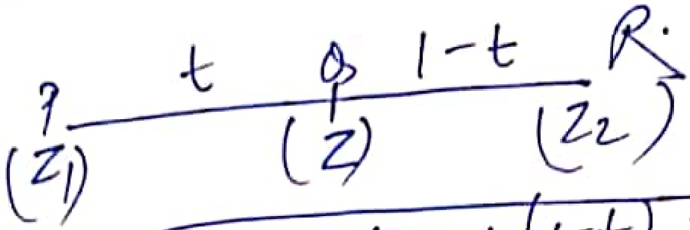
(46.)



$$\boxed{z = tz_2 + (1-t)z_1}$$

z lies b/w line segment joining z_1 & z_2

So, $\boxed{PQ + QR = PR}$

$$\boxed{|z-z_1| + |z-z_2| = |z_1-z_2|}$$ proved

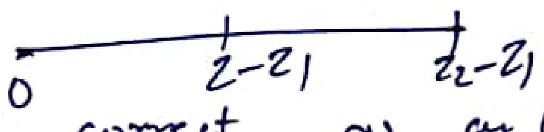
(A) is correct

$$\frac{z-z_1}{\bar{z}-\bar{z}_1} = \frac{z_2-z_1}{\bar{z}_2-\bar{z}_1} \quad \text{(complex slope)}$$

$$\Rightarrow (z-z_1)(\bar{z}_2-\bar{z}_1) - (\bar{z}-\bar{z}_1)(z_2-z_1) = 0$$

(C) is correct

On shifting origin to P



(D) is also correct as $(z-z_1) = a(z_2-z_1)$

$$|z_1+z_2|^2 = |z_1|^2 + |z_2|^2 + 2\,\mathrm{Re}\, z_1\bar{z_2}$$

$$\Rightarrow |z_1+z_2|^2 = 2 \Rightarrow (a+c)^2 + (b+d)^2 = 2$$

$$|\omega_1+\omega_2|^2 = |\omega_1|^2 + |\omega_2|^2 + 2\,\mathrm{Re}\,\omega_1\bar{\omega_2}$$

$$(a+c)^2 + (b+d)^2 = 2$$

$$\underbrace{a^2+c^2} + \underbrace{b^2+d^2} + 2(ac+bd) = 2$$

$$|\omega_1|^2 + |\omega_2|^2 = 2 \quad \left(\text{as } \mathrm{Re}\, z_1\bar{z_2} = 0\right)$$

So, $|\omega_1| = 1 \Rightarrow |\omega_2| = 1$ & $\mathrm{Re}\,\omega_1\bar{\omega_2} = 0$

Hence, (A), (B), (C) are correct.

48. $\arg(z-a) - \arg(z+a) = \pi/2$

$\boxed{AD}$ $\Rightarrow \arg\left(\frac{z-a}{z+a}\right) = \pi/2 \Rightarrow \frac{z-a}{z+a}$ = purely imaginary

$$\frac{z-a}{z+a} = -\frac{\bar{z}-a}{\bar{z}+a}$$

$\Rightarrow z\bar{z} + \cancel{az} - \cancel{a\bar{z}} - a^2 = -z\bar{z} + \cancel{az} - \cancel{a\bar{z}} + a^2$

$\Rightarrow |z|^2 = a^2$ let, $z = x+iy$

$\Rightarrow |z| = a$ On solving by using $\arg z = \tan^{-1}\left(\frac{y}{x}\right)$

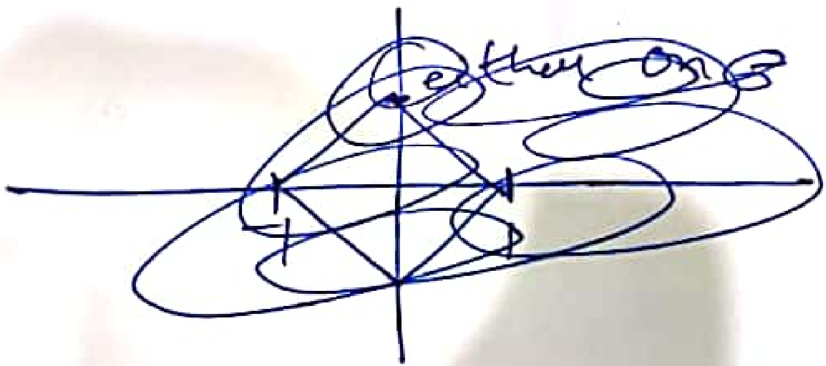
So, we get $y = \sqrt{3}x \Rightarrow \arg z = \pi/3$ Ans

(49.) $|z_1^2 - z_2^2| = |(\bar{z}_1 - \bar{z}_2)^2|$

$\Rightarrow |z_1 - z_2|\,|z_1 + z_2| = |\bar{z}_1 - \bar{z}_2|^2 = |z_1 - z_2|^2$

$\Rightarrow |z_1 + z_2| = |z_1 - z_2|$ (as $z_1 \neq z_2$)

$\Rightarrow \left|\frac{z_1}{z_2} + 1\right| = \left|\frac{z_1}{z_2} - 1\right|$ $\quad$ $\frac{z_1}{z_2}$ is equidistant from -1 & 1 so



$\frac{z_1}{z_2}$ = purely imaginary

& $\arg\left(\frac{z_1}{z_2}\right) = \pm \frac{\pi}{2}$

$\Rightarrow \arg z_1 - \arg z_2 = \pm \frac{\pi}{2}$

$\Rightarrow |\arg z_1 - \arg z_2| = \frac{\pi}{2}$

(A) & (D)

Scanned with CamScanner

(41.) Let, $z = kj$ be the root

$$a(kj)^2 + b(kj) + c = 0$$
$$\Rightarrow -ak^2 + bjk + c = 0$$
$$\Rightarrow \boxed{ak^2 - bjk - c = 0} \longrightarrow (1)$$

then, $\bar{z} = -kj$

$$az^2 + bz + c = 0$$
$$\Rightarrow \bar{a}\bar{z}^2 + \bar{b}\bar{z} + \bar{c} = 0$$
$$\Rightarrow \bar{a}(-kj)^2 + \bar{b}(-kj) + \bar{c} = 0$$
$$\Rightarrow -\bar{a}k^2 - \bar{b}kj + \bar{c} = 0$$
$$\Rightarrow \boxed{\bar{a}k^2 + \bar{b}kj - \bar{c} = 0} \longrightarrow (2)$$

(1) & (2) have one common root

$$(a\bar{b}j + \bar{a}bj)(b\bar{c}j + \bar{b}jc) = (-a\bar{c} + \bar{a}c)^2$$

$$\Rightarrow (c\bar{a} - a\bar{c})^2 = -(b\bar{c} + \bar{b}c)(a\bar{b} + \bar{a}b)$$

My answer is (D) None of these

Scanned with CamScanner

(42) $2\left|1+\alpha+\alpha^2+\frac{1}{\alpha}\right|$

$$= \frac{2\left|1+\alpha+\alpha^2+\alpha^3\right|}{|\alpha|}$$

$$= 2\left|1+\alpha+\alpha^2+\alpha^3\right|$$

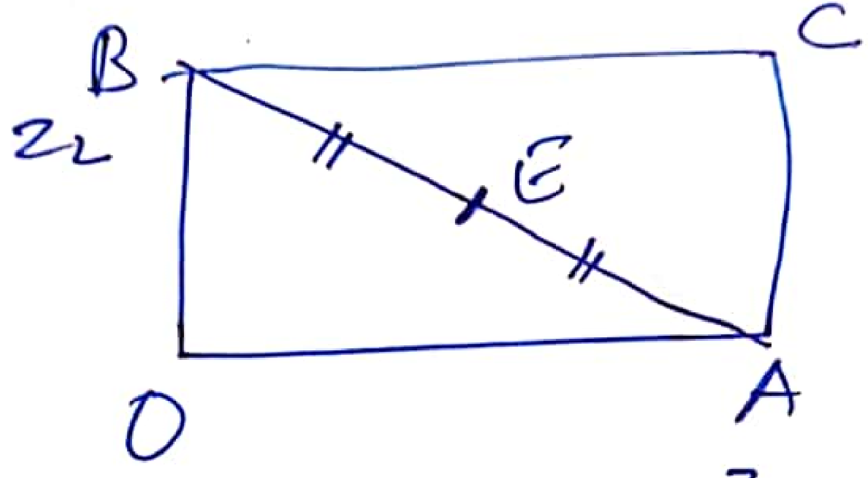
$$= 2\left|-\alpha^4\right| \quad \text{(as sum of roots of unity} = 0\text{)}$$

$$= 2|\alpha|^4$$

$$= 2$$

My Ans (B) but given answer (C)

43.



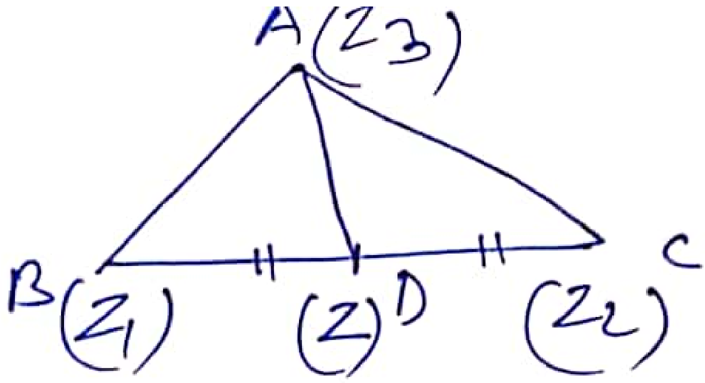
$AB = |z_1 - z_2|$

$OC = |z_1 + z_2|$

then $\triangle AOB$ is right $\triangle$.

$E = \dfrac{z_1 + z_2}{2}$ is the circumcenter

44.



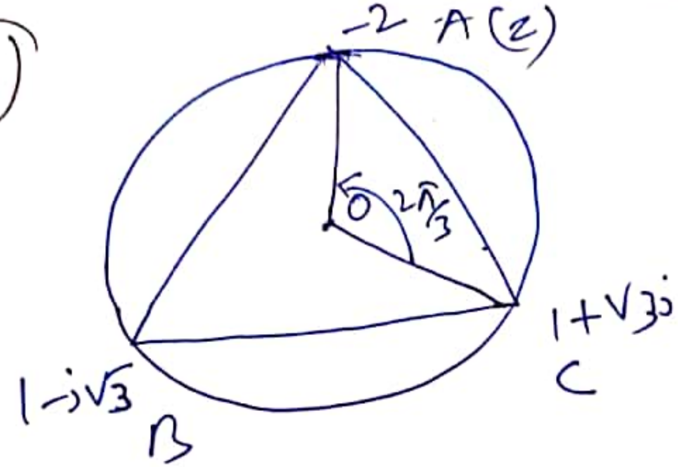
D is the circumcentre &

Δ is right triangle

$$\angle A = \arg\left(\frac{z_3 - z_2}{z_3 - z_1}\right) = \boxed{\frac{\pi}{2}}$$

(A) is the correct choice.

45.



$|z| = 2$

$$\frac{z - 0}{1+\sqrt{3}i} = e^{j2\pi/3}$$

$$z = \left(-\frac{1}{2} + \frac{j\sqrt{3}}{2}\right)\left(1+\sqrt{3}i\right)$$

$$= -\frac{\cancel{4}}{\cancel{2}}\,2 = -2$$

lly using coni method other vertex will $1 - j\sqrt{3}$

(57) $\cos z = \dfrac{e^{jz} + e^{-jz}}{2}$

$$= \frac{e^{-\log(2-\sqrt{3})} + e^{\log(2-\sqrt{3})}}{2}$$

$$= \frac{2-\sqrt{3} + 2+\sqrt{3}}{2}$$

$$= 2$$

$\boxed{\cos z = 2}$ Ans

58. put, $z = i$

59. $|\bar{z}_1 + \bar{z}_2 + \bar{z}_3| = \left|\frac{1}{z_1} + \frac{4}{z_2} + \frac{9}{z_3}\right|$ as $|z_1| = 1$, $|z_2| = 2$, $|z_3| = 3$

$$= \frac{|9z_1z_2 + 4z_1z_3 + z_2z_3|}{|z_1||z_2||z_3|}$$

$$= \frac{12}{1 \times 2 \times 3} = 2 \quad \text{Ans}$$

Scanned with CamScanner