PRACTICAL – 1
APPLICATIONS OF LOGIC

Q1. Express the following circuit in symbolic form and write its switching table

Solution:

The lamp L will be on if and only if either switches $S_2$ and $S_2$ are closed or $S_3$ is closed if:

- $p$: The switch $S_1$ is closed
- $q$: The switch $S_2$ is closed
- $r$: Switch $S_3$ is closed and $L$: The lamp L is on

Then the circuit in the given diagram can be expressed as $(p \land q) \lor r$.

Switching table is:

<table>
<thead>
<tr>
<th>p</th>
<th>q</th>
<th>r</th>
<th>$(p \land q)$</th>
<th>$(p \land q) \lor r$</th>
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Q2. Give an alternative arrangement of the following circuit such that the new circuit has two switches.

Solution:

From the fig it is clear that if both $S_1$ and $S_2$ are closed or $S_1$ and $S_2$ are closed or both $S_1$ and $S_2$ are closed then the lamp (L) will be on.
If \( p \): The switch \( S_1 \) is closed, \( q \): The switch \( S_2 \) is closed and \( L \): The lamp is on, then the given figure can be expressed as
\[
(- p \land - q) \lor (p \land - q) \lor (- p \land q) \equiv L
\]
i.e.,
\[
L \equiv [(- q \land - p) \lor (- q \lor p)] \lor (- p \land q)
\]
(by commutative law)
\[
\equiv [- q \land (- p \lor p)] \lor (- p \land q)
\]
(by distributive law)
\[
\equiv (- q \land t) \lor (- p \land q)
\]
(\( \therefore - p \lor p \equiv t \))
\[
\equiv - q \lor (- p \land q)
\]
(\( \therefore - q \land t \equiv - q \))
\[
\equiv (- q \lor - p) \land (- q \lor q)
\]
(Distributive law)
\[
\equiv (- q \lor - p) \land t
\]
(\( \therefore - q \lor q \equiv t \))
\[
\equiv - q \lor - p
\]
(Identity law)

\( \therefore \) The simplified circuit is

Q3. Give an alternate arrangement of the circuit, given in the figure such that the new circuit has two switches only.

Solution:
The lamp will be on if the switches \( S_1 \) or \( S_2 \) are closed and \( S_1 \) and \( S_2 \) are closed or \( S_1 \) or \( S_3 \) are closed

If \( p : S_1 \) closed, \( q : \) switch \( S_2 \) is closed, \( r : S_3 \) closed, \( L = \) the lamp \( L \) is on,
then we can write the circuit in logical form as
\[
[(- p \lor - q) \land (p \lor q)] \lor (p \lor r) \equiv L
\]
On simplification
\[
[(- p \lor - q) \land (p \lor q)] \lor (p \lor r)
\]
(De Morgan law \( - (p \land q) \equiv (- p \lor - q) \))
\[
\equiv c \lor (p \lor r)
\]
(\( p \lor - p \equiv c \))
\[
\equiv (p \lor r)
\]
(Identity law)

\( \therefore \) Simplified circuit is
PRACTICAL 2-
INVERSE OF A MATRIX BY ADJOINT METHOD AND HENCE SOLUTION OF
SYSTEM OF LINEAR EQUATIONS

Q1. Find the inverse of \( \begin{bmatrix} 3 & 2 & 6 \\ 1 & 1 & 2 \\ 2 & 2 & 5 \end{bmatrix} \) by adjoint method.

Solution:
Step (1) Let \( A=\begin{bmatrix} 3 & 2 & 6 \\ 1 & 1 & 2 \\ 2 & 2 & 5 \end{bmatrix} \)

\[ |A| = 3(5-4) - 2(5-4) + 6(2-2) = 3 - 2 = 1 \]

Step (2) \( A_{21} = (-1)^{1+1} (5 - 4) = 1, A_{42} = (-1)^{1+2} (5 - 4) = -1, A_{33} = (-1)^{1+3} (2 - 2) = 0 \)

\( A_{21} = (-1)^{2+1} (10 - 12) = 2, A_{22} = (-1)^{2+2} (15 - 12) = 3, A_{23} = (-1)^{2+3} (6 - 4) = -2 \)

\( A_{31} = (-1)^{3+1} (4 - 6) = -2, A_{32} = (-1)^{3+2} (6 - 6) = 0, A_{33} = (-1)^{3+3} (3 - 2) = 1 \)

Step (3) \( \text{adj } A = \begin{bmatrix} A_{11} & A_{12} & A_{13} \\ A_{21} & A_{22} & A_{23} \\ A_{31} & A_{32} & A_{33} \end{bmatrix}^T = \begin{bmatrix} 1 & -1 & 0 \\ 2 & 3 & -2 \\ -2 & 0 & 1 \end{bmatrix} \)

Step (4) \( A^{-1} = \frac{\text{adj } A}{|A|} = \frac{1}{1} \begin{bmatrix} 1 & 2 & -2 \\ -1 & 3 & 0 \\ 0 & -2 & 1 \end{bmatrix} \)

Q2. Find the solution set of the following system of equation using inverse method.
\( x + 2y + 3z = 3 \quad 2x + 4y + 5z = 5 \quad 3x + 5y + 6z = 7 \)

Solution:
The given system of equations can be written in the matrix form as
\[
\begin{bmatrix} 1 & 2 & 3 \\ 2 & 4 & 5 \\ 3 & 5 & 6 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 3 \\ 5 \\ 7 \end{bmatrix}
\]

Step (1)

Step (2) Let \( A = \begin{bmatrix} 1 & 2 & 3 \\ 2 & 4 & 5 \\ 3 & 5 & 6 \end{bmatrix} \).

\[ |A| = 1(-1) + 2(3) + 3(-2) = -1 + 6 - 6 = -1 \neq 0 \]
adj \( A \) = 
\[
\begin{bmatrix}
-1 & -(12-15) & (10-12) \\
-(12-15) & 6-9 & -(5-6) \\
10-12 & -(5-6) & (4-4)
\end{bmatrix}
\]

\[\begin{bmatrix}
-1 & 3 & -2 \\
3 & -3 & 1 \\
-2 & 1 & 0
\end{bmatrix}
\]

\[\begin{bmatrix}
-1 & 3 & -2 \\
3 & -3 & 1 \\
-2 & 1 & 0
\end{bmatrix}
\]

\[A^{-1} = \frac{\text{adj} A}{|A|} = \begin{bmatrix}
1 & -3 & 2 \\
3 & -3 & -1 \\
-2 & 1 & 0
\end{bmatrix} = \begin{bmatrix}
1 & -3 & 2 \\
-3 & 3 & -1 \\
2 & -1 & 0
\end{bmatrix}
\]

\[\text{Step } (3) \quad \therefore X = A^{-1}B
\]

\[\begin{bmatrix}
1 & -3 & 2 \\
-3 & 3 & -1 \\
2 & -1 & 0
\end{bmatrix} \begin{bmatrix}
x \\
y \\
z
\end{bmatrix} = \begin{bmatrix}
3 \\
5 \\
7
\end{bmatrix}
\]

\[\therefore x = 2, \quad y = -1, \quad z = 1
\]

Q3. The cost of 4 kg of potato, 3 kg wheat and 2 kg rice is Rs. 60. The cost of 1 kg potato, 2 kg wheat and 3 kg rice is Rs. 45. The cost of 6 kg potato, 2 kg wheat and 3 kg rice is Rs. 70. Find the cost of each item per kg by matrix and adjoint method.

Solution:

Let the cost of one kg of potato be \( x \), one kg of wheat be \( y \) and that of rice be \( z \). Then, the given information can be written in the matrix form as:

\[
\begin{bmatrix}
4 & 3 & 2 \\
1 & 2 & 3 \\
6 & 2 & 3
\end{bmatrix}
\begin{bmatrix}
x \\
y \\
z
\end{bmatrix} = \begin{bmatrix}
60 \\
45 \\
70
\end{bmatrix}
\]

Let \( A = \begin{bmatrix}
4 & 3 & 2 \\
1 & 2 & 3 \\
6 & 2 & 3
\end{bmatrix} \), \(|A| = 25 \neq 0, \quad \therefore A^{-1}\text{exists.}
\]

\[
\text{adj} A = \begin{bmatrix}
(6-6) & -(3-18) & (2-12) \\
-(9-4) & (12-12) & -(8-18) \\
9-4 & -(12-2) & 8-3
\end{bmatrix}
\]

\[\begin{bmatrix}
0 & 15 & -10 \\
-5 & 0 & 10 \\
5 & -10 & 5
\end{bmatrix}
\]

\[\begin{bmatrix}
0 & -5 & 5 \\
15 & 0 & -10 \\
-10 & 10 & 5
\end{bmatrix}
\]
\[
A^{-1} = \frac{\text{adj} A}{|A|} = \frac{1}{25} \begin{bmatrix}
0 & -5 & 5 \\
15 & 0 & -10 \\
-10 & 10 & 5
\end{bmatrix} = \begin{bmatrix}
\frac{-1}{5} & \frac{1}{5} \\
\frac{3}{5} & 0 & \frac{-2}{5} \\
\frac{-2}{5} & \frac{2}{5} & \frac{1}{5}
\end{bmatrix}
\]

\[
\begin{bmatrix}
x \\
y \\
z
\end{bmatrix} = \begin{bmatrix}
0 & -1 & 1 \\
5 & 5 & 5 \\
-2 & 2 & 1
\end{bmatrix}^{-1} \begin{bmatrix}
60 \\
45 \\
70
\end{bmatrix} = \begin{bmatrix}
\frac{-9+14}{5} \\
\frac{36+0-28}{8} \\
\frac{-24+18+14}{8}
\end{bmatrix} = \begin{bmatrix}
5 \\
8 \\
8
\end{bmatrix}
\]

\[\therefore\] Rate of potato is Rs 5, wheat is Rs 8, and rice is Rs 8.

**PRACTICAL-3**

**INVERSE OF A MATRIX BY ELEMENTARY TRANSFORMATION AND HENCE SOLUTION OF SYSTEM OF LINEAR EQUATIONS**

Q1. Find the inverse of \( \begin{bmatrix}
1 & 2 \\
2 & 3
\end{bmatrix} \).

Solution:

Step (1) Let \( A = \begin{bmatrix}
1 & 2 \\
2 & 3
\end{bmatrix} \), \( |A| = 3-4 = -1 \neq 0 \) \therefore inverse of A i.e., \((A^{-1})\) exists.

Step (2) i.e. \( \begin{bmatrix}
1 & 2 \\
2 & 3
\end{bmatrix} A^{-1} = \begin{bmatrix}
1 & 0 \\
0 & 1
\end{bmatrix} \)

Step (3) By elementary transformation convert A to I.

\[
R_2 - 2R_1 \quad \begin{bmatrix}
1 & 2 \\
0 & -1
\end{bmatrix} A^{-1} = \begin{bmatrix}
1 & 0 \\
-2 & 1
\end{bmatrix}
\]

\[-R_2\]

\[
\begin{bmatrix}
1 & 2 \\
0 & 1
\end{bmatrix} A^{-1} = \begin{bmatrix}
1 & 0 \\
2 & -1
\end{bmatrix}
\]

\[
R_1 - 2R_2 \quad \begin{bmatrix}
1 & 0 \\
0 & 1
\end{bmatrix} A^{-1} = \begin{bmatrix}
-3 & 2 \\
2 & -1
\end{bmatrix}
\]

Step (4) \( \therefore A^{-1} = \begin{bmatrix}
-3 & 2 \\
2 & -1
\end{bmatrix} \).
Q2. Find the inverse of the matrix
\[
\begin{bmatrix}
1 & -1 & 1 \\
0 & 1 & 1 \\
3 & 2 & -4
\end{bmatrix}
\]
by elementary transformation.

Solution:

Step (1) Let \( A = \begin{bmatrix} 1 & -1 & 1 \\ 0 & 1 & 1 \\ 3 & 2 & -4 \end{bmatrix} \)

\[\therefore |A| = (-4 - 2) + 1(-3) + 1(-3) = -6 - 3 = -12 \neq 0\]

Step (2) \( A^{-1} \) exists and \( AA^{-1} = I \)

\[
\begin{bmatrix} 1 & -1 & 1 \\ 0 & 1 & 1 \\ 3 & 2 & -4 \end{bmatrix}^{-1} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}
\]

\[
\begin{bmatrix} 1 & -1 & 1 \\ 0 & 1 & 1 \\ 0 & 5 & -7 \end{bmatrix}^{-1} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ -3 & 0 & 1 \end{bmatrix}
\]

\[R_1 - 3R_3\]

\[
\begin{bmatrix} 1 & -1 & 1 \\ 0 & 1 & 1 \\ 0 & 0 & -12 \end{bmatrix}^{-1} = \begin{bmatrix} 1 & 0 & 2 \\ 0 & 1 & 1 \\ 0 & 0 & 1 \end{bmatrix}
\]

\[R_1 + R_2 \text{ and } R_3 - 5R_2\]

\[
\begin{bmatrix} 1 & 0 & 2 \\ 0 & 1 & 1 \\ 0 & 0 & 1 \end{bmatrix}^{-1} = \begin{bmatrix} 1 & 1 & 0 \\ 0 & 1 & 0 \\ 3 & 5 & -1 \end{bmatrix}
\]

\[\frac{1}{12} R_3\]

\[
\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}^{-1} = \begin{bmatrix} 1 & 1 & 0 \\ 0 & 1 & 0 \\ \frac{1}{12} & \frac{7}{12} & \frac{1}{12} \end{bmatrix}
\]

\[R_2 - R_1, R_3 - 2R_1\]

\[
\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}^{-1} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ \frac{1}{12} & \frac{5}{12} & \frac{1}{12} \end{bmatrix}
\]

\[\therefore A^{-1} = \begin{bmatrix} \frac{1}{2} & \frac{1}{6} & \frac{1}{6} \\ \frac{1}{12} & \frac{7}{12} & \frac{1}{12} \\ -3 & 3 & -1 \end{bmatrix}\]
Q3. Find the inverse of the matrix \[
\begin{bmatrix}
1 & 1 & 1 \\
2 & 5 & 7 \\
2 & 1 & -1 \\
\end{bmatrix}
\]
by elementary transformation.

Hence, solve the system of equations \( x + y + z = 9, 2x + 5y + 7z = 52, 2x + y - z = 0 \).

Solution:

Step (1) Let \( A = \begin{bmatrix} 1 & 1 & 1 \\ 2 & 5 & 7 \\ 2 & 1 & -1 \end{bmatrix} \)

\( \therefore [A] = 1(-5 - 7) - (2 - 14) + 1(2 - 10) = -12 + 16 - 8 = -4 \neq 0. \quad \text{\( A^{-1} \) exists.} \)

We shall obtain \( A^{-1} \) by elementary row transformation.

\[
\begin{bmatrix}
1 & 1 & 1 \\
0 & 3 & 5 \\
0 & -1 & -3
\end{bmatrix}
\]

\[ A^{-1} = \begin{bmatrix} 1 & 0 & 0 \\ -2 & 1 & 0 \\ -2 & 0 & 1 \end{bmatrix} \quad R_2 - 2R_1, \quad R_1 - 2R_1 \]

\[
\begin{bmatrix}
1 & 1 & 1 \\
0 & 3 & 5 \\
0 & 0 & 4
\end{bmatrix}
\]

\[ A^{-1} = \begin{bmatrix} 1 & 0 & 0 \\ -2 & 1 & 0 \\ -8 & 1 & 3 \end{bmatrix} \quad 3R_3 + R_2 \]

\[
\begin{bmatrix}
3 & 0 & -2 \\
0 & 3 & 5 \\
0 & 0 & -4
\end{bmatrix}
\]

\[ A^{-1} = \begin{bmatrix} 5 & -1 & 0 \\ -2 & 1 & 0 \\ -8 & 1 & 3 \end{bmatrix} \quad 3R_1 - R_2 \]

\[
\begin{bmatrix}
3 & 0 & 0 \\
0 & 3 & 0 \\
0 & 0 & -4
\end{bmatrix}
\]

\[ A^{-1} = \begin{bmatrix} 9 & \frac{3}{2} & 0 \\ -12 & \frac{9}{4} & \frac{15}{4} \\ -8 & 1 & 3 \end{bmatrix} \quad R_1 - \frac{R_3}{2}, R_2 + \frac{5R_3}{4} \]

\[
\begin{bmatrix}
1 & 0 & 0 \\
0 & 1 & 0 \\
0 & 0 & 1
\end{bmatrix}
\]

\[ A^{-1} = \begin{bmatrix} 3 & \frac{-1}{2} & 0 \\ -4 & \frac{3}{4} & \frac{5}{4} \\ -2 & \frac{-1}{4} & \frac{-3}{4} \end{bmatrix} \quad \frac{R_1, R_2, R_3}{3, 3, -4} \]
\[
A^{-1} = \begin{bmatrix}
3 & -\frac{1}{2} & 0 \\
-4 & \frac{3}{4} & \frac{5}{4} \\
2 & -\frac{1}{4} & -\frac{3}{4}
\end{bmatrix}
\]

\[
X = A^{-1}B
\]

\[
X = \begin{bmatrix}
3 & -\frac{1}{2} & 0 \\
-4 & \frac{3}{4} & \frac{5}{4} \\
2 & -\frac{1}{4} & -\frac{3}{4}
\end{bmatrix} \begin{bmatrix} 9 \\ 52 \\ 0 \end{bmatrix}
\]

\[
\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 1 \\ 3 \\ 5 \end{bmatrix}
\]

\[
\therefore x = 1, y = 3, z = 5
\]

**PRACTICAL - 4**

**SOLUTIONS OF A TRIANGLE**

Q1. If \(\frac{\sin A}{\sin C} = \frac{\sin (A - B)}{\sin (B - C)}\), prove that \(a^2, b^2, c^2\) are in A.P.

Solution.

We have the results:

\[
\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C} = 2R,
\]

\[
\sin A = \frac{a}{2R}, \quad \sin C = \frac{c}{2R} \quad \text{and}
\]

\[
\cos A = \frac{b^2 + c^2 - a^2}{2bc}, \quad \cos B = \frac{c^2 + a^2 - b^2}{2ca}, \quad \cos C = \frac{a^2 + b^2 - c^2}{2ab}
\]

*Given:* \(\frac{\sin A}{\sin C} = \frac{\sin (A - B)}{\sin (B - C)}\)

*i.e.* \(\frac{\sin A}{\sin C} = \frac{\sin A \cos B - \cos A \sin B}{\sin B \cos C - \cos B \sin C}\)

Substituting the results, we have:
\[
\frac{a}{c} = \frac{c^2 + a^2 - b^2 - b^2 - c^2 + a^2}{c}
\]

i.e. \[\frac{a}{c} = \frac{2a^2 - 2b^2}{2b^2 - 2c^2}\]

i.e. \[b^2 - c^2 = a^2 - b^2\]

i.e. \[c^2 - b^2 = b^2 - a^2\]

\(a^2, b^2, c^2\) are in A.P.

Q2. The sides of triangle are \(3x + 4y, 4x + 3y, 5x + 5y\) units where \(x, y \geq 0\). Show that the triangle is obtuse angle.

Solution: Let \(a = 3x + 4y, b = 4x + 3y, c = 5x + 5y\).

\[c^2 - (a^2 + b^2) = (5x + 5y)^2 - [(3x + 4y)^2 + (4x + 3y)^2]\]

\[= (25x^2 + 25y^2 + 50xy) - (9x^2 + 16y^2 + 24xy + 4x^2 + 9y^2 + 24xy)\]

\[= (25x^2 + 25y^2 + 50xy) - (25x^2 + 25y^2 + 48xy)\]

\[= 2xy \geq 0 \quad (\because x, y > 0)\]

\[\therefore \angle c > 90^\circ\]

\[\therefore \text{The triangle with given sides is an obtuse angled triangle.}\]

Q3. In \(\triangle ABC\), if \(b = 24, c = 20\) and \(\sin \left(\frac{A}{2}\right) = \frac{1}{\sqrt{8}}\), find \(a\).

Solution:

Given: \(\sin \left(\frac{A}{2}\right) = \frac{1}{\sqrt{8}}\).

\[\sin^2 \left(\frac{A}{2}\right) = \frac{1}{8}\]

\[\cos A = 1 - 2\sin^2 \frac{A}{2} = 1 - 2 \times \frac{1}{8} = 1 - \frac{1}{4} = \frac{3}{4}\]

We have \(a^2 = b^2 + c^2 - 2bc \cos A\)

Given: \(b = 24, c = 20\)
Substituting,
\[ a^2 = 576 + 400 - 2 \times 24 \times 20 \times \frac{3}{4} = 256 \]
\[ \therefore a = 16 \]

**PRACTICAL - 5
APPLICATIONS OF SCALAR TRIPLE PRODUCT OF VECTORS**

Q1. Show that the points A (2, 3, -1), B (-2, -3, -3), C (1, 7, 2) and D (-6, 2, 2) are coplanar.

Solution:

The points A, B, C, D are coplanar if \( \overrightarrow{AB}, \overrightarrow{AC}, \text{and} \overrightarrow{AD} \) are coplanar.

i.e. \[ [\overrightarrow{AB} \overrightarrow{AC} \overrightarrow{AD}] = 0 \]

\[ \overrightarrow{AB} = \overrightarrow{b} - \overrightarrow{a} = (-2i - 3j - 3k) - (2i + 3j - k) = -4i - 6j - 2k \]

\[ \overrightarrow{AC} = \overrightarrow{c} - \overrightarrow{a} = (i + 7j + 2k) - (2i + 3j - k) = -i + 4j + 3k \]

\[ \overrightarrow{AD} = \overrightarrow{d} - \overrightarrow{a} = (-6i + 2j + 2k) - (2i + 3j - k) = -8i - j + 3k \]

\[ [\overrightarrow{AB} \overrightarrow{AC} \overrightarrow{AD}] = \begin{vmatrix} -4 & -6 & -2 \\ -1 & 4 & 3 \\ -8 & -1 & 3 \end{vmatrix} = -4(12 + 3) + 6(-3 + 24) - 2(1 + 32) = -80 + 126 - 66 \]

\[ = -126 + 126 = 0 \]
\[ \therefore \text{The points are coplanar.} \]

Q2. Find the volume of the tetrahydron, whose vertices are A(1, 1, 1), B(2, 1, 3), C(4, 5, 2), D(3, 1, 2).

Solution:

\[ \overrightarrow{a} = \overrightarrow{AB} = \overrightarrow{b} - \overrightarrow{a} = (2i + j + 3k) - (i + j + k) = i + 2k \]

\[ \overrightarrow{b} = \overrightarrow{AC} = \overrightarrow{c} - \overrightarrow{a} = (4i + 5j + 2k) - (i + j + k) = 3i + 4j + k \]

\[ \overrightarrow{c} = \overrightarrow{AD} = \overrightarrow{d} - \overrightarrow{a} = (3i + j + 2k) - (i + j + k) = 2i + k \]

Volume of the tetrahydron \[ = \frac{1}{6} [\overrightarrow{a} \overrightarrow{b} \overrightarrow{c}] = \frac{1}{6} \begin{vmatrix} 1 & 0 & 2 \\ 3 & 4 & 1 \\ 2 & 0 & 1 \end{vmatrix} = \frac{1}{6} [1(4 - 0) - 0 + 2(0 - 8)] \]

\[ = \frac{1}{6} \begin{vmatrix} 4 & -16 \end{vmatrix} = \frac{-12}{6} = -2 \]

Volume of the tetrahydron = \(|-2| = 2\) cubic unit
Q3. Prove that the normal to the plane containing the 3 points whose position vectors are \( \vec{i} \), \( \vec{j} \) lies in the direction \( \vec{i} \times \vec{j} + \vec{j} \times \vec{k} + \vec{k} \times \vec{i} \).

Solution:

Let A, B, C be the points with position vectors \( \vec{i} \), \( \vec{j} \) respectively.

Then \( \overrightarrow{AB} = \vec{j} - \vec{i} \), \( \overrightarrow{AC} = \vec{k} - \vec{i} \)

\[ \overrightarrow{AB} \cdot (\vec{i} \times \vec{j} + \vec{j} \times \vec{k} + \vec{k} \times \vec{i}) \]

\[ = (\vec{j} - \vec{i}) \cdot (\vec{i} \times \vec{j} + \vec{j} \times \vec{k} + \vec{k} \times \vec{i}) \]

\[ = \vec{j} \cdot (\vec{i} \times \vec{j}) + \vec{j} \times (\vec{j} \times \vec{k}) + \vec{k} \cdot (\vec{k} \times \vec{i}) - \vec{i} \cdot (\vec{i} \times \vec{j}) - \vec{j} \cdot (\vec{j} \times \vec{k}) - \vec{k} \cdot (\vec{k} \times \vec{i}) \]

\[ = [\vec{i} \cdot \vec{j}] + [\vec{j} \cdot \vec{k}] + [\vec{k} \cdot \vec{i}] - [\vec{i} \cdot \vec{j}] - [\vec{j} \cdot \vec{k}] - [\vec{k} \cdot \vec{i}] \]

\[ = 0 + [\vec{k} \cdot \vec{j}] + 0 - [\vec{i} \cdot \vec{k}] - 0 - 0 = 0 \]

Similarly we can show that \( \overrightarrow{AC} \cdot (\vec{i} \times \vec{j} + \vec{j} \times \vec{k} + \vec{k} \times \vec{i}) = 0 \)

Hence \( \vec{i} \times \vec{j} + \vec{j} \times \vec{k} + \vec{k} \times \vec{i} \) is perpendicular to both \( \overrightarrow{AB} \) and \( \overrightarrow{AC} \) and hence to the plane containing A, B, C. Hence the result.

**PRACTICAL - 6**

**3 DIMENSIONAL GEOMETRY - LINE**

Q1. Find the perpendicular distance of the point (2, 3, 4) from \( \frac{x-2}{2} = \frac{y-3}{6} = \frac{z-4}{2} \). Also find the foot of the perpendicular.

Solution:

Equation of the given line in symmetric form is \( \frac{x-2}{2} = \frac{y-3}{6} = \frac{z-4}{2} = r \).

Then, any point on the above line can be taken as \((4 - 2r, 6r, 1 - 3r)\).

The direction ratios of the given line are \(-2, 6, -3\).

Also, the direction ratios of \( PQ \) are \(4 - 2r - 2, 6r - 3, 1 - 3r - 4\), i.e. \(2 - 2r, 6r - 3, -3r - 3\).

Since \( PQ \) is perpendicular to the given line, we have:

\((2 - 2r) \cdot (-2) + (6r - 3) \cdot 6 + (-3r - 3) \cdot (-3) = 0\)

i.e. \(49r = 13\), \(r = 13/49\)

putting this \( r \) in the coordinates of \( Q \)

\[ Q = \left( \frac{4 - 2 \times 13}{49}, \frac{6 \times 13}{49}, \frac{1 - 3 \times 13}{49} \right) \]

\( \vec{Q} \) = length of the perpendicular from \( P \)

\[ = \sqrt{\left( \frac{170}{49} - 2 \right)^2 + \left( \frac{78}{49} - 3 \right)^2 + \left( \frac{10}{49} - 3 \right)^2} = \frac{2}{\sqrt{7}} \sqrt{101} \]
Q2. Show that the two lines \( \frac{x-3}{1} = \frac{y-2}{-1} = \frac{z-1}{2} \) and \( \frac{x-3}{2} = \frac{y-2}{-4} = \frac{z-1}{4} \) intersect each other. Also find the coordinates of the point of intersection.

Solution:
Let \( \frac{x-3}{1} = \frac{y-2}{-1} = \frac{z-1}{2} = \lambda \) and \( \frac{x-3}{2} = \frac{y-2}{-4} = \frac{z-1}{4} = \gamma \)

Then, any point on the lines can be of the form \( (\lambda + 3, 2\lambda + 5, -\lambda + 1) \) and \( (2\gamma + 4, -\gamma + 2, 2\gamma + 4) \).
If the lines are intersecting then,
\[
\lambda + 3 = 2\gamma + 4, 2\lambda + 5 = -\gamma + 2, -\lambda + 1 = 2\gamma + 4
\]
i.e.
\[
\begin{align*}
\lambda - 2\gamma - 1 &= 0, \quad \ldots \ (1) \\
2\lambda + \gamma + 3 &= 0, \quad \ldots \ (2) \\
-\lambda - 2\gamma - 3 &= 0 \quad \ldots \ (3)
\end{align*}
\]
Since the determinant is equal to zero, (1), (2), and (3) are consistent. Therefore the lines intersect.
Solving (1) and (2), we get \( \lambda = -1, \gamma = -1 \)
Therefore the point of intersection is \( (-1 + 3, -2 + 5, 1 + 1) \) i.e. \( (2, 3, 2) \).

Q3. Find the shortest distance between the lines \( \frac{x-3}{2} = \frac{y-2}{1} = \frac{z-1}{1} \) and \( \frac{x+3}{2} = \frac{y+2}{1} = \frac{z+4}{1} \).

Solution:
The given lines are
\[
\begin{align*}
\frac{x-3}{2} &= \frac{y-2}{1} = \frac{z-1}{1} \quad \ldots \ (1) \quad \text{and} \\
\frac{x+3}{2} &= \frac{y+2}{1} = \frac{z+4}{1} \quad \ldots \ (2)
\end{align*}
\]
The line (1) passes through \( A (3, 8, 3) \) and is parallel to the vector \( 3\hat{i} - \hat{j} + 3\hat{k} \).
Let \( \vec{a} \) be the position vector of \( A = 3\hat{i} + 8\hat{j} + 3\hat{k} \) and \( \vec{b} = 3\hat{i} - \hat{j} + 3\hat{k} \).

Therefore vector equation of the line (1) is \( \vec{r} = \vec{a} + \lambda \vec{b} \) \( \ldots \) (3)
The line (2) passes through \( C (-3, -2, 6) \) and is parallel to the vector \( -3\hat{i} + 7\hat{j} + 4\hat{k} \).
Let \( \vec{c} \) be the position vector of \( C = -3\hat{i} - 2\hat{j} + 6\hat{k} \) and \( \vec{d} = -3\hat{i} + 7\hat{j} + 4\hat{k} \).

Therefore vector equation of the line (2) is \( \vec{r} = \vec{c} + \lambda \vec{d} \) \( \ldots \) (4)
\[
\vec{b} \times \vec{d} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 3 & -1 & 1 \\ -3 & 7 & 4 \end{vmatrix} = \hat{i}(-4 - 7) - \hat{j}(12 - 3) + \hat{k}(21 - 3) = -11\hat{i} - 15\hat{j} + 18\hat{k}
\]
The direction ratio of the line \(| \vec{b} \times \vec{c} | = \sqrt{(-11)^2 + (-15)^2 + (18)^2} = \sqrt{670} \)
\[ \vec{c} - \vec{a} = -6\vec{i} - 10\vec{j} + 3\vec{k} \]
\[ (\vec{c} - \vec{a}) \cdot (\vec{b} \times \vec{c}) = (-6\vec{i} - 10\vec{j} + 3\vec{k}) \cdot (-11\vec{i} - 15\vec{j} + 18\vec{k}) = 66 + 150 + 54 = 270 \]

Therefore the shortest distance between (3) and (4) is \( \frac{(\vec{s} - \vec{a}) \cdot (\vec{b} \times \vec{c})}{|\vec{b} \times \vec{c}|} = \frac{270}{\sqrt{670}} \)

**PRACTICAL – 7**

**THREE DIMENSIONAL GEOMETRY - PLANE**

Q.1 Find the foot of the perpendicular drawn from the origin to the plane \( 2x + 3y + 4z = 6 = 0 \)

Solution:- Let the co-ordinate of the perpendicular from the origin to the plane is \((x_i, y_i, z_i)\).

\[ \therefore \text{ The direction ratio of the line } \overrightarrow{OP} \text{ is } x_i, y_i, z_i \]

Writing the equation of the plane in normal form gives
\[ \frac{2}{\sqrt{29}} x + \frac{3}{\sqrt{29}} y + \frac{4}{\sqrt{29}} z = \frac{6}{\sqrt{29}} \]

\[ \therefore \text{ Direction cosines of the line } \overrightarrow{OP} \text{ are } \frac{2}{\sqrt{29}}, \frac{3}{\sqrt{29}}, \frac{4}{\sqrt{29}} \]

Since direction cosines and direction ratios are in proportion.

\[ \frac{x_i}{2/\sqrt{29}} = \frac{y_i}{3/\sqrt{29}} = \frac{z_i}{4/\sqrt{29}} = k \quad \therefore \quad x_i = \frac{2k}{\sqrt{29}}, \quad y_i = \frac{3k}{\sqrt{29}}, \quad z_i = \frac{4k}{\sqrt{29}} \quad \text{ (I)} \]

\((x_i, y_i, z_i)\) satisfies the equation of the plane.

\[ \therefore 2 \cdot \frac{2k}{\sqrt{29}} + 3 \cdot \frac{3k}{\sqrt{29}} + 4 \cdot \frac{4k}{\sqrt{29}} = 6 \]

\[ k \left( \frac{4 + 9 + 16}{\sqrt{29}} \right) = 6, \]

\[ k = \frac{6}{\sqrt{29}} \]

Substituting in (I) gives the foot of the perpendicular as
\[ (x_i, y_i, z_i) = \left( \frac{12}{29}, \frac{18}{29}, \frac{24}{29} \right) \]

Q.2 Find the vector and cartesian equations of the plane which passes through the point \((5, 2, 4)\) and perpendicular to line with direction ratios \(-2, 3, 1\)

Solution:

The position vector of the point \((5, 2, 4)\) is \( \vec{a} = 5\vec{i} + 2\vec{j} + 4\vec{k} \) and the normal \( \vec{n} \) to the plane...
\[ \vec{n} = -2\vec{i} + 3\vec{j} + \vec{k} \] is

\[ \therefore \text{The vector of the plane is given by } (\vec{r} - \vec{a}) \cdot \vec{n} = 0, \quad \vec{r} = x\vec{i} + y\vec{j} + z\vec{k} \]

\[ [(x - 5)\vec{i} + (y - 2)\vec{j} + (z - 4)\vec{k}] \cdot (-2\vec{i} + 3\vec{j} + \vec{k}) = 0 \]

\[-2(x - 5) + 3(y - 2) + 1(z - 4) = 0 \]

\[-2x + 10 + 3y - 6 + z - 4 = 0 \]

\[-2x + 3y + z = 0 \]

i.e. \( 2x - 3y - z = 0 \), is the cartesian equation of the plane.

Q.3 Show that the lines \( \frac{x + 1}{-3} = \frac{y - 3}{2} = \frac{z + 2}{1} \) and \( \frac{x - 7}{-3} = \frac{y + 7}{2} = \frac{z + 7}{1} \) are coplanar.

Solution-

Here, we have \( (x_1, y_1, z_1) = (-1, 3, 2) \) and \( (x_2, y_2, z_2) = (0, 7, -7) \), \( l_1 = -3, m_1 = 2, n_1 = 1 \)

\( l_2 = 1, m_2 = -3, n_2 = 2 \)

\[ \begin{vmatrix} x_1 - x_2 & y_1 - y_2 & z_1 - z_2 \\ l_1 & m_1 & n_1 \\ l_2 & m_2 & n_2 \end{vmatrix} = \begin{vmatrix} -1 & -4 & 5 \\ -3 & 2 & 1 \\ 1 & -5 & 2 \end{vmatrix} \]

\[ = -1(4 + 3) + 4(-6 - 1) + 5(9 - 2) \]

\[ = -7 - 20 + 35 = 0 \]

Hence, the lines are coplanar.

**PRACTICAL - 8**

**LINEAR PROGRAMMING PROBLEMS**

Q.1 A company manufactures two products A and B with Rs.12 and Rs.8 as the profit per unit respectively. The processing of products requires times on two machines M1 and M2. The maximum available time for these machines is 160Hrs. and 120Hrs. respectively.

One unit of product A requires 2 hrs, on machine M1 and 3 hrs on machine M2 one unit of product B requires 4 hrs, on machine M1 and 1 hrs on machine M2. Formulate the linear programming problem to maximize the profit the company and solve it.

Solution –

Step (1)

<table>
<thead>
<tr>
<th>Product A</th>
<th>Product B</th>
<th>Maximum availability</th>
</tr>
</thead>
<tbody>
<tr>
<td>Machine M1</td>
<td>2</td>
<td>4</td>
</tr>
<tr>
<td>Machine M2</td>
<td>3</td>
<td>1</td>
</tr>
<tr>
<td>Profit</td>
<td>12</td>
<td>8</td>
</tr>
</tbody>
</table>
Step (2) The L.P.P. is to maximize
Profit \( P = 12x + 8y \)

Step (3) Under the constraints

\[
\begin{align*}
2x + 4y & \leq 160 \\
3x + y & \leq 120 \\
x & \geq 0, y & \geq 0
\end{align*}
\]

The shaded portion is the feasible region.

Step (5) Solving \( 3x + y = 120 \) and \( 2x + 4y = 160 \) gives \( B = (321, 24) \)

Step (6) \( P \) is maximum at one of the vertices of feasible region

<table>
<thead>
<tr>
<th>Points</th>
<th>Profit ( P = 12x + 8y )</th>
</tr>
</thead>
<tbody>
<tr>
<td>A (40, 0)</td>
<td>( 12 \times 40 + 8 \times 0 = 480 )</td>
</tr>
<tr>
<td>B (32, 24)</td>
<td>( 12 \times 32 + 8 \times 24 = 576 )</td>
</tr>
<tr>
<td>C (0,40)</td>
<td>( 12 \times 0 + 8 \times 40 = 320 )</td>
</tr>
</tbody>
</table>

Step (7) Therefore profit is maximum when \( x = 32, y = 24 \) and the maximum profit is Rs. 576.

Q2. A farm is engaged in breeding pigs. The pigs are fed on various products grown on the farm. Because of the need to ensure certain nutrient constituents it is necessary to buy additional one or two products which we shall call A and B. The nutrient constituents (Vitamins and Proteins) in each unit of the product are given below.

<table>
<thead>
<tr>
<th>Nutrient Constituents</th>
<th>Product</th>
<th>Minimum Requirements</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>A</td>
<td>B</td>
</tr>
<tr>
<td>1</td>
<td>36</td>
<td>6</td>
</tr>
<tr>
<td>2</td>
<td>3</td>
<td>12</td>
</tr>
<tr>
<td>3</td>
<td>20</td>
<td>10</td>
</tr>
</tbody>
</table>

Product A costs Rs. 20, product B costs Rs. 40 per unit. Determine how much of products A and B must be purchased so as to provide the pigs nutrients not less than the minimum requirement at lowest possible cost. Solve graphically.

Solution.
Let the number of products of A is \( x \) and B is \( y \). Then the L.P.P. is to minimize cost \( C = 20x + 40y \) under the constraints.

\[
\begin{align*}
36x + 6y & \geq 108. \\
3x + 12y & \geq 36.
\end{align*}
\]
\[20x + 10y \geq 100, \quad x \geq 0, \quad y \geq 0\]

\[36x + 6y \geq 108.\]

\[3x + 12y \geq 36.\]

\[20x + 10y \geq 100\]

<table>
<thead>
<tr>
<th>x</th>
<th>3</th>
<th>0</th>
<th>x</th>
<th>12</th>
<th>0</th>
<th>x</th>
<th>5</th>
<th>0</th>
</tr>
</thead>
<tbody>
<tr>
<td>y</td>
<td>0</td>
<td>18</td>
<td>y</td>
<td>0</td>
<td>3</td>
<td>y</td>
<td>0</td>
<td>10</td>
</tr>
</tbody>
</table>

The minimum value lies at A, B, C or D

<table>
<thead>
<tr>
<th>Points</th>
<th>C = 20x + 40y</th>
</tr>
</thead>
<tbody>
<tr>
<td>A (12, 0)</td>
<td>20(12) + 40(0) = 240</td>
</tr>
<tr>
<td>B (4, 2)</td>
<td>20(4) + 40(2) = 160</td>
</tr>
<tr>
<td>C (2, 6)</td>
<td>20(2) + 40(16) = 280</td>
</tr>
<tr>
<td>D (0, 18)</td>
<td>20(0) + 40(18) = 720</td>
</tr>
</tbody>
</table>

Therefore cost is minimized when \(x = 4, \ y = 2\) and minimum cost is Rs. 160

Q. 3 Minimize \(Z = 8x_1 + 4x_2\)

Subject to \(x_1 + 2x_2 \geq 2, \ 3x_1 + x_2 \geq 3, \ 4x_1 + 3x_2 \geq 6, \ x_1 \geq 0, \ x_2 \geq 0\)

Solution: Convert the inequations into equations and draw the corresponding lines.

\[\frac{x_1}{2} + \frac{x_2}{1} = 1 \quad \frac{x_1}{1} + \frac{x_2}{3} = 1 \quad \frac{x_1}{1.5} + \frac{x_2}{2} = 1\]

As \(x_1, x_2 \geq 0\), the solution lies in the first quadrant.
The shaded region is the feasible region. A, B, C, D are the corner points (vertices). Consider any one point of the feasible region say P(1, 2). Draw initial isocost line \( z_1 \) passing through P(1, 2).

\[
\therefore \quad z_1 = 8(1) + 4(2) = 8 + 8 = 16
\]

\[
\therefore \quad \text{Initial iso cost line is } 8x_1 + 4x_2 = 16
\]

i.e. \[
\frac{x_1}{2} + \frac{x_2}{4} = 1
\]

This line is shown in the graph by dotted line \( z_1 \). As it can be seen from the graph that if the iso-cost line is moved parallel to itself towards the origin, the cost \( z \) will reduce. Hence, we can see when it is moved parallel to itself towards origin, the last point it passes through is C.

\[
\therefore \quad \text{The point C gives optimal solution.}
\]

\[
\therefore \quad \text{Co-ordinates of C are (0.6, 1.2)}
\]

\[
\therefore \quad z \text{ has minimum value when } x_1 = 0.6 \text{ and } x_2 = 1.2
\]

And Minimum value of \( z \) is \( 8(0.6) + 4(1.2) = 9.6 \) (Unique optimal solution)

**PRACTICAL – 9**

**GEOMETRICAL APPLICATIONS OF DERIVATIVES**

Q1. Find equations of tangent and normal to the curve \( y = 3x^2 - 4x + 7 \) at the point whose abscissa is 1.

Solution:

Given equation of the curve is \( y = 3x^2 - 4x + 7 \)

\[
\frac{dy}{dx} = 6x - 4
\]

Therefore the slope of the tangent at \( x = 1 \) is \( m_1 = 6 \times 1 - 4 = 2 \)

Slope of the normal = \( m = \frac{-1}{2} \).

When \( x = 1 \), \( y = 3 \times 1^2 - 4 \times 1 + 7 = 6 \)

Therefore equation of tangent at \( (1, 6) \) is \( y - y_1 = m_1 \) \( (x - x_1) \)

ie. \( y - 6 = 2 \) \( (x - 1) \)

ie. \( 2x - y + 4 = 0 \)

Equation of the normal is \( y - y_1 = m \) \( (x - x_1) \)

ie. \( y - 6 = \frac{-1}{2} \) \( (x - 1) \)

ie. \( 2y - 12 = -x + 1 \)

ie. \( x + 2y - 13 = 0 \)
Q2. Verify Rolle’s theorem for 
\[ f(x) = \frac{x^2 - 4x}{x + 2} \text{ in } [0, 4] \]
Solution:
\[ f(x) = \frac{x^2 - 4x}{x + 2} \text{ is continuous in } [0, 4] \text{ and differentiable in } (0, 4) \]
\[ f(0) = 0 \text{ and } f(4) = 0 \]

since all conditions of Rolle’s theorem are satisfied, there is a point \( c \in [0, 4] \) such that \( f'(c) = 0 \)

but 
\[ f'(x) = \frac{(x+2)(2x-4)-(x^2-4x)}{(x+2)^2} \]

therefore \( f'(c) = 0 \) gives

\[ (c + 2)(2c - 4) - (c^2 - 4c) = 0 \]
\[ 2c^2 - 4c + 4c - 8 - c^2 + 4c = 0 \]
\[ c^2 + 4c - 8 = 0 \]
\[ c = \frac{-4 \pm \sqrt{4^2 - 4 \cdot 1 \cdot (-8)}}{2} = -2 \pm 2 \sqrt{3} \]
\[ c = -2 - 2 \sqrt{3} \in [0, 4] \]
But \( c = -2 + 2 \sqrt{3} \in (0, 4) \).

Hence the proof.

Q3. \( f(x) = x (1 - \log x), x > 0 \). Show that \( (a - b) \log c = b (1 - \log b) - a (1 - \log a) \), where \( 0 < a < c < b \)
Solution:
Given \( f(x) = x (1 - \log x), x > 0 \) in \([a, b]\)
\[ f(a) = a (1 - \log a), \]
\[ f(b) = b (1 - \log b), \]

\[ f'(c) = x \left( \frac{1}{x} \right) + (1 - \log x) = -1 + 1 - \log x = -\log x \]

\( f(x) \) is continuous in \([a, b]\) and \( f(x) \) is \((a, b)\).
Therefore, by Lagrange’s Mean Value Theorem, there is a \( c \in (a, b) \) such that:
\[ f''(c) = \frac{f(b) - f(a)}{b - a} \]

ie. \[ \log c = \frac{b (1 - \log b) - a (1 - \log a)}{b - a} \]

ie. \( (a - b) \log c = b (1 - \log b) - a (1 - \log a), 0 < a < c < b \)

Hence the proof.
PRACTICAL - 10
APPLICATION OF DERIVATIVES RATE MEASURES

Q1. The law of motion of a particle is given by \( s = t^3 - 2t^2 + 5t + 2 \). Find the velocity and acceleration at \( t = 2 \).

Solution:

Given \( s = t^3 - 2t^2 + 5t + 2 \).

Velocity \( v = \frac{ds}{dt} = \frac{d}{dt}(t^3 - 2t^2 + 5t + 2) \)

\[ v = 3t^2 - 4t + 5. \]

\[ \therefore \text{velocity at } t = 2 \text{ is} \]

\[ V_{t=2} = 3(2)^2 - 4(2) + 5 = 12 - 8 + 5 = 9 \text{ unit/sec} \]

Acceleration = \( a = \frac{dv}{dt} = \frac{d}{dt}(3t^2 - 4t + 5) = 6t - 4 \)

\[ \therefore a_{t=2} = 6(2) - 4 = 8 \text{ units/ sec}^2 \]

Q2. A water tank has the shape of an inverted right circular cone with its axis vertical and vertex lowermost. Its semi vertical angle is \( \tan^{-1}(0.5) \). Water is poured into it at constant rate of 4 cu. ft / min. Find the rate at which the level of the water is rising at the instant when the depth of the water in the tank is 8 ft.

Solution:

Let at an instant \( t \) the depth of the water in the tank be \( h \) and radius be \( r \).

Given semi vertical angle, \( \theta = \tan^{-1}(0.5) \)

\[ \therefore \tan \theta = 0.5 \]

From the triangle ABC,

\[ \tan \theta = \frac{r}{h} \therefore r = h \tan \theta = 0.5h \]

Also, \( \frac{dv}{dt} = 4 \text{ cu. ft / min} \)

Volume of the water at the instant \( t \) is
Q3. A ladder of length 17 m. rest with one end against a vertical wall and the other end on ground level. If the lower end slips at the rate of 1 m/s. Find how fast is its upper end coming down when the lower end is 8 m from the wall?

Solution:

Let at an instant \( t \) lower part \( B \) of the ladder be \( x \) m from the wall and upper part is at a height of \( y \) m from ground.

From fig. \( x^2 + y^2 = 17^2 \)

Given \( \frac{dx}{dt} = 1 \text{ m/s} \). We have to find \( \frac{dy}{dt} \) \( x = 8 \)

We have \( x^2 + y^2 = 289 \)

Diff. w.r.t. \( t \)

\[
2x\frac{dx}{dt} + 2y\frac{dy}{dt} = 0
\]

\[
\frac{dy}{dt} = -\frac{x}{y} \frac{dx}{dt}
\]

When \( x = 8 \), \( y^2 = 289 - 8^2 = 289 - 64 = 225 \)

Therefore \( y = 15 \)

Therefore \( \left(\frac{dy}{dt}\right)_{x=8} = -\frac{8}{15} \times 1 = -\frac{8}{15} \text{ m/s} \)
PRACTICAL – 11
APPLICATION OF DERIVATIVES, MAXIMA & MINIMA

Q1. Examine for maxima & minima of a function \(2x^3 + 9x^2 - 24x + 18\). Also find the maximum & minimum values of \(f(x)\).

Solution:
\[ f(x) = 2x^3 + 9x^2 - 24x + 18 \]

Step 1: \( f'(x) = 6x^2 + 18x - 24 \)
At maxima & minima \( f'(x) = 0 \)
Therefore \( 6x^2 + 18x - 24 = 0 \)
\( \text{ie. } x^2 + 3x - 4 = 0 \)
\( \text{ie. } (x - 1)(x + 4) = 0 \)
\( \text{ie. } f(x) \text{ is maximum or minimum at } x = 1 \text{ and } x = -4 \)

\( f''(x) = 12x + 18 \)

Step 2: \((f''(x))_{x=1} = f''(1) = 12 \times 1 + 18 = 30 > 0 \)
Therefore \( f(x) \text{ is minimum at } x = 1 \)

Step 3: \((f''(x))_{x=-4} = f''(-4) = 12 \times (-4) + 18 = -30 < 0 \)
Therefore \( f(x) \text{ is maximum at } x = -4 \)

The maximum value is \( f(-4) = 2(-4)^3 + 9(-4)^2 - 24(-4) + 18 = 130 \)
The minimum value is \( f(1) = 2(1)^3 + 9(1)^2 - 24(1) + 18 = 5 \)

Q2. The perimeter of a triangle is 8 cm. If one of the sides is 3 cm, what are the other two sides for maximum area of the triangle?

Solution:
Let \( a, b, c \) be the length of the sides of \( \triangle ABC \). Given \( a + b + c = 8 \) cm, then \( s = \frac{a+b+c}{2} \)
Also given \( a = 3 \) cm.
Therefore \( b + c = 8 - 3 = 5 \) cm, \( c = 5 - b \) cm.

Area of triangle with sides \( a, b, c \) is
\[ A = \sqrt{s(s-a)(s-b)(s-c)} = \sqrt{4(4-b)(4-b)(4-5)} = 2\sqrt{(4-b)(b-1)} \]

At maximum \( A \),
\[ \frac{dA}{db} = 0, \quad \frac{dA}{db} = \frac{2[(4-b)+(b-1)(-1)]}{2\sqrt{(4-b)(b-1)}} \]
\[ \frac{dA}{db} = \frac{5-2b}{\sqrt{(4-b)(b-1)}}, \quad \frac{dA}{db} = 0 \rightarrow b = 2.5 \]
\[
\frac{d^2A}{db^2} = \frac{\sqrt{(4-b)(b-1)}(-2)-(5-2b)\frac{5-2b}{2\sqrt{(4-b)(b-1)}}}{(4-b)(b-1)}
\]
\[
\left(\frac{d^2A}{db^2}\right)_{b=2.5} = -2 < 0
\]

Therefore A is maximum when b = 2.5, c = 5 - 2.5 = 2.5

**Q3.** The figure shows a junction of two corridors of width 9 m & 4 m which are at right angle. P & Q are variable points such that PBQ is a straight line. Express AQ in terms of CP & find the value of CP for which PA + AQ is minimum.

**Solution:**

Let length of CP be x m & that of DQ be y m.

From the figure, AC = 9 m & AD = 4 m.

PB = \sqrt{CB^2 + CP^2} = \sqrt{16 + x^2} \quad \text{... (1)}

QB = \sqrt{DB^2 + DQ^2} = \sqrt{81 + y^2} \quad \text{... (2)}

PQ = PB + BQ \quad \text{... (3)}

But PQ = \sqrt{AQ^2 + AP^2}

ie. PQ = \sqrt{(9 + x)^2 + (4 + y)^2} \quad \text{... (4)}

From eq (1), (2), (3), and (4)

\[
\sqrt{(y + x)^2 + (4 + y)^2} = \sqrt{81 + y^2} + \sqrt{16 + x^2}
\]

Squaring both sides

\[
(9 + x)^2 + (4 + y)^2 = 81 + y^2 + 16 + x^2 + 2\sqrt{81 + y^2} \sqrt{16 + x^2}
\]

ie. \(81 + 18x + x^2 + 16 + 0y + y^2 = 81 + y^2 + 16 + x^2 + 2\sqrt{81 + y^2} \sqrt{16 + x^2}\)

ie. \(18x + 8y = 2\sqrt{81 + y^2} \sqrt{16 + x^2}\)

ie. \(9x + 4y = \sqrt{81 + y^2} \sqrt{16 + x^2}\)

Squaring both sides

\[
9x + 4y = \sqrt{81 + y^2} \sqrt{16 + x^2}
\]

ie. \((9x + 4y)^2 = (81 + y^2)(16 + x^2)\)

ie. \(72xy = x^2y^2 + 1296\)

ie. \(x^2y^2 - 72xy + 1296 = 0\)

ie. \((xy - 36)^2 = 0\)

die. \(xy - 36 = 0\)

ie. \(xy = 36\)
ie. \( y = \frac{26}{x} \)
therefore \( AQ = \frac{26}{AP} \)

Let \( S = PA + AQ \)
therefore \( S = 9 + x + 4 + \frac{26}{x} \)

ie \( S = 13 + x + \frac{26}{x} \)

At minimum \( S \),
\[
\frac{dS}{dx} = 0, \quad \text{ie. } 1 - \frac{36}{x^2} = 0, \quad \text{ie. } x = 6 \quad (> x \text{ cannot be negative})
\]
\[
\left( \frac{d^2S}{dx^2} \right) = \frac{72}{x^3}, \quad \left( \frac{d^2S}{dx^2} \right)_{x=6} > 0
\]

Therefore \( S \) is minimum when \( x = 6 \)
Therefore For minimum \( AQ + PA \), CP must be equal to 6 m.

PRACTICAL – 12

APPLICATIONS OF DEFINITE INTEGRAL AS LIMIT OF A SUM

Q1. Find \( \int_{0}^{5} (x+1)dx \) as the limit of a sum.

Solution:

By definition
\[
\int_{a}^{b} f(x)dx = \lim_{n \to \infty} h \sum_{r=0}^{n-1} f(a + rh), \text{ where } h = \frac{b-a}{n}
\]

In this example \( a = 0, b = 5, f(x) = x + 1 \)

\[
h = \frac{b-a}{n} = \frac{5-0}{n} = \frac{5}{n}
\]

\[ \therefore \int_{0}^{5} (x+1)dx = \lim_{n \to \infty} h \sum_{r=0}^{n-1} f(0 + rh) = \lim_{n \to \infty} \sum_{r=0}^{n-1} hf(rh) \]

\[ = \lim_{n \to \infty} h \sum_{r=0}^{n-1} (rh + 1) \]
\[
\lim_{n \to \infty} \left[ h \sum_{r=0}^{n-1} rh + h \sum_{r=0}^{n-1} 1 \right] = \lim_{n \to \infty} \left\{ \frac{5n}{n} \left[ \frac{5(n-1)}{2} \right] + \frac{5}{n} (n) \right\} \\
= \lim_{n \to \infty} \left\{ \frac{25}{2} \left[ 1 - \frac{1}{n} \right] + 5 \right\} \\
= \frac{25}{2} + 5 = 17.5
\]

Q2. Find \(\int_{\frac{3}{2}}^{2} x^2 \, dx\) as the limit of the sum.

Solution:

\[
\int_{a}^{b} f(x) \, dx = \lim_{n \to \infty} \sum_{r=0}^{n-1} hf(a + rh) , h = \frac{b-a}{n}
\]

In this example

\(a=2, \ b=3, \ f(x) = x^2\) \(\therefore \ f(a + rh) = (2 + rh)^2 = 4 + 4rh + r^2h^2, \ h = \frac{1}{n}\)

\(\therefore \int_{\frac{3}{2}}^{2} x^2 \, dx = \lim_{n \to \infty} \sum_{r=0}^{n-1} h \left[ 4 + 4rh + r^2h^2 \right] \)

\(= \lim_{n \to \infty} \left[ 4h \sum_{r=0}^{n-1} 1 + 4h^2 \sum_{r=0}^{n-1} r + h^3 \sum_{r=0}^{n-1} r^2 \right] \)

\(= \lim_{n \to \infty} \left[ 4 + \frac{2 \times n^2}{n^2} \left( 1 - \frac{1}{n} \right) + \frac{1}{n^3} \left( 1 - \frac{1}{n} \right) \left( 1 - \frac{2}{n} \right) \left( 2 - \frac{1}{n} \right) \right] \left( \lim_{n \to \infty} \frac{1}{n} = 0 \right) \)

\(= 4 + 2 \times \frac{6}{3} = 6 \times \frac{1}{3} = \frac{19}{3}\)
Q3. What is the area of the shaded region? Confirm analytically the same result by limiting process.

Solution:

Area of the shaded portion = Area of the rectangle ABCD + Area of right triangle DCE

\[ = (b-a)a + \frac{1}{2}(b-a)(b-a) \]

\[ = ab - a^2 + \frac{1}{2}(b^2 - 2ba + a^2) \]

\[ = \frac{b^2 - a^2}{2} \]

By limiting process,

Area of the shaded portion= Area under the line y=x between x=a, x=b = \[ \int_a^b x \, dx \]

\[ = \lim_{n \to \infty} \sum_{r=0}^{n-1} \left( \frac{b-a}{n} \right)(a + rh) \]

\[ = \lim_{n \to \infty} \left( \frac{b-a}{n} \right) \left[ \frac{a}{n} + \frac{n-1}{2} \frac{a}{n} \right] \]

\[ = \lim_{n \to \infty} \left( \frac{b-a}{n} \right) \left[ a + \frac{(b-a)(n-1)n \frac{1}{2}}{2} \right] \]

\[ = \lim_{n \to \infty} \left( \frac{b-a}{n} \right) \left[ a + \frac{(b-a)(1-1)}{2} \frac{1}{n} \right] \]

\[ = \lim_{n \to \infty} \left( \frac{b-a}{n} \right) \left[ a + \frac{(b-a)(1-1)}{2} \frac{1}{n} \right] \]

\[ = (b-a) \frac{b+a}{2} \]

\[ = \frac{b^2 - a^2}{2} \]
PRACTICAL – 13
APPLICATION OF DEFINITE INTEGRALS: AREA

Q1. Find the area of the region bounded by the curve \( y^2 = x \) and the lines \( x = 1, \ x = 4 \) and the x-axis

Solution:

Putting \( y = 0 \) in \( y^2 = x \) \( \Rightarrow \) \( x = 0 \)

\( \therefore \) In between \( x = 1, \ x = 4 \) the curve does not cut the x-axis.

\( \therefore \) Area of the region bounded by the curve \( y^2 = x \) and the lines \( x = 1, \ x = 4 \) and the x-axis is

\[
A = \left| \int_1^4 y \, dx \right| = \left| \int_1^4 \sqrt{x} \, dx \right| = \left| \left[ \frac{x^{3/2}}{3/2} \right]_1^4 \right|
\]

\[
= \frac{2}{3} \left( 4^{3/2} - 1^{3/2} \right) = \frac{2}{3} \left( 2^3 - 1 \right) = \frac{14}{3} \text{ sq. units.}
\]

Q2. Find the area of the region in the first quadrant enclosed by the x-axis, the line \( y = x \) and the circle \( x^2 + y^2 = 32 \)

Solution:

Putting \( y = x \) in \( x^2 + y^2 = 32 \) gives \( x^2 + x^2 = 32 \),

\( 2x^2 = 32, \ x^2 = 16, \ x = \pm 4 \)

Since the point is in 1st quadrant \( x = 4 \).

\( \therefore \) The required Area= \( A (OABO) + A (ABCA) \)

\( A (OABO) = \text{Area bounded by the line } y = x, \text{ x-axis and the ordinate } x=4 \)

\[
= \int_0^4 y \, dx = \int_0^4 x \, dx = \left[ \frac{x^2}{2} \right]_0^4 = \frac{16}{2} = 8 \text{ sq. units}
\]

\( A (ABCA) = \text{Area bounded by the circle } x^2 + y^2 = 32, \text{ x axis, ordinate } x=4, \ x = \sqrt{32} = 4\sqrt{2} \)

\[
= \int_4^4 \sqrt{32 - x^2} \, dx
\]

\[
= \left[ \frac{x}{2} \sqrt{32 - x^2} + \frac{32}{2} \sin^{-1} \left( \frac{x}{\sqrt{32}} \right) \right]_4^{4\sqrt{2}}
\]

\[
= 16 \sin^{-1} (1) - \left( \frac{4}{2} \sqrt{32 - 16 + 16 \sin^{-1} \left( \frac{4}{4\sqrt{2}} \right)} \right)
\]
\[
\begin{align*}
= 16 \times \frac{\pi}{2} \left[ 8 + 16 \left( \frac{\pi}{4} \right) \right] = 8\pi - 8 - 4\pi = 4\pi - 8 \\
\therefore \text{Required area} = 8 + 4\pi - 8 = 4\pi \text{ sq. units}
\end{align*}
\]

Q3. Calculate the area between the parabolas \( y = x^2 \) and \( x = y^2 \).

Solution:

The given parabolas are: \( y = x^2 \) \( \ldots \ldots \) (1)
And \( x = y^2 \) \( \ldots \ldots \) (2)

Eliminating \( y \) between (1) and (2), we get:
\[
x = x^4 \quad \therefore x^4 - x = 0
\]
\[
x(x^3 - 1) = 0 \quad \therefore x = 0, y = 1.
\]

When \( x=0 \), \( y=0 \).
When \( x=1 \), \( y=1 \).
Hence, the two parabolas intersect at \((0,0)\) and \((1,1)\).

The required area is the area of the shaded portion.

\[
\text{Required area} = \int_0^1 \sqrt{x} \, dx - \int_0^1 x^2 \, dx = \left[ \frac{x^{3/2}}{3/2} \right]_0^1 - \left[ \frac{x^3}{3} \right]_0^1 = \frac{2}{3} - \frac{1}{3} = \frac{1}{3} \text{ sq.units}
\]

PRACTICAL - 14
APPLICATIONS OF DIFFERENTIAL EQUATIONS

Q.1. If \( y \) represents the amount by which the temperature of a body exceeds that of the surrounding air, then the rate at which \( y \) decreases is proportional to \( y \). If \( y \) is initially 8° & was 7° after 1 minute, what will it be after 2 minutes?

Solution:

Given \( \frac{dy}{dt} \propto y \)

\[
\therefore \frac{dy}{dt} = ky,
\]
\[
\therefore \frac{dy}{y} = k \, dt,
\]
\[
\therefore \int \frac{dy}{y} = k \int dt
\]
\[
\therefore \log y = kt + c
\]

Given when \( t = 0 \), \( y = 8 \) and when \( t = 1 \), \( y = 7 \).

\[
\therefore \log 8 = c,
\]
\[ \log y = k \cdot t + \log 8, \]
Substitute \( t = 1 \), gives
\[ \log 7 = k \cdot \log 8, \]
ie. \( k = \log 7/8 \)
\[ \log y = \left( \log \left( \frac{7}{8} \right) \right) t + \log 8 = \log \left( 8 \cdot \left( \frac{7}{8} \right)^t \right) \]
\[ y = 8 \cdot \left( \frac{7}{8} \right)^t \]

When \( t = 2, \ y = 8 \times \left( \frac{7}{8} \right)^2 = \frac{49}{8} = 6.125 \]

\[ Q2. \] The population of US in 1980 was 225 million and rate of increase of population is proportional to the population at that time. If the population was 300 million in year 1990, find the population in 2015.

Solution:

Let \( P \) denote the population

\[ \frac{dP}{dt} = k \cdot P, \]

0
\[ \Rightarrow \int \frac{dP}{P} = k \int dt, \]
\[ \Rightarrow \log P = k \cdot t + c \quad \text{.... (1)} \]

If for 1980 we take \( t = 0 \) for 1990, we take \( t = 10 \) for 2015, \( t = 35 \) Substitute in (1)

\[ \Rightarrow \log 225 = c, \]
\[ \Rightarrow \log P = k \cdot t + \log 225, \]
\[ \log 300 = k \times 10 + \log 225 \]

ie. \( k = \frac{\log 300}{10} - \log \left( \frac{225}{225} \right) = \frac{\log 4}{\log 3} \)
\[ \Rightarrow \log P = \left( \frac{t}{10} \log \left( \frac{4}{3} \right) \right) + \log 225 = \log \left[ \left( \frac{4}{3} \right)^{t/10} \times 225 \right] \]
\[ \Rightarrow P = \left( \frac{4}{3} \right)^{t/10} \times 225 \]

When \( t = 35 \) years \( , \ P = \left( \frac{4}{3} \right)^{3.5} \times 225 \) million.

\[ \Rightarrow \text{Population for the year 2005} = P = \left( \frac{4}{3} \right)^{3.5} \times 225 \text{ million}. \]
Q3. The engine of a motorboat moving at 10 m/sec is shut off. Given that the retardation at any subsequent time (after shutting off the engine) equals the velocity at that time, find (i) the velocity \( v \) at \( t \) sec after switching off engine, (ii) the velocity after 2 secs of switching off the engine.

Solution:
(i) Given
\[
- \frac{dv}{dt} = v
\]
\[
\Rightarrow - \frac{dv}{v} = dt
\]
\[
\Rightarrow - \log v = t + c
\]
\[
\Rightarrow \log v = -t + c
\]
\[
\Rightarrow v = e^{-t+c}
\]
\[
\Rightarrow v = ke^{-t}
\]

When \( t = 0, v = 10 \Rightarrow k = 10 \)
\[
\Rightarrow \text{The required equation is } v = 10e^{-t}
\]

(ii) The velocity after 2 sec of switching off the engine is \( v = 10e^{-2} = \frac{10}{e^2} \).

PRACTICAL - 15
EXPECTED VALUE, VARIANCE AND STANDARD DEVIATION OF RANDOM VARIABLE

Q1. 2 cards are drawn successively without replacement from a well-shuffled deck of 52 cards. Find the probability distribution of the number of aces.

Solution.
The random variable is the no. of aces. Let \( X \) be the random variable. Then \( X \) can take the values 0, 1 and 2.

\[
P (x = 0) = P (\text{no ace cards out of 2 cards}) = \frac{\binom{48}{2}}{\binom{52}{2}} = \frac{188}{221}
\]

\[
P (x = 1) = P (\text{1 ace and 1 other cards}) = \frac{\binom{4}{1} \binom{48}{1}}{\binom{52}{2}} = \frac{4 \times 48}{221} = \frac{32}{221}
\]

\[
P (x = 2) = P (\text{both are ace cards}) = \frac{\binom{4}{2}}{\binom{52}{2}} = \frac{1}{221}
\]
Therefore required probability distribution is

<table>
<thead>
<tr>
<th>X</th>
<th>0</th>
<th>1</th>
<th>2</th>
<th>Total</th>
</tr>
</thead>
<tbody>
<tr>
<td>P(X)</td>
<td>1/221</td>
<td>2/221</td>
<td>1/221</td>
<td>1</td>
</tr>
</tbody>
</table>

Q2. Let $X$ denote the number of hours you study during randomly selected school day. The probability that $X$ can take the values $x$ has the following form, where $k$ is some unknown constant.

$$P(X = x) = f(x) = \begin{cases} \ 0.1, & x = 0 \\ \ kx, & x = 1 \text{ or } 2 \\ \ k(5 - x), & x = 3 \text{ or } 4 \\ \ 0.1, & \text{otherwise} \end{cases}$$

(i) Find the value of $k$
(ii) What is the probability that you study at least two hours? Exactly two hours? Atmost two hours?

Solution: The probability distribution of $X$ is

<table>
<thead>
<tr>
<th>X</th>
<th>0</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>Total</th>
</tr>
</thead>
<tbody>
<tr>
<td>P(X)</td>
<td>0.1</td>
<td>$k$</td>
<td>$2k$</td>
<td>$2k$</td>
<td>$k$</td>
<td>1</td>
</tr>
</tbody>
</table>

Since total probability is 1

$0.1 + 6k = 1$, i.e. $6k = 0.9$, $k = 0.9/6 = 0.15$

(1) Therefore $k = 0.15$
(2) $P(\text{you study at least 2 hrs}) = P(X = 2) + P(X = 3) + P(X = 4)$

$= 5k = 5 \times 0.15 = 0.75$

$P(\text{you study exactly 2 hrs}) = P(X = 2) = 2k = 2 \times 0.15 = 0.3$

$P(\text{you study almost 2 hrs}) = P(X = 0) + P(X = 1) + P(X = 2)$

$= 0.1 + 3k = 0.1 + 3 (0.15) = 0.55$

Q3. Let two pair of dice be thrown and the random variable $X$ is the sum of the numbers on the two dice. Find probability distribution, the mean and variance of $X$.

Solution.

Sample space $S = \{(1, 1), (1, 2)\ldots (1, 6), (2, 1), (2, 2)\ldots (2, 6)\ldots (6, 1), (6, 6)\}$

$X$ takes the values 2, 3, 4, 5, 6, 7, 8, 9, 10, 11 and 12.

Therefore probability distribution is

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<tr>
<th>X</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
<th>8</th>
<th>9</th>
<th>10</th>
<th>11</th>
<th>12</th>
<th>Total</th>
</tr>
</thead>
<tbody>
<tr>
<td>P(X = x)</td>
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<td>$\frac{2}{36}$</td>
<td>$\frac{3}{36}$</td>
<td>$\frac{4}{36}$</td>
<td>$\frac{5}{36}$</td>
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<tr>
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<td>xP(x)</td>
<td>x²P(x)</td>
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<tr>
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</tbody>
</table>

\[
E(X) = \sum xP(x) = \frac{252}{36} = 7
\]

\[
E(X^2) = \sum x^2P(x) = \frac{1974}{36} = 54.83
\]

\[
V(X) = E(X^2) - [E(X)]^2 = 54.83 - 7^2 = 54.83 - 49 = 5.83
\]

**PRACTICAL – 16**

**BINOMIAL DISTRIBUTION**

Q1. An unbiased coin is tossed 5 times. Find the probability of getting (i) 3 heads (ii) atleast 1 head.

Solution.

Here, we have 5 independent trials. Success of getting a head

\( n = 5, \ p = \frac{1}{2}, \ q = 1 - \frac{1}{2} = \frac{1}{2} \)

(i) \( P(3 \text{ heads}) = P(X = 3) = \binom{5}{3} \left(\frac{1}{2}\right)^3 \left(\frac{1}{2}\right)^2 = 10 \times \left(\frac{1}{2}\right)^5 = \frac{10}{32} = \frac{5}{16} \)

(ii) \( P(\text{atleast 1 head}) = 1 - P(\text{no head}) = 1 - P(X = 0) \)

\[
P(X = 0) = \binom{5}{0} \left(\frac{1}{2}\right)^0 \left(\frac{1}{2}\right)^5 = \frac{1}{32} \times P(\text{atleast 1 head}) = 1 - \frac{1}{32} = \frac{31}{32}
\]

Q2. Find the mean & variance of a binomial distribution if \( n = 12, \ p = \frac{1}{2} \)

Solution.

Mean = \( np = 12 \times \frac{1}{2} = 6 \)

Variance = \( npq = 12 \times \frac{1}{2} \times \left(1 - \frac{1}{2}\right) = 12 \times \frac{1}{2} \times \frac{1}{2} = \frac{6}{5} \)
Q3. If the probability that a man aged 60 years will live up to 70 years is 0.65, what is the probability that out of 10 men, now at 60 years of age, selected randomly, 8 will live up to 70 years?

Solution.

Given:

\[ p = 0.65, n = 10, x = 8, \]

\[ q = 1 - p = 0.35 \]

To find \( P(X = 8) \)

\[
P(X = 8) = \binom{n}{x} p^x q^{n-x} = \binom{10}{8} (0.65)^8 \cdot (0.35)^2 = 45 \cdot \left(\frac{65}{100}\right)^8 \cdot \left(\frac{35}{100}\right)^2 = 0.18
\]