1. (C)

2. (B)
\[ \Delta x_{cm} = \frac{M_1 \Delta x_1 + M_2 \Delta x_2}{M_1 + M_2} \]

3. (C)
Initially particle will remain stationary till time \( t_1 \),

Here \( t_1 = \frac{L}{V} \)

Centre of mass velocity \( V_{cm} = \frac{MV}{m + M} \)

After this time particle will fall down vertically and reaches ground in time \( t_2 \)

\[ t_2 = \sqrt{\frac{2h}{g}} \]

\( x_{cm} = V_{cm} (t_1 + t_2) \)

4. (C)

\[ h_{cm} = \int y \, dm = \int y \, dV \]
\[ dV = (\pi x^2) \, dy \]

5. (A)

\[ \bar{x}_{cm} = \frac{M_1 (0) - M_2 \left( \frac{R}{10} \right)}{M_1 + M_2} \]

\[ \bar{y}_{cm} = \frac{M_1 \left( \frac{3R}{8} \right) - M_2 \left( \frac{3R}{16} \right)}{M_1 + M_2} \]

6. (B)

Hint: L-conservation \( \Rightarrow MV_{cm} \frac{L}{2} + \frac{ML^2}{12} = \frac{MV}{2} \Rightarrow v_{cm} + \frac{\omega L}{6} = \omega \)

P-conservation \( \Rightarrow \frac{\omega L}{2} - v_{cm} = v \Rightarrow \omega \left( \frac{L}{2} + \frac{L}{6} \right) = 2v \)

\[ \therefore \omega = \frac{3v}{L} \]
7. (A)

8. (D)
Let $f$ be the force of friction on the ring towards right. For pure rolling instantaneous point of contact is at rest.

\[ F + f = ma ; \]
\[ \text{Torque } T = mR^2 \alpha \]
\[ R(F - f) = mR^2 \frac{a}{R} ; \]
So, $f = 0$

9. (C)
Conservation of angular momentum
\[ I, w_1 + m(r-h)v = I, w_2 + mrw_2 \]
\[ w_2 = \frac{(2r-h)v}{2r^2} \]
Conservation of energy
\[ \frac{1}{2}(I + mr^2)w_2^2 \geq mgh \]

10. (C)
\[ mg \sin \theta - f = ma \]
\[ fR = mR^2 \alpha \]
\[ a = R\alpha \]
\[ \text{or } f = \frac{mg \sin \theta}{2} . \]

11. (A, C, D)
In both CM and ground frame, $K_{\text{max}}$ is there, when $x$ is zero in spring, which occurs simultaneously.
\[ v_{CM} = \frac{m(v_0) + 0}{5m} = \frac{v_0}{5} \]
\[ K_{\text{max CM}} = \frac{1}{2} m \left( \frac{4v_0}{5} \right)^2 + \frac{1}{2} (4m) \left( \frac{v_0}{5} \right)^2 = \frac{2}{5} mv_0^2 \]
\[ K_{\text{max ground}} = \frac{1}{2} mv_0^2 \]
\[ K_{\text{min CM}} = 0 \]
\[ K_{\text{min ground}} = \frac{1}{2} (m + 4m) v_{CM}^2 = \frac{mv_0^2}{10} \]
\[ K_{\text{max m}} = \frac{1}{2} mv_0^2 \text{ (ground frame)} \]
\[ K_{\text{min m}} = 0 \text{ (ground frame when energy is shared by spring and 4m and m will reverse direction of motion)} \]

12. (A, B, C, D)
Let the weight of each brick be $W$ and length $l$. As bricks are homogeneous, the center of gravity of each brick must be at the mid point. Therefore, the topmost brick will be in equilibrium of its center of gravity lies at the edge of brick below it,
i.e., II brick. Thus the topmost brick can have maximum equilibrium extension of $l/2$.

![Diagram showing equilibrium extension](image)

$C_1$ is the center of mass of the top two bricks which lies on the edge of the third brick.

$C_2$ is the center of mass of the top three bricks which lies on the edge of the fourth brick.

Now let the center of gravity of the top two bricks be located at a distance $x$ from the edge of third brick. Then for equilibrium $W_x = W \left( \frac{l}{2} - x \right)$ or $x = l/4$

Thus the second brick from the top overhangs the third brick by $l/4$.

Now we find the combined center of gravity of the top three bricks, let it be located at a distance $x'$ from the edge of the fourth brick. The center of gravity of the third brick along is at a distance $l/2$ from the edge. Then for equilibrium

$$W \left( \frac{l}{2} - x' \right) = 2Wx' \quad \text{or} \quad x' = \frac{l}{6}$$

Thus maximum overhanging length to top from the edge of bottom brick is

$$\frac{l}{2} + \frac{l}{4} + \frac{l}{6} = \frac{11l}{12}$$

13. **(B,C,D)**

Centre of mass always lies b/w centre of mass of segments and always lies on axis of symmetry.

14. **(A,D)**

15. **(A,B,C)**

$$y_{cm} = \frac{A_1 y_1 - A_2 y_2}{A_1 - A_2} = \frac{0 - \pi \left( \frac{R}{4} \right)^2 \times \frac{R}{2}}{\pi R^2 - \pi \left( \frac{R}{4} \right)^2} = -R \frac{30}{\pi R^2}$$

$$I = \frac{1}{2} \left( \frac{16m}{15} \right) R^2 - \left[ \frac{1 m}{2} \left( \frac{R}{4} \right)^2 + \frac{m}{15} \left( \frac{R}{2} \right)^2 \right] \approx \frac{1}{2} mR^2$$

On slight rolling

Total energy associated is

$$E = \frac{1}{2} I \omega^2 + \frac{1}{2} mv^2 + mg \left( 1 - \cos \theta \right) \frac{R}{30}$$

$$\frac{dE}{dt} = 0 \Rightarrow \alpha = \frac{-mgR \theta}{30 \left( I + mR^2 \right)} = -\omega^2 \theta$$

$$\therefore \quad T = 6\pi \sqrt{\frac{5R}{g}}$$

16. **(A,B,C,D)**

At A:
for a rolling wheel, \( a = R \alpha \)
\[ \therefore \text{(A) is correct.} \]
At B :
\[
\begin{array}{c}
\text{If } \frac{V^2}{R} = a \text{ then } a_B \text{ may be vertically downwards} \\
\therefore \text{(B) is correct.}
\end{array}
\]
At C :
\[ \therefore \text{(C) is correct.} \]
Consider this
\[ \therefore \text{(D) is correct.} \]

17. **(A,B,D)**

Apply COLM

\[
|J| = m(u - v - \frac{\omega \ell}{4}) = mv
\]
\[ \Rightarrow u = 2v + \frac{\omega \ell}{4} \quad \text{... (i)} \]

Apply COAM (about O)
\[ 0 = m \left( u - v - \frac{\omega \ell}{4} \right) \cdot \frac{\ell}{4} - \frac{m \ell^2}{12} \cdot \omega \]
\[ u = v + \frac{7\omega \ell}{12} \quad \text{... (ii)} \]

equation (i) = equation (ii)
\[ 2v + \frac{\omega \ell}{4} = v + \frac{7\omega \ell}{12} \]
\[ v = \frac{\omega \ell}{3} \]
\[ \omega = \frac{12u}{11\ell}, \quad v = \frac{4u}{11}. \]

18. (A,B,D)
\[ J = m v \]
\[ J \cdot (h - R) = \frac{2}{5} m R^2 \cdot \omega \]
\[ v \cdot (h - R) = \frac{2}{5} m R^2 \cdot \omega. \]

19. (D)
\[ a_\alpha = \frac{1}{2} \alpha + \alpha x \cos 30^\circ \]
\[ 2\alpha y = l\alpha + \sqrt{3} \alpha x \quad \text{and} \quad \alpha x = \frac{x}{2} \]

20. (A,C,D)

21. (A)
\[ \bar{V}_{cm} = \frac{m_1 \bar{V}_1 + m_2 \bar{V}_2 + \ldots + m_n \bar{V}_n}{m_1 + m_2 + \ldots + m_n} \]
\[ \Rightarrow M \bar{V}_{cm} = \bar{p}_1 + \bar{p}_2 + \ldots + \bar{p}_n \]

22. (A)
\[ \Delta x_{cm} = 0 = \frac{m_1 x d + m_2 x}{m_1 + m_2} \]
\[ \rightarrow x = - \left( \frac{m_1}{m_2} \right) d \]
Is \( x \) should be opposite to \( d \).

23. (D)

24. 25. 26
A,C,C
Let \( \alpha \) be the angular acceleration of rod about hinge and \( a \) be the acceleration of block just after the system is released from rest.
The free body diagram of red, pulley and block are as shown in figure. In free body diagram of rod, the force exerted by hinge on rod is not shown.
Initial velocity of particle is,
For rod,
\[ mg \times \frac{L}{2} \cos 37^\circ - T \times L \cos 37^\circ = -I \alpha \left\lbrack I = \frac{ml^2}{3} \right\rbrack \]
For block,
\[ 2mg - T = 2ma \]
From constraint, \( a = \frac{4L\alpha}{5} \cos 37^0 \)

Solving above equations, we get
\[ a = \frac{72g}{121}, \alpha = \frac{90g}{121L}, T = \frac{98mg}{121} \]

Reaction force exerted by \( H_2 \) on pulley is,
\[ N_1 = 2T + 2mg = \frac{438}{121}mg \]

Now draw the complete free body diagram of rod as follows
\[ \Rightarrow R_1 + T \cos 37^0 - mg \cos 37^0 = \frac{mL}{2} \alpha \]
\[ \Rightarrow R_1 + T \sin 37^0 = mg \sin 37^0 \]

Net reaction force on rod due to hinge is,
\[ F = \sqrt{R_1^2 + R_2^2} \]

27. (8)

FBD of cylinder and block are as shown by Newton’s laws
\[ 40 - 2T - f_s = 8a \tag{1} \]
\[ T - f_s = 4a/2 \tag{2} \]

Subtracting equation (2) from equation (1)
\[ 40 - 3T = 6a \]
\[ T = \frac{40 - 6a}{3} \]

Also, by \( \tau = I\alpha \), we get
\[ T \times R + f_s \times R = I \frac{3a}{2R} \]
\[ \Rightarrow T + f_s = I \frac{3a}{2R^2} \]
\[ \Rightarrow (40 - 6a)/3 + (40 - 12a)/3 = 3Ia/2R^2 \Rightarrow a = \frac{80}{18 + 9I/2R^2} \]

As \( \tau = I\alpha = I \times (3a/2R) \)
\[ \therefore \tau = \frac{3I}{2R} \frac{80}{18 + 9I/2R^2} \Rightarrow \tau = \frac{3 \times 80}{18 + 9} = \frac{80}{9} \text{ N-m} \]

28. (3 m/s)

applying conservation of angular momentum about point P
\[ mv_R - \frac{2}{5} MR^2 \times \frac{V_0}{R} = mv_R + \frac{2}{5} mR^2 \times \frac{V}{R} \]

\[ \Rightarrow mv_R \left( 1 - \frac{2}{5} \right) = mv_R \left( 1 + \frac{2}{5} \right) \]

\[ \Rightarrow v = \frac{3v_0}{7} = \frac{3 \times 7}{7} = 3 \text{m/s} \]

29. (3)

\[ v_1 = \left( R + \frac{R}{2} \right) w \]

\[ = \frac{3}{2} RW = \frac{3}{2} v \]

\[ v_2 = \left( R - \frac{R}{2} \right) W \]

\[ = \frac{R}{2} W = \frac{v}{2} \]

\[ \frac{v_1}{v_2} = \frac{3}{1} \]

30. (1)

The torque applied to the wheel is

\[ \tau = \bar{r} \times (20 \text{Nm}) \times (0.20 \text{m}) = 4.0 \text{Nm} \]

The angular acceleration produced is

\[ \alpha = \frac{\tau}{I} = \frac{4.0 \text{Nm}}{0.20 \text{kgm}^2} = 20 \text{rad/s}^2 \]

The angular velocity after 5.0 seconds is

\[ \phi = \alpha t = (20 \text{ rad/s}^2) (5.0 \text{ s}) = 100 \text{ rad/s} \]

31. (1)

Before cutting string

\[ N_1 = T = Mg/2 \]

After cutting string

\[ Mg - N_2 = Ma \quad \text{(i)} \]

\[ N_2 l \sin \theta = Ma \left( 2l \right)^2 / 12 \quad \text{(ii)} \]

By constraint that lower end will have acceleration only in horizontal direction.
32. (4)
The rod will rotate about point A. Let \( a \) be the linear acceleration of centre of mass of the rod is \( a \) and \( \alpha \) be the angular acceleration of the rod about A. Then

\[ \ell \sin \theta = a \]  

from (i), (ii) and (iii)

\[ \Rightarrow N_2 = \frac{mg}{\left(3 \sin^2 \theta + 1\right)} \]

\[ \Rightarrow \frac{N_1}{N_2} = \frac{\left(3 \sin^2 \theta + 1\right)}{2} = 1 \]

33. (5)
In ground frame

\[ MV' = MV \]
\[ R\omega - V = V' \]

\[ \therefore V = \frac{\omega R}{2} \]
\[ \frac{1}{x} = \frac{\frac{1}{2} MV^2 + \frac{1}{2} \frac{1}{5} MR^3 \omega^2}{\frac{1}{2} MV^2} = 1 + \frac{8}{5} = \frac{13}{5} \]

\[ \Rightarrow 13x = 5. \]

34. (4)
\[ dm = \frac{m}{\pi R} \left( \frac{d\ell}{2} \right) \]
\[ = \frac{2M}{\pi R} (Rd\theta) \]
\[ = \frac{2Md\theta}{\pi} \]

\[ y_{cm} = \frac{\int ydm}{M} = \frac{2MR}{\pi} \int_{-\pi/4}^{\pi/4} \cos \theta d\theta \]
\[ = \frac{2R}{\pi} \left[ \sin \theta \right]_{-\pi/4}^{\pi/4} = \frac{2R}{\pi} \left[ \frac{1}{\sqrt{2}} + \frac{1}{\sqrt{2}} \right] \]
\[ = \frac{2\sqrt{2}R}{\pi} = \frac{2\sqrt{2} \times \sqrt{2} \pi}{\pi} = 4 \]

35. (2)
\[ dm = \left[ \frac{M_0}{L} x \right] dx \]

For centre of mass \[ x_{cm} = \frac{\int_0^L dxm}{\int_0^L dm} = \frac{2L}{3} \]

36. (4)

Mass of cut out disc = \[ m = \frac{M}{\pi R^2} \left( \frac{R}{2} \right)^2 = \frac{M}{4} \]

From centre of original uniform disc the distance of centre of mass of final disc
\[ X_{cm} = \frac{M \times 0 - \frac{M}{4} \left( -\frac{R}{2} \right) + \frac{M}{4} \left( \frac{R}{2} \right)}{M - \frac{M}{4} + \frac{M}{4}} = \frac{R}{4} \]

37. A-S ; B-QT ; C-PQR ; D-PGR
38. A – QT ; B – P ; C – RS ; D – RS
From toppling about A
\[ F \cos \theta = \frac{mg}{2} \]
\[ \Rightarrow F \geq \frac{mg}{2 \cos \theta} \]
Minimum force that can overturn cube is equal to \( mg/2 \) and directed horizontal.
In this condition for no slipping
\[ F \leq \mu \cdot mg \]
\[ \Rightarrow \frac{mg}{2} \leq \mu \cdot mg \]
\[ \Rightarrow \mu \leq 1/2 \]
But for \( \mu < 1/2 \)
\[ \frac{mg}{2 \cos \theta} \leq F \cos \theta \leq \mu \cdot (mg + F \sin \theta) \]
\[ \Rightarrow \tan \theta \geq \left( \frac{1-2\mu}{\mu} \right) \]
\[ F_{\text{min}} = \frac{mg}{2 \cos \theta} \]
\( \cos \theta \) is maximum when \( \tan \theta = \left( \frac{1-2\mu}{\mu} \right) \) for condition \( \mu < 1/2 \)
\[ \Rightarrow F_{\text{min}} = \frac{mg(\sqrt{\mu^2 + (1-2\mu)^2})}{2\mu} = \frac{mg(\sqrt{5\mu^2 - 4\mu + 1})}{2\mu} \]
\[ \Rightarrow \text{For } \mu = 1/3 \]
\[ F_{\text{min}} = \frac{mg}{\sqrt{2}} \text{ and } \theta = \tan^{-1}(1) = 45^\circ \]
For \( \mu = 1/4 \) \( F_{\text{min}} = \sqrt{5} (mg/2) \text{ and } \theta = \tan^{-1}(2) \)
For \( \mu = 3/4 \text{ and } \mu = 2/3 \)
\[ F_{\text{min}} = mg/2 \text{ and } \theta = 0^\circ \]

39. A – PRS ; B – QRS ; C – QRS ; D – PRS

40. A – S ; B – P ; C – P ; D – QR