IIT – JEE: 2022 **TW TEST (FLAGSHIP)** **DATE: 02/08/20**

TOPIC: QUADRATIC EQUATIONS

SOLUTION

1. (C)

$D = 16k + 16k^2$

$= 16\left(k + k^2\right)$

$= 16k\left(1 + k\right)$

2. (D)

$x \to \frac{x}{3}$

3. (C)

Sum of coefficients is same, so $1 + a + b = 0$ ensures $x = 1$ is a common root

4. (A)

Clearly sum of roots > 0 & product of roots < 0 indicates that positive root is numerically greater than negative root.

5. (C)

$x^4 - x^3 - 4x^2 - x + 1 = 0$

$\Rightarrow \left(x^2 + \frac{1}{x^2}\right) - \left(x + \frac{1}{x}\right) - 4 = 0$

$\Rightarrow t^2 - t - 6 = 0;\ x + \frac{1}{x} = t$

$\Rightarrow t = 3, -2$

$\therefore\ x + \frac{1}{x} = 3$

$\Rightarrow x^2 - 3x + 1 = 0$

$\Rightarrow$ sum of irrational roots = 3.

6. (D)

Since coefficients are real, $3 + \sqrt{5}\,i$ must be a root giving $3 + 3 + \gamma = \frac{23}{3}$

$\Rightarrow \gamma = \frac{23}{3} - 6 = \frac{5}{3}$

7. (D)

$(2x-1)(x+3)$ is a factor

$\Rightarrow\ x=\frac{1}{2}$ & $x=-3$ are roots of the given polynomial

8. (A)

$x^3-x=x(x+1)(x-1)$

Let remainder be ax^2+bx+c

$x^{87}+x^{69}+x^{51}+x^{33}+x^{15}=x(x+1)(x-1)Q(x)+(ax^2+bx+c)$

$$\left.\begin{aligned}&\Rightarrow\ 0=c\\&\Rightarrow\ 5=a+b+c\\&\Rightarrow\ -5=a-b+c\end{aligned}\right\}\ \therefore\ a=0,\ b=5,\ c=0$$

9. (C)

New sum of roots

$=\alpha^2+\beta^2=(\alpha+\beta)^2-2\alpha\beta$

$$=\left(-\frac{b}{a}\right)^2-\frac{2c}{a}$$

$$=\frac{b^2}{a^2}-\frac{2c}{a}$$

$$=\frac{b^2-2ac}{a^2}$$

New product of roots $=\alpha^2\beta^2=\frac{c^2}{a^2}$

10. (D)

$x^2+(k-29)x-k=0$ .....(1)

& $2x^2+(2k-43)x+k=0$ .....(2)

Must have a common root multiply equation (1) by 2 & substract to get common root as $x=-\frac{k}{5}$

$\therefore\ k=0$ or 30

11. (B)

$$\frac{1}{\alpha^2}+\frac{1}{\beta^2}+\frac{1}{\gamma^2}=\left(\frac{1}{\alpha}+\frac{1}{\beta}+\frac{1}{\gamma}\right)^2-2\left(\frac{1}{\alpha\beta}+\frac{1}{\beta\gamma}+\frac{1}{\gamma\alpha}\right)$$

$$=\left(\frac{\Sigma\alpha\beta}{\alpha\beta\gamma}\right)^2-2\left(\frac{\Sigma\alpha}{\alpha\beta\gamma}\right)$$

$$=2$$

12. (B)

$D \ge 0$

& $f(3) > 0$

& $\dfrac{-b}{2a} > 3$

13. (B)

$$y = \frac{(x^2 - 3x + 4) - 8}{x^2 - 3x + 4}$$

$$= 1 - \frac{8}{x^2 - 3x + 4}$$

Now $x^2 - 3x + 4 \in \left[\frac{7}{4}, \infty\right)$

$$\Rightarrow \frac{1}{x^2 - 3x + 4} \in \left(0, \frac{4}{7}\right]$$

$$\Rightarrow \frac{-8}{x^2 - 3x + 4} \in \left[-\frac{32}{7}, 0\right)$$

$$\Rightarrow 1 - \frac{8}{x^2 - 3x + 4} \in \left[-\frac{24}{7}, 1\right)$$

14. (D)

Since $x^2 + 2x + 3 = 0$ has real coeffcients & non-real roots, so both the roots of $ax^2 + bx + c = 0$ are also non-real & hence both roots common.

$\therefore\ a : b : c \equiv 1 : 2 : 3$

15. (C)

$ax^2 + hxy + by^2 + 2gx + 2fy + c$ has two linear factros if

$$\begin{vmatrix} a & h & g \\ h & b & g \\ g & f & c \end{vmatrix} = 0$$

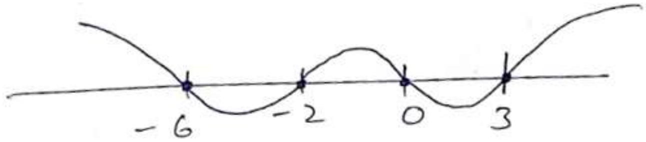
16. (C)

$$\alpha + \beta = \frac{-3a}{1-a} = \frac{3a}{a-1} > 0$$

$$\alpha\beta = \frac{-1}{1-a} = \frac{1}{a-1} > 0$$

17. (C)

$(x-3)(x+2)x(x+6) \ge 0$



Clearly $x = 3$.

18. (B)

Rearranging as quadratic and applying sum of roots = 0
i.e. coefficient of $x = 0$ gives results.

19. (D)

$D = (a-10)^2 - 4$ should be a perfect square

$\Rightarrow\ a-10 = \pm 2$

$\Rightarrow\ a = 12, 8$

20. (B)

$\alpha^2 - 3\alpha = -5$

$\beta^2 - 3\beta = -5$

$\therefore$ Roots are 2, 2.