{dx} \right| \]

   \( E \) is maxima at \[ x = \frac{R}{\sqrt{2}} \]

   So, \[ \frac{dE}{dx} = 0 \]

   \[ \therefore F = 0 \]
4. (C)

\[ A = 4^\circ \]

\[ \theta = 120^\circ = \frac{120}{180} \pi = \frac{2\pi}{3} \]

\[ t = \frac{\theta}{\omega} = \frac{T}{3} = \frac{2\pi}{3} \sqrt{\frac{md}{2qE}} \]

5. (C)

Quartering ring (charge \( +q \), radius \( R \)) find dipole moment of system.

\[ dq = \frac{q}{\pi/2} d\theta = \frac{2q}{\pi} d\theta \]

\[ dp = (dq)R = \frac{2q}{\pi} d\theta R \]

\[ p_x = \int dp \cos \theta = \frac{2qR}{\pi} \int_0^{\pi/2} \cos \theta dQ = \frac{2qR}{\pi} \]

\[ p_y = \int dp \sin \theta = \frac{2qR}{\pi} \int_0^{\pi/2} \sin \theta d\theta = \frac{2qR}{\pi} \]

\[ \vec{p} = \frac{2qR}{\pi} \hat{i} + \frac{2qR}{\pi} \hat{j}; \quad p = \frac{2\sqrt{2}qR}{\pi} \]
6. (B)

7. (ABCD)
   By an external force in case of SHM only equilibrium position changes. Time period remains same.
   As speed of block at mean position is same amplitude will be same in all cases. In case-4 equilibrium
   position $x_o = \frac{3mg}{k}$ which is maximum among all cases. Thus all the given options are correct.

8. (C)
   Electric field at each point of the cavity is normal to its surface but not necessarily equal and being a
   conductor at every point of conductor potential is same. An electric line of force does not penetrate a
   metal surface hence total flux through the cavity surface is always zero. Thus only option (C) is correct.

9. (BD)
   The electric field at any point inside the shell due to the point charge can be calculated by standard
   relation but that due to induced charges cannot be calculated. The electric field at points outside the
   outer surface is only due to the charge distribution on the outer surface of the shell so on outer surface
   it is given as
   \[ E = \frac{1}{4\pi \varepsilon_o} \frac{q}{(2R)^2} \]
   The options (B) and (D) are correct.

10. (CD)

11. (BD)
   As per the given information we can calculate the electric field component in y direction in the system
   which is given as
   \[ E_y = \frac{100 - 50}{2} = 25 \text{ V/m} \]
   As net electric field in the system is given as
   \[ E = \sqrt{E_x^2 + E_y^2 + E_z^2} \]
   So, net electric field in the region may be greater than 25 V/m or equal to 25 V/m if other components
   are zero.

12. (C)
   The capacitance of the system decreases as oil flows out so the charge on the tubes will also decrease
   with time and a current flows through the battery.

   Instantaneous charge on tubes is given as
   \[ q = CV \]
\[ q = \left[ \frac{\varepsilon_0 (l-x) + \varepsilon_0 x k}{\ln \left( \frac{b}{a} \right) \ln \left( \frac{b}{a} \right)} \right] V \]

Current through battery can be given as
\[ i = \frac{\varepsilon_0}{\ln \left( \frac{b}{a} \right)} (k-1) \left( \frac{dx}{dt} \right) V \]

As \( \frac{dx}{dt} \) depends on the area of the hole, so \( i \) in the circuit depends on area of the hole.

13. (ACD)
When \( x \) or \( z \) is replaced by a conductor, the effective separation between the capacitor plates decreases to its capacitance increases. If we replace \( y \) by a dielectric, then it will act as another capacitor in series so overall capacitance will decrease. If \( x \) is replaced by a dielectric, then also the capacitance of this section increases which will result in overall increase of capacitance. Thus options (A), (C) and (D) are correct.

14. (ABC)
Across points 1 and 2 equivalent capacitance can be calculated as
\[ C_{12} = C \left| P \right| \left( \frac{C}{2} \left| S \right| \frac{4C}{3} \right) = \frac{15C}{11} \]
Where \( \left| P \right| \) and \( \left| S \right| \) stands for parallel and series combinations respectively. Across points 3 and 6 equivalent capacitance can be calculated as
\[ C_{36} = \frac{C}{3} \left| P \right| \left( \frac{C}{3} \right) = \frac{5C}{3} \]
Across points 1 and 3 equivalent capacitance can be calculated as
\[ C_{12} = \frac{C}{2} \left| P \right| \left( \frac{C}{2} \left| S \right| \frac{4C}{3} \right) = \frac{4C}{15} \]
Across points 3 and 5 equivalent capacitance can be calculated as
\[ C_{35} = \frac{C}{2} \left| P \right| \left( \frac{C}{2} \left| S \right| \frac{4C}{3} \right) = \frac{4C}{15} \]
Thus options (A), (B) and (C) are correct.

15. (ABC)
The end of plates where the plates are closer the electric field will be higher as conducting plates are at same potential difference between all their points. So the metal surface out of which electric field is higher surface charge density is also higher. Thus options (A), (B) and (C) are correct.
16.  
Total charge coming out of battery = 0

Writing nodal equation for \( y \) gives

\[
5\left[ y + 20 - (x - 20) \right] + 10\left[ y + 20 - (x + 10) \right] + 10\left[ y - (x + 10) \right] + 5\left[ y - (x - 10) \right] = 0
\]

\[
\Rightarrow y - x + 40 + 2y - 2x + 20 + 2y - 2x - 20 + y - x + 10 = 0
\]

\[
\Rightarrow 6y - 6x + 50 = 0 \quad \ldots (1)
\]

Writing nodal equation for 0 V point gives

\[
10(20 - x) + 5(20 - x + 10) + 5(0 - x) + 10(0 - x + 20) = 0
\]

\[
\Rightarrow 40 + 30 + 40 - x - x + 2x = 0
\]

\[
\Rightarrow 6x = 110 \quad \ldots (2)
\]

\[
\Rightarrow x = \frac{110}{6} \text{V}
\]

From equation \(- (1)\)

\[
y = \frac{6x - 50}{6} = \frac{60}{6} = 10 \text{ V}
\]

17.  
Let \( C_0 \) be the initial capacitance of the condenser. Then

\[
C_0 = \frac{K\varepsilon_0 \left( \pi R^2 \right)}{2d}
\]

In the rotated condition, let \( C_1 \) be the new capacity of inner condenser. Then

\[
C_1 = \frac{K\varepsilon_0 \left[ \frac{\pi R^2}{3} - \frac{R^2\phi}{2} \right]}{d}
\]

Where outside plate are \( \frac{R^2\phi}{2} \), because the circumference is

\[
R\phi \quad \text{and area is } \frac{1}{2} \times R \times (R\phi) \text{ i.e., } \frac{R^2\phi}{2}
\]

If \( C_2 \) be the capacity of outside condenser, then

\[
C_2 = \frac{\varepsilon_0 R^2\phi}{2d}
\]

In rotated position, the arrangement in equivalent to two capacitors \([\text{of capacity } C_1 \text{ and } C_2] \) connected in parallel. Hence

\[
C = C_1 + C_2
\]
\[
\Rightarrow C = \frac{K\varepsilon_0 (\pi R^2)}{2d} - \frac{K\varepsilon_0 R^2\phi}{2d} + \frac{\varepsilon_0 R^2\phi}{2d}
\]
\[
\Rightarrow C = \frac{\varepsilon_0 R^2}{2d} \left[ K\pi + (1 - K)\phi \right]
\]

Initial energy \( U_i = \frac{1}{2} C_0 V^2 = \frac{K\varepsilon_0 (\pi R^2)}{4d} V^2 \)

Final energy \( U_f = \frac{1}{2} \frac{\varepsilon_0 R^2}{2d} \left[ K\pi + (1 - K)\phi \right] V^2 \)

\[
\Rightarrow \Delta U = U_i - U_f = \frac{\varepsilon_0 R^2}{2d} (K-1)\phi V^2
\]

If \( \tau \) be the moment of force, then
\[
\tau \phi = \frac{\varepsilon_0 R^2}{4d} (K-1)\phi V^2
\]
\[
\Rightarrow \tau = \frac{\varepsilon_0 R^2}{4d} (K-1)V^2
\]

18. (1.00)

Since collision is elastic the velocity of dielectric after collision is \( v_0 \).

Dielectric will move and when it is coming out of capacitor a force is exerted on it by the capacitor in inward direction which is given as
\[
F = \frac{E^2\varepsilon_0 b(K-1)}{2d}
\]
Which decreases its speed to zero, till it comes out it travels a distance \( a \) so we use
\[
\frac{1}{2}Mv_0^2 = \frac{E^2\varepsilon_0 b(K-1)a}{2d}
\]

19. (1.00)

When \( S_1 \) is closed for first time if \( q_{10} \) is the charge comes on outer shell, we have
\[
V_{outer} = K \left( \frac{Q}{2r} + \frac{q_{10}}{2r} \right)
\]
\[
\Rightarrow q_{10} = -Q
\]

Then \( S_2 \) is closed for first line and a charge \( q_{1i} \) comes on inner shell, we have
\[
V_{inner} = K \left( \frac{q_{10}}{2r} + \frac{q_{1i}}{r} \right) = 0
\]
\[
\Rightarrow q_{1i} = \frac{Q}{2}
\]

Now when \( S_1 \) is closed for the second time, if charge on outer shell is \( q_{20} \), then we have
\[
V_{outer} = K \left( \frac{q_{20}}{2r} + \frac{(Q/2)}{2r} \right) = 0
\]
\[
\Rightarrow q_{20} = -\frac{Q}{2}
\]
Thus with similar analysis we can state that after $n$ times closing and opening switches $S_1$ and $S_2$ final charge on inner shell becomes

$$q_{n}\rangle = \frac{Q}{(2)^n}$$

Thus final potential difference between shells is given as

$$V_n = K\left(\frac{q_{n1}}{r} - \frac{q_{n2}}{2r}\right) = \frac{Kq_{n1}}{2r}$$

$$\Rightarrow V_n = \frac{1}{(2)^n+1}\left(\frac{Q}{4\pi \varepsilon_0 r}\right)$$

20. (6.00)

The direction of electric field inside the cavity leftward is in direction and of constant magnitude given as

$$E_{\text{cavity}} = \frac{\rho a}{3 \varepsilon_0}$$

For touching the sphere again, electron must move a distance $2r \cos 45^\circ$ and time taken by electron for this is given as

$$t = \sqrt{\frac{2l}{a}}$$
SOLUTION

41. (D)
\[
\lim_{x \to 0} \frac{x(e^x - 1)}{1 - \cos x} = \lim_{x \to 0} \frac{2x(e^x - 1)}{4 \sin^2 \frac{x}{2}} \\
= 2 \lim_{x \to 0} \left[ \frac{(x/2)^2}{\sin^2 \frac{x}{2}} \right] \left( e^x - 1 \right) = 2.
\]

42. (D)
\[
\lim_{x \to 0^-} f(x) = \lim_{h \to 0} \frac{0 - h}{h + h^2} = \lim_{h \to 0} \frac{-1}{1 + h} = -1 \\
\text{and } \lim_{x \to 0^+} f(x) = \lim_{h \to 0} \frac{h}{h + h^2} = \lim_{h \to 0} \frac{1}{1 + h} = 1
\]
Hence, limit does not exist.

43. (A)
As \( f \) is a positive increasing function, we have \( f(x) < f(2x) < f(3x) \)
Dividing by \( f(x) \) leads to \( 1 < \frac{f(2x)}{f(x)} < \frac{f(3x)}{f(x)} \)
As \( \lim_{x \to \infty} \frac{f(3x)}{f(x)} = 1 \), we have by Squeez theorem or Sandwich theorem, \( \lim_{x \to \infty} \frac{f(2x)}{f(x)} = 1 \)

44. (C)
\[
f(0) = 0, \ \lim_{x \to 0^-} f(x) = \lim_{x \to 0^+} f(x) = \lim_{x \to 0} x \left[ \frac{\sin x^2}{x^2} \right] = 0.
\]

45. (C)
\[
f(x) = \frac{2x^2 + 7}{x^2 (x + 3) - 1(x + 3)} \\
= \frac{2x^2 + 7}{(x-1)(x+1)(x+3)}
\]
Hence points of discontinuity are
\[ x = 1, x = -1 \text{ and } x = -3 \text{ only.} \]

46. (BD) \[ f(x) = \begin{cases} e^x ; & x \leq 0 \\ 1 - x; & 0 < x \leq 1 \\ x - 1; & x > 1 \end{cases} \]

\[ Rf'(0) = \lim_{h \to 0} \frac{f(0 + h) - f(0)}{h} = \lim_{h \to 0} \frac{1 - h - 1}{h} = -1 \]

\[ Lf'(0) = \lim_{h \to 0} \frac{f(0 - h) - f(0)}{-h} = \lim_{h \to 0} \frac{e^{-h} - 1}{-h} = 1 \]

So, it is not differentiable at \( x = 0 \).
Similarly, it is not differentiable at \( x = 1 \).
But it is continuous at \( x = 0, 1 \).

47. (D)

48. (A) If \( f \) is continuous at \( x = 0 \), then

\[ \lim_{x \to 0} f(x) = \lim_{x \to 0} f(0) \Rightarrow f(0) = \lim_{x \to 0} f(x) \]

\[ k = \lim_{h \to 0} f(0 - h) = \lim_{h \to 0} \frac{\cos \frac{\pi}{2} [0 - h]}{[0 - h]} \]

\[ k = \lim_{h \to 0} \frac{\cos \frac{\pi}{2} [-h]}{-h} = \lim_{h \to 0} \frac{\cos \frac{\pi}{2} [-h - 1]}{[-h - 1]} \]

\[ k = \lim_{h \to 0} \frac{\cos \left( \frac{\pi}{2} \right)}{-1} ; k = 0. \]

49. (AD)

We have, \( f(x) = |x| + |x - 1| \)

\[ = \begin{cases} -2x + 1, & x < 0 \\ x - x + 1, & 0 \leq x < 1 \\ x + x - 1, & x \geq 1 \end{cases} = \begin{cases} -2x + 1, & x < 0 \\ 1 & 0 \leq x < 1 \\ 2x - 1, & x \geq 1 \end{cases} \]

Clearly, \( \lim_{x \to 0^+} f(x) = 1 \), \( \lim_{x \to 0^-} f(x) = 1 \), \( \lim_{x \to 1^+} f(x) = 1 \)

And \( \lim_{x \to 1^-} f(x) = 1 \). So, \( f(x) \) is continuous at \( x = 0, 1 \).

\[ f'(x) = \begin{cases} -2, & x < 0 \\ 0, & 0 \leq x < 1 \\ 2, & x \geq 1 \end{cases} \]

Here \( x = 0, f'(0^+) = 0 \) while \( f'(0^-) = -2 \)
and at \( x = 1, f'(1^+) = 2 \) while \( f'(1^-) = 0 \)
Thus, \( f(x) \) is not differentiable at \( x = 0 \) and \( 1 \).

50. (AC)

\[ f(x) = \max \{ (1-x), (1+x), 2 \} , \quad \forall \ x \in (-\infty, \infty). \]

\[ f(x) = \begin{cases} 
1+x; & x > 1 \\
2; & -1 \leq x \leq 1 \\
1-x; & x < -1
\end{cases} \]

Since \( f(x) = 1-x \) or \( 1+x \) are polynomial functions and \( f(x) = 2 \) is a constant function.

\[ \therefore \text{These are continuous at all points} \quad \text{.....(i)} \]

\[ \therefore f(x) \text{ is differentiable at all the points, except at } x = 1 \text{ and at } x = -1. \]

51. (B)

For \( \frac{-\pi}{2} < x \leq 0 \), \( f(x) = -p \sin x + q e^{-x} - rx^3 \), so

\[ f'(0-) = \lim_{x \to 0^-} \frac{f(x) - f(0)}{x - 0} = \lim_{x \to 0^-} \left[ - \frac{p \sin x}{x} - q \left( \frac{e^{-x} - 1}{-x} \right) - r x^2 \right] = -p - q \]

For \( 0 < x < \frac{\pi}{2} \), \( f(x) = p \sin x + q e^{x} + rx^3 \),

\[ f'(0+) = \lim_{x \to 0^+} \frac{f(x) - f(0)}{x - 0} = \lim_{x \to 0^+} \left[ \frac{p \sin x}{x} + q \left( \frac{e^{x} - 1}{x} \right) - r x^2 \right] = p + q \]

\[ \therefore -(p + q) = p + q \implies p + q = 0 \]

52. (B)

Given limit \( \lim_{n \to \infty} (4^n + 5^n)^{1/n} \)

\[ = \lim_{n \to \infty} \left[ 1 + \left( \frac{4}{5} \right)^n \right]^{(5/4)^n} \]

\[ = \lim_{n \to \infty} \left[ 1 + \left( \frac{4}{5} \right)^n \right]^{(1/(1+1))^{n}} \]

\[ = 5 \cdot e^0 = 5 \left( \frac{4}{5} \right)^n \to 0 \text{ as } n \to \infty \]

53. (ABD)

\[ \lim_{x \to 0} \sqrt[+]{\frac{1}{x} - \cos 2x} = \lim_{x \to 0} \frac{\sin x}{x} \]

So, \( \lim_{x \to 0} \frac{\sin x}{x} = 1 \) and \( \lim_{x \to 0^-} \frac{\sin x}{x} = -1 \)

Hence, limit does not exist.
54. (A) 
\[ \lim_{x \to 0} \frac{\log x}{x^4} = \lim_{x \to 0} \frac{1}{nx^4} = 0 \quad \text{(By L-Hospital's rule)} \]

55. (CD) 
\[ \lim_{x \to 2^-} \frac{|x - 2|}{x - 2} = \lim_{h \to 0} \frac{2 - h - 2}{2 - h - 2} = -1 \]
and 
\[ \lim_{x \to 2^+} \frac{|x - 2|}{x - 2} = \lim_{h \to 0} \frac{2 + h - 2}{2 + h - 2} = 1 \]

Hence limit does not exist.

56. (2.00) 
\[ \lim_{x \to 0} \left( \frac{1 + 5x^2}{1 + 3x^2} \right)^{1/2} = \frac{\left(1 + 5x^2\right)^{1/2}}{\left(1 + 3x^2\right)^{1/2}} = \frac{e^5}{e^2} = e^3. \quad \text{[\because \lim_{x \to 0} (1 + x)^{1/2} = e]} \]

57. (5.00) 
Now expanding \( e^x \) and \( \cos x \), we get
\[ \lim_{x \to 0} \frac{3x^2 + x^4 \left( \frac{1}{2!} - \frac{1}{4!} \right) + \ldots}{x^2} = \frac{3}{2} \]

**Aliter:** Apply L-Hospital’s rule,
\[ \lim_{x \to 0} \frac{2xe^x + \sin x}{2x} = \lim_{x \to 0} e^x + \frac{\sin x}{2x} = 1 + \frac{1}{2} = \frac{3}{2}. \]
\( p + q = 5 \) Ans.

58. (5.00) 
\[ \lim_{x \to 0} \frac{f \left( \frac{2 \sin^2 \frac{x}{2}}{2} \right)}{g(x) \left( \frac{2 \sin^2 \frac{x}{2}}{2} \right)^2 \left( \frac{\sin \frac{x}{2}}{2} \right) \left( \cos \frac{x}{2} \right)} \]
\[ = \lim_{x \to 0} \frac{a}{g(x)} \times \tan^2 \frac{x}{2} = b \]
\[ \lim_{x \to 0} \frac{x^2}{g(x)} = \frac{4b}{a} \]

Then, \( \lim_{x \to 0} g \left( \frac{1 - \cos 2x}{x^4} \right) = \frac{a}{b} \)

59. (5.00) 
\[ \lim_{x \to \frac{\pi}{2}} \frac{(\cos x + \sin x)^3 - 2\sqrt{2}}{1 - \sin 2x} \]
\[
= \lim_{x \to \frac{\pi}{4}} 2\sqrt{2} \frac{\cos^3\left(x - \frac{\pi}{4}\right) - 1}{1 - \sin 2x}
\]
\[
= -2\sqrt{2} \lim_{x \to \frac{\pi}{4}} \frac{1 - \cos\left(x - \frac{\pi}{4}\right)}{1 - \cos\left(\frac{\pi}{2} - 2x\right)} \lim_{x \to \frac{\pi}{4}} \left[1 + \cos^2\left(x - \frac{\pi}{4}\right) + \cos\left(x - \frac{\pi}{4}\right)\right]
\]
\[
= -6\sqrt{2} \lim_{x \to \frac{\pi}{4}} \frac{2\sin^2\left(\frac{\pi}{4} - x\right)}{2\sin^2\left(\frac{\pi}{4} - x\right)} = -6\sqrt{2}
\]

\[
= -6\sqrt{2} \lim_{x \to \frac{\pi}{4}} \frac{\sin^2\left(\frac{\pi}{4} - x\right)}{4\sin^2\left(\frac{\pi}{4} - x\right) \cos^2\left(\frac{\pi}{4} - x\right)} = -6\sqrt{2} \frac{1}{4}
\]

\[
= (4.00)
\]

For \( f(x) \) to be continuous at \( x = 0 \), we should have \( \lim_{x \to 0} f(x) = f(0) = 12 (\log 4)^3 \)

\[
\lim_{x \to 0} f(x) = \lim_{x \to 0} \left(\frac{4^x - 1}{x}\right)^3 \times \left(\frac{x}{p}\right) \cdot \frac{px^2}{\sin\left(\frac{x}{p}\right) \log\left(1 + \frac{1}{3} x^2\right)}
\]
\[
= (\log 4)^3 \cdot \frac{x^2}{\frac{1}{3} x^2 - \frac{1}{18} x^4 + \ldots \ldots}
\]
\[
= 3p (\log 4)^3.
\]

Hence \( p = 4 \).