1. Solve for \( x \); 
\[
\left( x^2 + x \right)^2 - 8\left( x^2 + x \right) + 12 = 0
\]

2. Find the value of \( a \), if the polynomial \( x^3 + 9x^2 + 26x + a \) is exactly divisible by \( x + 4 \)?

3. Find the smallest natural number by which 8640 must be multiplied, so that it becomes a perfect square?

4. Express 3.27 as a rational number?

5. If \( a, b, c \) are consecutive natural numbers, then \( 2^a + 2^b + 2^c \) is divisible by 7. State whether this statement is true or false? Give reasons in support of your answer.

6. What are the values of angles \( \theta \) and \( \alpha \) in the given figure?

7. In an isosceles triangle ABC, AB=AC. If \( \angle ACB = 30^\circ \), find the value of \( \angle BAC \)?

8. Factorise \( 8x^3 - 8x^2 - 9 \)?

9. If \( x^3 -(a + 4)x^2 + (4a + c)x + d \) is divisible by \( x-a \), then find the value of ‘\( a \)’, in terms of \( c \) and \( d \)?

10. A circle has circumference of \( \pi \) units, find its area?

11. Solve for \( x \):
\[
\frac{4}{x-1} - \frac{5}{x+2} = \frac{3}{x}
\]

12. Divide \( \left( x^4 + x^3 - 9x^2 - 3x + 5 \right) \) by \( x^2 + 4x + 2 \)?

13. If \( 4x - 5y = 1 \) and \( xy = 2 \), find the value of \( 64x^3 - 125y^3 \)?

14. Solve for \( x \):
\[
\frac{x}{x+1} + \frac{x+1}{x} = \frac{169}{60}
\]

15. If \( x + \frac{1}{x} = 7 \), find the value of \( x^2 + \frac{1}{x^2} \)?
Q. 1.

\[(x^2 + x)^2 - 8 (x^2 + x) + 12 = 0 \quad (1)\]

Let \( (x^2 + x) = a \).

Substituting it in equation (1), we get,

\[a^2 - 8a + 12 = 0\]
\[a^2 - 6a - 2a + 12 = 0\]
\[\therefore a(a-6) - 2(a-6) = 0\]
\[\therefore (a-2)(a-6) = 0\]
\[\therefore a = 2 \text{ or } a = 6.\]

Now resubstituting the value of \( a = x^2 + x \), we have

\[x^2 + x = 2 \text{ or } x^2 + x = 6\]
\[\therefore x^2 + x - 2 = 0 \text{ or } x^2 + x - 6 = 0\]
\[\therefore x^2 + 2x - x - 2 = 0 \text{ or } x^2 + 3x - 2x - 6 = 0\]
\[\therefore x(x+2) - 1(x+2) = 0 \text{ or } x(x+3) - 2(x+3) = 0\]
\[\therefore (x-1)(x+2) = 0 \text{ or } (x-2)(x+3) = 0\]
\[\therefore x = 1, -2 \text{ or } x = 2, -3.\]

\[\therefore \text{ Solution set } = \{-3, -2, 1, 2\} \]

Q. 2. \( p(x) = x^3 + qx^2 + 26x + a \)

It is given that \( p(x) \) is divisible by \((x+4)\).

By Remainder theorem,

\[p(-4) = 0\]
\[\therefore p(-4) = (-4)^3 + q(-4)^2 + 26(-4) + a = 0\]
\[\therefore -64 + 16q - 104 + a = 0\]
\[\therefore a = 24.\]
6.3. \[ 8640 = 2 \times 4320 \]
\[ = 2 \times 2 \times 2160 \]
\[ = 2 \times 2 \times 2 \times 1080 \]
\[ = 2 \times 2 \times 2 \times 2 \times 540 \]
\[ = 2 \times 2 \times 2 \times 2 \times 2 \times 270 \]
\[ = 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 135 \]
\[ = 2^6 \times 3 \times 45 \]
\[ = 2^6 \times 3^2 \times 15 \]
\[ = 2^6 \times 3^2 \times 3 \times 5 \]

\[ \therefore \text{To make 8640, a perfect square, we will have to multiply it by } 3 \times 5 = 15. \]

8.4.

\[ \frac{3.7}{\text{a}} \]
\[ x = \frac{3.727272727}{100} = \frac{37.272727}{100} \]
\[ \frac{x}{100} = \frac{37.2727}{100} \]
\[ 50x = \frac{3727.27}{10} \]
\[ x = \frac{3727.27}{50} \]
\[ x = \frac{3727}{50} \]
\[ x = \frac{36}{10} \]

8.5. \(a, \ b\) and \(c\) are consecutive natural numbers.

\[ \therefore a = a, \quad b = a + 1, \quad c = a + 2. \]

We have \[ 2^a + 2^b + 2^c \]
\[ = 2^a + 2^a + 1 + 2^a + 2 \]
\[ = 2^a + 2 \times 2^a + 2^a \times 2^2 \]
\[ = 2^a (1 + 2 + 4) \]
\[ = 2^a \times 7 \quad \rightarrow \text{This number is definitely divisible by } 7 \text{ as } 7 \text{ is one of the factors.} \]

\[ \therefore \text{The given statement is true.} \]
Q.6. From the given figure,

\[ \angle ZABE \]
\[ \angle \angle \text{ACD} + \alpha = 180^\circ \]
\[ \text{...(pair of linear angles)} \]
\[ \therefore 110^\circ + \alpha = 180^\circ \]
\[ \therefore \alpha = 180^\circ - 110^\circ = 70^\circ. \]

Now in \( \triangle ABC \),
\[ m\angle \text{ACD} = m\angle \text{CAB} + m\angle \text{ABC} \]
\[ \therefore 110^\circ = 30^\circ + \theta \]
\[ \therefore \theta = 110^\circ - 30^\circ = 80^\circ \]
\[ \therefore \alpha = 70^\circ, \quad \theta = 80^\circ. \]

Q.7. In \( \triangle ABC \), \( \overline{AB} = \overline{AC} \).
\[ \therefore \angle ABC = \angle ACB \] (opposite angles to congruent sides of isosceles triangle)
\[ \therefore \text{We have} \]
\[ m\angle ABC + m\angle ACB + m\angle BAC = 180^\circ \]
\[ \text{...(sum of all angles in a triangle)} \]
\[ \therefore 30^\circ + 30^\circ + m\angle BAC = 180^\circ \]
\[ \therefore m\angle BAC = 120^\circ. \]

Q.8. 
\[ 8x^3 - 8x^2 - 9 \]
\[ = 8x^3 - 8x^2 - 8 - 1 \]
\[ = 8x^3 - 8 - 8x^2 - 1 \]
Q.9. \( p(x) = x^3 - (a+4)x^2 + (4a+c)x + d \)

\( p(a) \) is divisible by \( (x-a) \).

\( \therefore \ p(a) = 0 \)

\( \therefore \ a^3 - (a+4)a^2 + (4a+c)a + d = 0 \)

\( \therefore \ a^2 - a^2 - 4a^2 + 4a^2 + ac + d = 0 \)

\( \therefore \ ac = -d \)

\( \therefore \ \frac{a}{c} = -\frac{d}{c} \)

Q.10. Circumference of circle = \( \pi \) units

\( 2\pi r = \pi \) units

\( \therefore \ r = \frac{1}{2} \) units

\( \therefore \ ) Area of the circle = \( \pi r^2 \)

\( = \pi \times \left(\frac{1}{2}\right) \times \left(\frac{1}{2}\right) \)

\( = \frac{\pi}{4} \) sq. units

Q.11. \( \frac{4}{x-1} - \frac{5}{x+2} = \frac{3}{x} \)

Taking LCM,

\( \frac{4(x+2) - 5(x-1)}{(x-1)(x+2)} = \frac{3}{x} \)

\( \therefore \ \frac{4x + 8 - 5x + 5}{(x-1)(x+2)} = \frac{3}{x} \)

\( \therefore \ \frac{13 - x}{x^2 + x - 2} = \frac{3}{x} \)

\( \therefore \ \alpha (13 - \alpha) = 3(\alpha^2 + \alpha - 2) \)
\[13x - x^2 = 3x^2 + 3x - 6\]
\[4x^2 - 10x - 6 = 0\]
\[2x^2 - 5x + 3 = 0\]
\[2x^2 - 2x - 3x + 3 = 0\]
\[2(x-1) - 3(x-1)\]
\[2x^2 - 6x + x - 3 = 0\]
\[2x(x-3) + 1(x-3) = 0\]
\[(x+1)(x-3) = 0\]
\[x = -1, 3\]

Solution Set = \[\frac{1}{2}, 3\]

Q. 12.

\[
\begin{array}{c}
x^2 - 3x + 1 \\
x^3 + 4x + 2
\end{array}
\]

\[
\begin{array}{c}
x^4 + x^3 - 9x^2 - 3x + 5 \\
x^4 + 4x^3 + 2x^2
\end{array}
\]

\[
\begin{array}{c}
-3x^3 - 11x^2 - 3x \\
-3x^3 + 12x^2 + 6x
\end{array}
\]

\[
\begin{array}{c}
x^2 + 3x + 5 \\
x^2 + 4x + 2
\end{array}
\]

Quotient = \(x^2 - 3x + 1\)

Remainder = \(-x + 3\)

Q. 13. \(4x - 5y = 1\) \(\frac{y}{x} = 2\) \(\text{Given}\)

\[64x^3 - 125y^3 = (4x)^3 - (5y)^3\]

\[= (4x - 5y)[(4x)^2 + (4x)(5y) + (5y)^2]\]

\[= (4x - 5y)^3 + 3(4x)(5y)(4x - 5y)\]

\[= (4x - 5y)^3 + 60(2)(1)\]

\[= 121\]
Q. 14. \[ \frac{x}{x+1} + \frac{x+1}{x} = \frac{169}{60} \]

Let \( \frac{x}{x+1} = a \)

\[ \therefore \ a + \frac{1}{a} = \frac{169}{60} \]

\[ \therefore \ \frac{a^2 + 1}{a} = \frac{169}{60} \]

\[ \therefore \ 60(a^2 + 1) = 169a \]

\[ \therefore \ 60a^2 + 60 = 169a \]

\[ \therefore \ 60a^2 - 169a + 60 = 0 \]

\[ \therefore \ 60a^2 - 144a - 25a + 60 = 0 \]

\[ \therefore \ 12a(5a - 12) - 5(5a - 12) = 0 \]

\[ \therefore \ (12a - 5)(5a - 12) = 0 \]

\[ \therefore \ a = \frac{5}{12} \text{ or } \frac{12}{5} \]

\[ \therefore \ \frac{x}{x+1} = \frac{5}{12} \text{ or } \frac{x}{x+1} = \frac{12}{5} \]

12x = 5x + 5

\[ \therefore \ 7x = 5 \]

\[ \therefore \ x = \frac{5}{7} \]

\[ \text{or} \quad 5x = 12x + 12 \]

\[ \therefore \ -7x = 12 \]

\[ \therefore \ x = \frac{-12}{7} \]

Q. 15. \( \frac{x + \frac{1}{x}}{x^2} = 7 \)

\[ x^2 + \frac{1}{x^2} = (x + \frac{1}{x})^2 - 2 \left( x \cdot \frac{1}{x} \right) = (x + \frac{1}{x})^2 - 2 \]

\[ = (7)^2 - 2 \]

\[ = 47 \]