IIT – JEE: 2019 **TW TEST (ADV)** **DATE: 04/03/18**

ANSWER KEY

PHYSICS : GRAVITATION

1.	**(D)**	**2.**	**(A)**	**3.**	**(B)**	**4.**	**(B)**	**5.**	**(C)**
6.	**(ACD**	**7.**	**(AD)**	**8.**	**(AD)**	**9.**	**(AD)**	**10.**	**(BD)**
11.	**(BCD)**	**12.**	**(C)**	**13.**	**(ABCD)**	**14.**	**(C)**	**15.**	**(ABD)**
16.	**(2)**	**17.**	**(0)**	**18.**	**(3)**	**19.**	**(8)**	**20.**	**(2)**

CHEMISTRY : VOLUMETRIC

21.	**(C)**	**22.**	**(A)**	**23.**	**(C)**	**24.**	**(B)**	**25.**	**(A)**
26.	**(CD**	**27.**	**(ACD)**	**28.**	**(AC)**	**29.**	**(ABC)**	**30.**	**(CD)**
31.	**(AB)**	**32.**	**(BCD)**	**33.**	**(ACD)**	**34.**	**(ABD)**	**35.**	**(BCD)**
36.	**(4)**	**37.**	**(5)**	**38.**	**(2)**	**39.**	**(2)**	**40.**	**(3)**

MATHEMATICS : LIMIT

41.	**(B)**	**42.**	**(D)**	**43.**	**(A)**	**44.**	**(B)**	**45.**	**(A)**
46.	**(ABCD)**	**47.**	**(BC)**	**48.**	**(ABC)**	**49.**	**(AD)**	**50.**	**(BC)**
51.	**(AC)**	**52.**	**(BC)**	**53.**	**(ACD)**	**54.**	**(AB)**	**55.**	**(BCD)**
56.	**(3)**	**57.**	**(0)**	**58.**	**(4)**	**59.**	**(2)**	**60.**	**(2)**

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TOPIC: GRAVITATION

SOLUTION

1. **(D)**

From modified Gauss's theorem for gravitation

$$\int \vec{E}.d\vec{s} = 4\pi G\left(\int_{r=0}^{r=r} \rho\, dv\right) \Rightarrow E4\pi r^2 = 4\pi G\int_{r=0}^{r=r} \frac{k}{r} 4\pi r^2\, dr$$

get E = constant

2. **(A)**

as E is constant, so the Potential $\left(V = -\int E\, dr\right)$ will be proportional to r

3. **(B)**

4. **(B)**

$$\Delta U = m\left(V_f - V_i\right)$$

$$\Delta U = m\left(\frac{-GM}{(4R)} - \left(\frac{-GM}{R}\right)\right) = \frac{3}{4}m\left(\frac{GM}{R}\right) = \frac{3}{4}mR\left(\frac{GM}{R^2}\right) = mRg \times \frac{3}{4}$$

5. **(C)**

6. **(ACD**

The reading of the beam balance will be independent of effective g, so $W_b = W'_b$ but the reading of the spring balance will Proportional to $g_{effactive}$

At equator due to centrifugal force of earth, $g_{effactive}$ is less so Reading of spring ballance is less $W_s < W'_b$

7. **(AD)**

8. **(AD)**

9. **(AD)**

10. **(BD)**

11. **(BCD)**

12. **(C)**

13. **(ABCD)**

14. **(C)**

15. **(ABD)**

16. **(2)**

$$g = \frac{GM}{R^2}$$

$$\Rightarrow \quad \frac{dg}{g} = -2\frac{dR}{R}$$

$$\frac{dR}{R} = -1\% \quad \Rightarrow \quad \frac{dg}{g} = 2\%$$

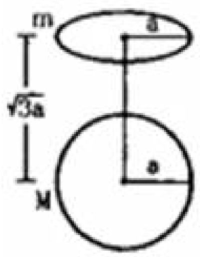
17. **(0)**

18. **(3)**

The acceleration due to gravity at earth's surface is g and at a distance R from earth's surface it is g/9. Hence

19. **(8)**

The gravitational field at any point on the ring due to the sphere is equal to the field due to a single particle of mass M placed at the centre of the sphere. Thus, the force on the ring due to the sphere is also equal to the force on it by a particle of mass M placed at this point. By Newton's third law it is equal to the force on the particle by the ring. Now the gravitational field due to the ring at a distance $d = \sqrt{3}a$ on its axis is



$$E = \frac{Gmd}{\left(a^2 + d^2\right)^{3/2}} = \frac{\sqrt{3}Gm}{8a^2}$$

The force on a particle of mass M placed here is

$$F = ME$$

$$= \frac{\sqrt{3}GMm}{8a^2}$$

This is also the force due to the sphere on the ring.

20. **(2)**

$$F = \frac{Gm(M-m)}{r^2}$$

For maximum force $\frac{dF}{dm} = 0$

$$\Rightarrow \frac{d}{dm}\left(\frac{GmM}{r^2} - \frac{Gm^2}{r^2}\right) = 0$$

$$\Rightarrow M - 2m = 0 \Rightarrow \frac{M}{m} = 2$$

SOLUTION

41. **(B)**

$$\lim_{n\to\infty}\sum_{r=1}^{n}\frac{n}{n^2+r^2}=\lim_{n\to\infty}\sum_{r=1}^{n}\frac{n}{n^2\left(1+\frac{r^2}{n^2}\right)}$$

$$=\int_0^1\frac{dx}{1+x^2}=\left[\tan^{-1}x\right]_0^1=\frac{\pi}{4}$$

42. **(D)**

$$f(x)=\begin{cases}\frac{\tan x^2}{x^2} & \text{left neighbourhood of } x=0\\ \sin x & \text{right neighbourhood of } x=a\end{cases}$$

$\therefore \lim_{x\to0^+} f(x)=0 \;\&\; \lim_{x\to0^-} f(x)=-1$

$\therefore$ Limit does not exist.

43. **(A)**

$$\lim_{n\to\infty}\left[\lim_{x\to0}\left(1+\sum_{k=1}^{n}\sin^2(kx)\right)^{\frac{1}{n^3x^2}}\right]=\lim_{n\to\infty}\left[\lim_{x\to0}\left(1+\sum_{k=1}^{n}\sin^2(kx)\right)^{\frac{1}{\sum_{k=1}^{n}\sin^2(kx)}}\right]^{\frac{\sum_{k=1}^{n}\sin^2(kx)}{n^3x^2}}$$

$$=\lim_{n\to\infty}\left[e^{\frac{1}{n^3}\lim_{x\to0}\frac{\sum_{k=1}^{n}\sin^2(kx)}{x^2}}\right]$$

$$=e^{\lim_{n\to\infty}\frac{1^2+2^2+.....+n^2}{n^3}}$$

$$=e^{\lim_{n\to\infty}\frac{(n+1)(2n+1)}{6n^2}}$$

$$=\sqrt[3]{e}$$

44. **(B)**

$$\lim_{x\to0}\frac{\frac{x^2}{\sqrt{a+x}}}{b-\cos x}=1$$

$\therefore\ b=1$

And $\lim\limits_{x\to 0}\dfrac{x^2}{\sqrt{a}\left(1-\cos x\right)}=1$

$\Rightarrow \dfrac{2}{\sqrt{a}}=1$

$\Rightarrow a=4$

45. (A)

$\sqrt{2}=2\cos\dfrac{\pi}{4}$

$\sqrt{2+\sqrt{2}}=\sqrt{2+2\cos\dfrac{\pi}{4}}=2\cos\pi/8$

$\sqrt{2+\sqrt{2+\sqrt{2}}}=\sqrt{2+2\cos\dfrac{\pi}{8}}=2\cos\dfrac{\pi}{16}$

$\sqrt{2+\sqrt{2+\text{................}}}=2\cos\dfrac{\pi}{2^{n+1}}$

$2.\cos\dfrac{\pi}{2^{n+1}}.2\cos\dfrac{\pi}{2^{n}}......................2\cos\dfrac{\pi}{2^{2}}=\dfrac{L}{2^{n}}$

Multiply divide by $\sin\dfrac{\pi}{2^{n+1}}$

$$\frac{\sin\frac{\pi}{2}}{\sin\frac{\pi}{2^{n+1}}}=\frac{L}{2^{n}}$$

If $n\to\infty$

$$\frac{1}{\dfrac{\sin\left(\frac{\pi}{2^{n+1}}\right)}{\left(\frac{\pi}{2^{n+1}}\right)}}\cdot\frac{\pi}{2^{n+1}}=\frac{L}{2^{n}}$$

$L=\dfrac{\pi}{2}$

46. (ABCD)

$$\lim_{n\to\infty}n^{k}\left(a^{\frac{1}{n}}-1\right)\left(\sqrt{\frac{n-1}{n}}-\sqrt{\frac{n+1}{n+2}}\right)=\lim_{n\to\infty}\frac{-n^{k}\left(a^{\frac{1}{n}}-1\right)}{n\left(n+2\right)}.\lim_{n\to\infty}\frac{2}{\sqrt{\frac{n-1}{n}}+\sqrt{\frac{n+1}{n+2}}}$$

$$=\lim_{n\to\infty}\frac{-n^{k-1}}{n\left(n+2\right)}.\lim_{n\to\infty}\frac{a^{\frac{1}{n}}-1}{\frac{1}{n}}$$

$$=\ln a.\lim_{n\to\infty}\frac{-n^{k-2}}{n+2}$$

$$=\begin{cases} 0 & \text{if } k\in\{0,1,2\} \\ -\ln a & \text{if } k=3 \\ \infty & \text{if } k\geq 4 \text{ and } a\in(0,1) \\ -\infty & \text{if } k\geq 4 \text{ and } a>1 \end{cases}$$

47. **(BC)**

$\lim_{x\to 0}\left[f(x)\right]$

$=\lim_{x\to 0}\left[\frac{f(x)}{x^2}\cdot x^2\right]$

$=0$

And $\lim_{x\to 0}\left[\frac{f(x)}{x}\right]$

$=\lim_{x\to 0}\left[\frac{f(x)}{x^2}\cdot x\right]$

$\therefore\ \lim_{x\to 0^+}\left[\frac{f(x)}{x^2}\cdot x\right]=0$

And $\lim_{x\to 0^-}\left[\frac{f(x)}{x^2}\cdot x\right]=-1$

48. **(ABC)**

(A) limit doesn't exist at x = 1, 2, 3, 4, 5

(B) f(x)=1 for all $x\in(0,6)$

(C) limit doesn't exist at $x=\frac{1}{6},\frac{5}{6},\frac{13}{6},\frac{17}{6},\frac{25}{6},\frac{29}{6}$

(D) if a=0 function is discontinuous at many point but limit still exist

49. **(AD)**

$\lim_{x\to 0}\frac{\sin[x]}{[x]}=\lim_{x\to 0^-}\frac{\sin[x]}{[x]}$

$=\sin 1$

$\because 0^+$ is not in domain and $\lim_{x\to 0}\sin^{-1}\left(e^{|x|}\right)$

0^- & 0^+ both are not in domain

$\lim_{x\to\infty}F(x)=e^{\lim_{x\to\infty}\ln F(x)}$

$=e^{\left[\lim_{x\to\infty}\frac{\ln\left(P_1a_1^{\,x}+P_2a_2^{\,x}+........P_na_n^{\,x}\right)}{x}\right]}$

$=e^{\left[\lim_{x\to\infty}\frac{P_1a_1^{\,x}\ln a_1+........P_n\left(a_n^{\,x}\right)\ln a_n}{P_1a_1^{\,x}+P_2a_2^{\,x}+........P_na_n^{\,x}}\right]}$

$=e^{\ln a_1}=a_1$

Similarly $\lim_{x\to-\infty}F(x)=a_n$

50. **(BC)**

51. **(AC)**

$$(A) e^{\lim_{x\to\pi}\frac{\sec 2x-1}{(x-\pi)^2}} = e^{\lim_{x\to\pi}\frac{1-\cos 2x}{(x-\pi)^2.\cos 2x}}$$

$$= e^{\lim_{x\to\pi}\frac{1-\cos(2x-2\pi)}{(x-\pi)^2.\cos 2x}} = e^{\frac{1}{2}.\frac{4}{1}} = e^2$$

$$(C) \lim_{x\to\infty}\left(1+\frac{2x-1}{x^2-4x+2}\right)^x = e^{\lim_{x\to\infty}\frac{x(2x-1)}{x^2-4x+2}} = e^2$$

$$(D) e^{\lim_{x\to 0}\frac{\cos x-1}{x^2}} = e^{-1/2}$$

52. (BC)

$$\lim_{x\to 0}\frac{(\cos x-1)(\cos x-e^x)-\frac{x^3}{2}}{x^n}$$

$$= \lim_{x\to 0}\frac{\left(-\frac{x^2}{2!}+\frac{x^4}{4!}....\right).\left(-x-x^2-\frac{x^3}{3!}-\frac{x^5}{5!}....\right)-\frac{x^3}{2}}{x^n}$$

$$= \lim_{x\to 0}\frac{\frac{x^4}{2}+\frac{x^5}{24}...}{x^n}$$

$$\Rightarrow n = 4, l = 1/2$$

53. (ACD)

$$\lim_{x\to 0}\left(2+\frac{f(x)}{x^2}-1\right)\frac{1}{x} = 2$$

$$\lim_{x\to 0}\left(\frac{a_0+a_1x+(a_2+1)x^2+a_3x^3+a_4x^4+....}{x^3}\right) = 2$$

If limits exists then

$a_0 = 0, a_1 = 0, a_2 = -1, a_3 = 2$

54. (AB)

$$f(0^-) = \lim_{h\to 0}\frac{\ln(\{-\sinh\}.\{\cosh\}+1)}{\{-\sinh\}.\{\cosh\}} = \lim_{h\to 0}\frac{\ln((1-\sinh).(\cosh)+1)}{(1-\sinh).(\cosh)} = \ln 2$$

$$f(0^+) = \lim_{h\to 0}\frac{\ln(\{\sinh\}.\{\cosh\}+1)}{\{\sinh\}.\{\cosh\}} = \lim_{h\to 0}\frac{\ln((\sinh.\cosh)+1)}{(\sinh.\cosh)} = 1$$

$$f\left(\frac{\pi}{2}^+\right) = \lim_{h\to 0}\frac{\ln\left(\left\{\sin\left(\frac{\pi}{2}+h\right)\right\}.\left\{\cos\left(\frac{\pi}{2}+h\right)\right\}+1\right)}{\left\{\sin\left(\frac{\pi}{2}+h\right)\right\}.\left\{\cos\left(\frac{\pi}{2}+h\right)\right\}} = \lim_{h\to 0}\frac{\ln(\{-\sinh\}.\{\cosh\}+1)}{\{-\sinh\}.\{\cosh\}} = f(0^-)$$

$$f\left(\frac{\pi}{2}^-\right) = \lim_{h\to 0}\frac{\ln\left(\left\{\sin\left(\frac{\pi}{2}-h\right)\right\}.\left\{\cos\left(\frac{\pi}{2}-h\right)\right\}+1\right)}{\left\{\sin\left(\frac{\pi}{2}-h\right)\right\}.\left\{\cos\left(\frac{\pi}{2}-h\right)\right\}} = \lim_{h\to 0}\frac{\ln(\{\sinh\}.\{\cosh\}+1)}{\{\sinh\}.\{\cosh\}} = f(0^+)$$

55. **(BCD)**

$$\lim_{x\to\infty} f(x) = \lim_{x\to\infty}\left(\frac{a+1/x}{b+2/x}\right)^x$$

$$if\, 0<a<b, \frac{a}{b}<1, \lim_{x\to\infty} f(x) = \frac{a}{b}^{\infty} = 0$$

$$fa>b>0,, \frac{a}{b}>1 \lim_{x\to\infty} f(x) = \frac{a}{b}^{\infty} = \infty$$

$$ifa=b, \lim_{x\to\infty} f(x) = \lim_{x\to\infty}\left(1-\frac{1}{ax+2}\right)^x = e^{\lim_{x\to\infty}\frac{-x}{ax+2}} = e^{-1/a}$$

56. **(3)**

$$\lim_{x\to\infty} n^2.\frac{4}{7}.\frac{5}{8}.\frac{6}{9}.\frac{7}{10}............\left(\frac{n-5}{n-2}\right)\left(\frac{n-4}{n-1}\right)\left(\frac{n-3}{n}\right)$$

$$=\lim_{x\to\infty} 120\frac{n^3}{(n-2)(n-1)n}$$

$$\therefore\ k=120$$

57. **(0)**

$$\lim_{n\to\infty}\cos\pi n\left(1+\frac{1}{n}\right)^{1/2}$$

$$=\lim_{n\to 0}\cos\pi n\left(1+\frac{1}{2n}\right)$$

$$=\lim_{n\to 0}\cos\left(\pi n+\frac{n}{2}\right)$$

$$=0$$

58. **(4)**

$$\sum_{k=1}^{n}\left(\frac{1}{(2k-1)^2}-\frac{3}{4k^2}\right)=\sum_{k=1}^{n}\left(\frac{1}{(2k-1)^2}+\frac{1}{(2k)^2}-\frac{1}{k^2}\right)$$

$$=\sum_{k=1}^{2n}\frac{1}{k^2}-\sum_{k=1}^{n}\frac{1}{k^2}$$

$$=\sum_{k=n+1}^{2n}\frac{1}{k^2} \qquad (1)$$

Using partial fractions and summing the telescoping series, we get

$$\frac{1}{n+1}-\frac{1}{2n+2}=\sum_{k=n+1}^{2n}\frac{1}{k(k+1)}\le\sum_{k=n+1}^{2n}\frac{1}{k^2}\le\sum_{k=n+1}^{2n}\frac{1}{k(k-1)}=\frac{1}{n}-\frac{1}{2n} \qquad (2)$$

Therefore, the Squeeze Theorem and (2) yield

$$\lim_{n\to\infty} n\sum_{k=n+1}^{2n}\frac{1}{k^2}=\frac{1}{2}=\lambda$$

59. **(2)**

60. **(2)**

Telescoping, we have

$$\lim_{n\to\infty}\left(n^2+n-\sum_{k=1}^{n}\frac{2k^3+8k^2+6k-1}{k^2+4k+3}\right)=\lim_{n\to\infty}\left(n^2+n-2\sum_{k=1}^{n}k+\frac{1}{2}\sum_{k=1}^{n}\frac{1}{k+1}-\frac{1}{2}\sum_{k=3}^{n}\frac{1}{k+3}\right)$$

$$=\frac{1}{2}\lim_{n\to\infty}\left(\sum_{k=1}^{n}\frac{1}{k+1}-\sum_{k=1}^{n}\frac{1}{k+3}\right)$$

$$=\frac{5}{12}-\frac{1}{2}\lim_{n\to\infty}\left(\frac{1}{n+2}+\frac{1}{n+3}\right)$$

$$=\frac{5}{12}$$