SOLUTION
TOPIC: LIMITS, CONTINUITY & DIFFERENTIABILITY

41. **(A)**

$$\lim_{x\to\frac{\pi}{2}}\left[\frac{[\sin x]-[\cos x]+1}{3}\right]=\left[\frac{0-(-1)+1}{3}\right]=0$$

$$\lim_{x\to\frac{\pi}{2}^-}\left[\frac{[\sin x]-[\cos x]+1}{3}\right]=\left[\frac{0-0+1}{3}\right]=0$$

$$\therefore\ \lim_{x\to\frac{\pi}{2}}\left[\frac{[\sin x]-[\cos x]+1}{3}\right]=0$$

42. **(B)**

$$\lim_{x\to\infty}\left(e^x+\pi^x\right)^{\frac{1}{x}}=\lim_{x\to\infty}\left(\left(\frac{e}{\pi}\right)^x+1\right)^{1/x}=\pi$$

Now, $\{\pi\}=\pi-3$

$$\therefore \lim_{x\to\infty}\left\{\left(e^x+\pi^x\right)^{\frac{1}{x}}\right\}=\pi-3$$

43. **(D)**

$$\lim_{x\to\frac{\pi}{4}}\frac{\sqrt{1-\sqrt{\sin 2x}}}{\pi-4x}\times\frac{\sqrt{1+\sqrt{\sin 2x}}}{\sqrt{1+\sqrt{\sin 2x}}}$$

$$=\quad \lim_{x\to\frac{\pi}{4}}\frac{\sqrt{1-\sin 2x}}{(\pi-4x)}\times\lim_{x\to\frac{\pi}{4}}\frac{1}{\sqrt{1+\sqrt{\sin 2x}}}$$

$$=\quad \lim_{x\to\frac{\pi}{4}}\frac{\sqrt{\sin^2\left(\frac{\pi}{4}-x\right)}}{(\pi-4x)}.1$$

$$=\quad \lim_{x\to\frac{\pi}{4}}\frac{\left|\sin^2\left(\frac{\pi}{4}-x\right)\right|}{4\left(\frac{\pi}{4}-x\right)}$$

Which gives RHL = $-\frac{1}{4}$ at x = $\frac{\pi}{4}$ and LHL = $\frac{1}{4}$ at x = $\frac{\pi}{4}$ so, limit does not exist

44. **(C)**

45. (B)

$$f(x) = \begin{cases} \dfrac{2\cos x - \sin 2x}{(\pi - 2x)^2}, & x \le \dfrac{\pi}{2} \\ \dfrac{e^{-\cos x} - 1}{8x - 4\pi}, & x > \dfrac{\pi}{2} \end{cases}$$

L.H.L. at x = $\frac{\pi}{2}$

$$\lim_{h \to 0} \frac{2\sin h - \sin 2h}{4h^2} = \lim_{h \to 0} \frac{2\sin h(1 - \cos h)}{4h^2} = 0$$

R.H.L. = $\lim_{h \to 0} \dfrac{e^{\sin h} - 1}{8\left(\left(\frac{\pi}{2}\right) + h\right) - 4\pi}$

$$= \lim_{h \to 0} \frac{e^{\sin h} - 1}{8h} . \frac{\sin h}{\sin h} = \frac{1}{8}$$

$\Rightarrow$ h(x) has irremovable discontinuity at x = $\frac{\pi}{2}$

46. (A, C, D)

$\forall 1 < x < \frac{\pi}{2}$, $\tan^{-1} \tan x = x$

And x $0 < \log_e x < \log_e \frac{\pi}{2} < 1$

$\Rightarrow f(x) = x$

$\forall \frac{\pi}{2} < x < e, \tan^{-1} \tan x = x - \pi$

And $0 < \log_e x < 1$

$\therefore (\log_e x)^n = 0$

$\Rightarrow f(x) = x - \pi$

And for x > e, $\log_e x > 1, \therefore (\log_e x)^n \to \infty$

$\Rightarrow f(x) = 0$

47. (B, C, D)

(A) Clearly f(x) is continuous at x = 1

(B) $g(1^+) = 0, g(1) = 1 \Rightarrow g(x)$ is discontinuous at x = 1

(C) $h(1^+) = 1$ and $h(1^-) = 0 \Rightarrow h(x)$ is discontinuous at x = 1

(D) $\phi(1^+) = 1$ and $\phi(1^-) = -1 \Rightarrow \phi(x)$ is discontinuous at x = 1

48. **(A, D)**

$$\lim_{h\to 0} f(0+h) = \lim_{h\to 0}\left(1+\frac{ah+bh^3}{h^2}\right)^{1/h}$$

$$= \lim_{h\to 0} e^{\frac{1}{h}\ln\left(1+\frac{ah+bh^3}{h^2}\right)}$$

For limit to exist, we must have

$$\lim_{h\to 0}\frac{ah+bh^3}{h^2} = 0$$

$\therefore$ a = 0

So, we have

$$\lim_{h\to 0} f(0+h) = \lim_{h\to 0}(1+bh)^{1/h}$$

$$= \lim_{h\to 0}(1+bh)^{(1/bh)^b} = e^b$$

For f(x) to be continuous at x = 0, we must have

$$\lim_{x\to 0^+} f(x) = f(0)$$

$$\Rightarrow e^b = 3$$

$$\therefore b = \log_e 3$$

49. **(B, D)**

$$\lim_{x\to 0^+}\left(3-\left[\cot^{-1}\left(\frac{2x^3-3}{x^2}\right)\right]\right) = 3-\left[\cot^{-1}(-\infty)\right] = 0$$

$$\lim_{x\to 0^+}\{x^2\}\cos\left(e^{1/x}\right) = 0\times(\text{value between } -1 \text{and} 1)$$

= 0

50. **(A, B, C, D)**

$$f(1^+) = \lim_{x\to 1^+}\left(x\left[\frac{1}{x}\right]+x[x]\right)$$

$$= \lim_{x\to 1^+}(x(0)+x(1))$$

= 1

$$f(1^-) = \lim_{x\to 1^-}\left(x\left[\frac{1}{x}\right]+x[x]\right)$$

$$= \lim_{x\to 1^-}(x(1)+x(0))$$

= 1

$$f\left(2^{+}\right)=\lim_{x\to 2^{+}}\left(x\left[\frac{1}{x}\right]+x[x]\right)$$

$$=\lim_{x\to 2^{+}}\left(x(0)+x(2)\right)$$

= 4

$$f\left(2^{-}\right)=\lim_{x\to 2^{-}}\left(x\left[\frac{1}{x}\right]+x[x]\right)$$

$$=\lim_{x\to 2^{-}}\left(x(0)+x(1)\right)$$

= 2

Obviously f(x) is discontinuous at all positive integers but at x = 1 it has removable discontinuity

51. **(A,B,)**

$$\lim_{x\to 0^{+}} x\left(\frac{e^{|x|+[x]}-2}{|x|+[x]}\right)$$

$$=\lim_{x\to 0^{+}} x\left(\frac{e^{x+0}-2}{x+0}\right)$$

$$=\lim_{x\to 0^{+}} x\left(e^{x}-2\right)$$

= 1 – 2 = 0

$$=\lim_{x\to 0^{-}} x\left(\frac{e^{|x|+[x]}-2}{|x|+[x]}\right)$$

$$=\lim_{x\to 0^{-}} x\left(\frac{e^{-x-a}-2}{-x-1}\right)=0$$

52. **(C)**

$$f(x)=x^{1/3}(x-2)^{2/3}$$

$$\therefore\ f'(x)=x^{1/3}.\frac{2}{3}(x-2)^{-1/3}+(x-2)^{2/3}.\frac{1}{3}x^{-2/3}$$

$$=\frac{1}{3}x^{-2/3}(x-2)^{-1/3}(3x-2)$$

$\therefore$ f ' is not defined at x = 0 and at x = 2

53. **(B)**

54. **(B,C)**

55. **(A,C)**

56. **(zero)**

57. **(1)**

58. **(1)**

59. **(4)**

60. **(5)**