## Answer Key & Solution

1. (C)
Non-superimposable mirror images.

2. (C)
Structural isomers.

3. (C)
14 sigma and one pi bonds.

4. (D)
Cis Form.

5. (B)
Gamma H will participate.

6. (D)
No plane of symmetry.

7. (D)

8. (D)
Degree of unsaturation different.

9. (A)
Geometrical isomers.

10. (C)
Double bond carbon.

11. (A)
$HCOOC_2H_5$ and $CH_3COOH_3$

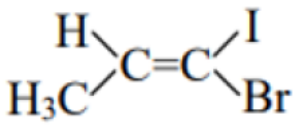
12. (D)
Double bond in the ring having more than 7 C atoms.

13. (D)
Identical

14. (A)
Neither superimposable nor mirror images.

15. (A)

16. (C)
Structural isomers.

17. (B)


18. (B)
Disubstituted ring.

19. (B)
Bottom most part decides D and L.

20. (B)
No alpha H.

21. (2)
Total 3 and one is meso.

22. (4)
Amines containing -NH2 group.

23. (4)
Two Chiral Carbon.

24. (8)
Three stereo-sites.

25. (7)
Four different groups.

26. (3)
Two terminal and one internal.

27. (5)
Benzene itself has 4 degree of unsaturation.

28. (3)
Structural and Cis trans.

29. (3)

30. (0)
It is meso compound.

## Answers & Solutions

**31. (B)**

$e = \dfrac{SS'}{PS + PS'}$ $\quad S(3,\ 4)$ $\quad S'(4, 3)$ $\quad P(0, 0)$

$e = \dfrac{\sqrt{2}}{5+5}$

$e = \dfrac{\sqrt{2}}{2\times 5} = \dfrac{1}{5\sqrt{2}}$

**32. (A)**

Eccentricity angle = $\theta$

$\left(\sqrt{6}\cos\theta, \sqrt{2}\sin\theta\right)$

$d = \sqrt{5}$

$6\cos^2\theta + 2\sin^2\theta = 5$

$6 - 4\sin^2\theta = 5$

$1 = 4\sin^2\theta \Rightarrow \sin\theta = \pm\dfrac{1}{2}$

$\Rightarrow \quad \tan\theta = \pm\dfrac{1}{\sqrt{3}}$

**33. (C)**

$T = 0$

$\dfrac{xh}{2} + yk - 1 = 0$

$x + 3y - 1 = 0$

$\dfrac{\frac{h}{2}}{1} = \dfrac{K}{3} = \dfrac{-1}{-1}$

$h = 2 \qquad K = 3$

**34. (C)**

$\dfrac{xx_1}{a^2} + \dfrac{yy_1}{b^2} = 1$ $\qquad$ $\dfrac{xx_2}{a^2} + \dfrac{yy_2}{b^2} = 1$

$m_1 = -\dfrac{x_1}{y_1}\times\dfrac{b^2}{a^2}$ $\qquad$ $m_2 = -\dfrac{x_2}{y_2}\times\dfrac{b^2}{a^2}$

$m_1 m_2 = -1$

$\dfrac{x_1 x_2}{y_1 y_2} = -\dfrac{a^4}{b^4}$

**35. (D)**

$\dfrac{x^2}{4} + y^2 = 1$ $\qquad$ $x^2 + \dfrac{y^2}{a^2} = 1$

$y^2 = a^2\left(1 - x^2\right)$

$x^2 + 4y^2 = 4$

$x^2 + 4a^2 - 4a^2x^2 = 4$

$x^2\left(1 - 4a^2\right) = 4 - 4a^2$

$x \in (1, \infty)$

$b^2 - 5b + 7 > 1$

$b^2 - 5b + 6 > 0$

$(b-2)(b-3) > 0$

$b \in (-\infty, 2) \cup (3, \infty)$

**36. (B)**

$25\left[(x-2) + (y+3)^2\right] = \dfrac{25}{4} \times \dfrac{(3x - 4y + 7)^2}{25}$

$\sqrt{(x-2)^2 + (y+3)^2} = \dfrac{1}{2}\left|\dfrac{3x - 4y + 7}{5}\right|$

$PS = ePM$

$e = \dfrac{1}{2}$

$\left(\dfrac{a}{e} - ae\right) = \left|\dfrac{3(2) - 4(-3) + 7}{5}\right|$

$a\left(\dfrac{3}{2}\right) = \dfrac{6 + 19}{5} = 5$

$a = \dfrac{10}{3} \Rightarrow 2a = \dfrac{20}{3}$

**37. (B)**

$x = t^2$ $\qquad$ $x^2 + \dfrac{y^2}{9} = 1$

$y^2 = 9\left(1 - x^2\right) \geq 0$

$1 - t^2 > 0$

$t^2 < 1$

$|t| < 1$

**38. (B)**

$S_1 + \lambda S_2 = 0$

$$\left(\frac{x^2}{4}+y^2-1\right)+\lambda\left(\frac{x^2}{a^2}+y^2-1\right)=0$$

$$x^2\left(\frac{1}{4}+\frac{\lambda}{a^2}\right)+y^2\left(1+\lambda\right)=1+\lambda$$

$$x^2\left(\frac{a^2+4\lambda}{4a^2\left(1+\lambda\right)}\right)+y^2=1$$

$$\Rightarrow \quad \frac{a^2+4\lambda}{4a^2\left(1+4\lambda\right)}=1$$

$$a^2+4\lambda=4a^2+16\lambda a^2$$

So circle is $x^2+y^2=1$

**39. (A)**

$R\left(h,k\right)$

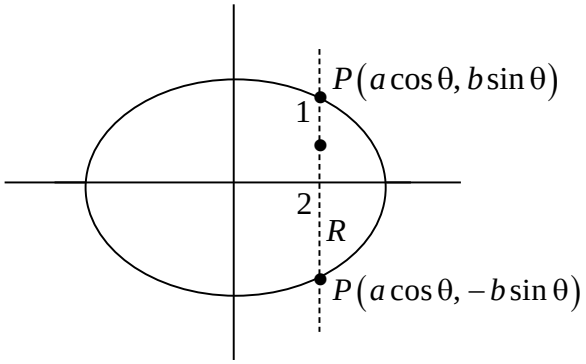
$h=a\cos\theta$

$K=\frac{b\sin\theta}{3}$

$\cos^2\theta+\sin^2\theta=1$

$$\left(\frac{h}{a}\right)^2+\left(\frac{3k}{b}\right)^2=1$$

$$\frac{x^2}{a^2}+\frac{9y^2}{b^2}=1$$




**40. (A)**

$x^2+4y^2=4$ and $x^2+1=y$

$$x^2=y-1$$

$y-1+4y^2=4$

$4y^2+y-5=0$

$4y^2+5y-4y-5=0$

$y\left(4y+5\right)-1\left(4y+5\right)=0$

$y=1$ or $y=-\frac{5}{4}$

$x^2=y-1>0$

$y>1$

$x=0$ or (0, 1)

$a\cos\theta, b\sin\theta$

**41. (A)**

Tangent $\frac{x}{a}\cos\theta+\frac{y}{b}\sin\theta=1$

$x$-intercept $x=a\sec\theta$

$y$-intercept $y=b\,\text{cosec}\,\theta$

$$A=\frac{1}{2}ab\frac{1}{\sin\theta\cos\theta}$$

$$= \frac{ab}{\sin 2\theta}$$

$$(\sin 2\theta)_{max} = 1$$

$$A_{min} = ab$$

**42. (C)**

$y = mx + \sqrt{9m^2 + 4}$

Passes trough (3, 4)

$4 = 3m + \sqrt{9m^2 + 4}$

$\Rightarrow \quad m = \frac{1}{2}, \infty$

Equation of tangents $y - 4 = \frac{1}{2}(x - 3)$

$x - 2y + 5 = 0$ and $x = 3$ orthocentre $\left(\frac{11}{5}, \frac{8}{5}\right)$

**43. (B)**

$$\left(\frac{x-1}{4}\right)^2 + \left(\frac{y-2}{3}\right)^2 = 1$$

$d = 2ae$

$$= 2 \times 4 \times \sqrt{1 - \frac{9}{16}}$$

$$= 2\sqrt{7}$$

**44. (B)**

$y = mx + \sqrt{25m^2 + 4}$

$y = mx + 4\sqrt{1 + m^2}$

$16 + 16m^2 = 25m^2 + 4$

$12 = 9m^2$

$m^2 = \frac{12}{4} = \frac{4}{3}$

$m = \pm \frac{2}{\sqrt{3}}$

**45. (A)**

$T = 0$ $\qquad\qquad$ $T = S_1$

$3x + 8y = 9$ $\qquad\qquad$ $xx_1 + 4yy_1 = x_1^2 + 4y_1^2$

$\frac{x_1}{3} = \frac{4y_1}{8} = \frac{x_1^2 + 4y_1^2}{9}$

**46. (D)**

**47. (B)**

**48.** **(A)**

Apply $SS_1 = T^2$

$$\tan\theta = \frac{2\sqrt{h^2 - ab}}{a+b}$$

**49.** **(C)**

$$\frac{1}{PS} + \frac{1}{SQ} = \frac{2}{\left(\frac{b^2}{a}\right)}$$

**50.** **(B)**

$$\frac{2b^2}{a} = \frac{1}{2}(2a)$$

$$a^2 = 2b^2$$

$$e = \sqrt{1 - \frac{b^2}{a^2}}$$

$$= \sqrt{1 - \frac{1}{2}} = \frac{1}{\sqrt{2}}$$

**51.** **(2)**

$$\frac{1}{PS} + \frac{1}{SQ} = \frac{2}{\left(\frac{b^2}{a}\right)} \qquad \frac{x^2}{25} + \frac{y^2}{16} = 1$$

$$\frac{1}{8} + \frac{1}{SQ} = \frac{5\times 2}{16}$$

$$\frac{1}{SQ} = \frac{5\times 2}{16} - \frac{1}{8} = \frac{8}{16} = \frac{1}{2}$$

$$SQ = 2$$

**52.** **(24)**

$$y = mx + \sqrt{a^2m^2 + b^2}$$

$$y = mx + \sqrt{18m^2 + 32}$$

$$m = -\frac{4}{3}$$

$$y = -\frac{4}{3}x + 8$$

Point on $x$-axis $\qquad y = 0 \Rightarrow x = 6 \quad (6, 0)$

Point on $y$-axis $\qquad x = 0 \Rightarrow y = 8 \quad (0, 8)$

$$\text{Area} = \frac{1}{2}\times 6\times 8$$

= 24 sq. units

**53.** **(4)**

$y = mx \pm \sqrt{a^2x^2 + b^2} \qquad m = 2$

$y = 2x \pm \sqrt{4a^2 + b^2}$

If is normal to curve $x^2 + y^2 + 4x + 1 = 0$ s

So passes through center (–2, 0)

$0 = -4 \pm \sqrt{4a^3 + b^2}$

$\Rightarrow \quad 4a^2 + b^2 = 16$

$\frac{4a^2 + b^2}{2} \geq \left(4a^2b^2\right)^{\frac{1}{2}}$

$\frac{16}{2} \geq 2ab$

$4 \geq ab$

**54.** **(3)**

$\frac{x^2}{169} + \frac{y^2}{25} = 1$

Equation of normal at the point $\left(13\cos\theta, 5\sin\theta\right)$ is

$\frac{13x}{\cos\theta} - \frac{5y}{\sin\theta} = 144$, it passes through (0, 6)

$\Rightarrow \quad \sin\theta = -\frac{5}{24}$

$\Rightarrow \quad \theta = \pi + \sin^{-1}\frac{5}{24}$ or $2\pi - \sin^{-1}\frac{5}{24}$ and $y$-axis is one of normal

**55.** **(10)**

$PF_1 + PF_2 = \alpha a$

$= 10$

**56.** **(4)**

$y = mx \pm \sqrt{a^2m^2 + b^2}$

$y = 2x \pm \sqrt{4a^2 + b^2}$

Normal to $x^2 + y^2 + 4x + 1 = 0$

Centre (–2, 0)

$0 = -4 \pm \sqrt{4a^2 + b^2}$

$4a^2 + b^2 = 16$

A.M. $\geq$ G.M.

$\frac{4a^2 + b^2}{2} \geq \left(4a^2b^2\right)^{\frac{1}{2}}$

$8 \geq 2ab$

$4 \geq ab$

**57.** **(8)**

Equation of chord with given mid point $T = S_1 \ \left(0, 3\right)$

$\frac{3y}{25}-1=\frac{9}{25}-1 \Rightarrow y=3$

Intercept the ellipse $\frac{x^2}{16}+\frac{y^2}{25}=1$

$\frac{x^2}{16}+\frac{9}{25}=1$

$x=\pm\frac{16}{5}$

$\therefore$ Length of chord $=\frac{32}{5}=\frac{4K}{5}$

$K=8$

**58. (8)**

**59. (2)**

**60. (2)**

Either points are $(\pm a, 0)$ or points are $(0, \pm b)$

Since $\sqrt{\frac{a^2+2b^2}{2}}>b$

$\Rightarrow \sqrt{\frac{a^2+2b^2}{2}}=a$

$a^2+2b^2=2a^2$

$a^2=2b^2$

$\frac{b^2}{a^2}=\frac{1}{2}$

$e=\sqrt{1-\frac{b^2}{a^2}}=\sqrt{1-\frac{1}{2}}$

$e=\frac{1}{\sqrt{2}}$