IIT – JEE 2019 **TW TEST (NAPJC – 1,2,3,4,5,6)** **DATE: 4/11/17**

TOPIC: PERMUTATION & COMBINATION

SOLUTION

1. **(D)**

$x^2+6x+y^2=4$

$(x+3)^2+y^2=13$

$x^2+y^2=13$

$x\pm2, y\pm3$

Or

$x=\pm3,\ y=\pm2$

2. **(A)**

1999 nos from 2 to 2000

Nos divisible by 3 are

3,6,9, …….1998 666 nos

Nos divisible by 2

2,4,6, …….. 2000 100 nos

Nos divisible by 6

6,12,………1998 333 nos

Nos 'n' uch that HCF (n, 36) = 1

$=1999-666-1000+333$

= 666

3. **(A)**

S Y T E M C

S T A

Select 2 vowels/3

(SE) (SA) Y T T I mC

group

$\frac{8!}{2!}$ arrangement

${}^{3}C_{2}\ {}^{8}P_{6}$

4. **(B)**

x_1 no of $\rightarrow$ x_3 no of $\uparrow$

x_2 no of x_4 no of $\downarrow$

$x_1 + x_2 + x_3 + x_4 = 6$

$x_1 = x_2 + 1$

$x_3 = x_4 + 1$

$2(x_2 + x_4) = 4$

$x_2 + x_4 = 2$

$120 + 180 = 300$

X_1	X_2	X_3	X_4	
1	0	3	2	$6!/1:3!2!$
2	1	2	1	$6!/2:2:1!1!$
3	2	1	0	$6!./1!3!2!$

5. **(B)**

$6^n = 5K + 1$

$9^m = 5K - 1$ if m = odd

n can be chosen in 50w

m can be chosen in 25w

1250w

6. **(C)**

A_1 marked

A_{16} marked

A_{991} marked

A_6 marked

A_{996} marked

A_{11} marked

A_{986} marked

A_1 marked

15 K + 1 marked

15 K + 6 marked

15K + 11 marked

Total marked

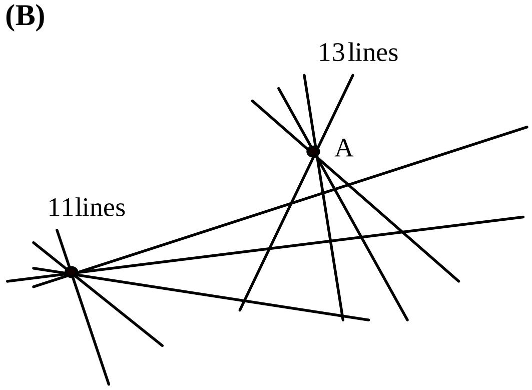
= 67 + 67 + 66

= 200

Total unmarked

= 800

7. **(B)**



Total Pts. = ${}^{37}C_2 - {}^{11}C_2 - {}^{13}C_2 + 2$

$= 37 \times 18 - 11 \times 5 - 13 \times 6 + 2$

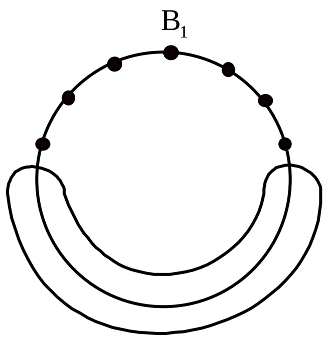
$= 540 + 70 + 56 - 55 - 78 + 2$

$= 541 - 6$

$= 535$

8. **(C)**

B_1 can sit in 1 way



B_2 can sit in 13w

13 seats remains 18!w

$13 \times 18!$

9. **(A)**

P_1 can sit in 2010 ways

Remain 4! Ways

So $2010 \times 4!$

$= 402 \times 5!$

10. **(C)**

If there are 1 nos after decimal pt then no of nos = ${}^9C_1$

If 2 nos after decimal = ${}^9C_2$

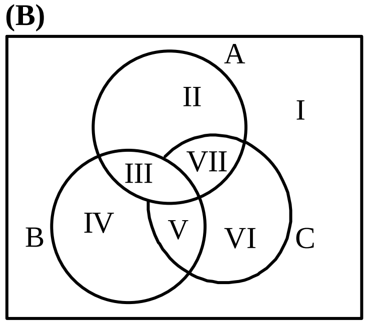
If 3 nos after decimal = ${}^9C_3$

If 9 nos after decimal = ${}^9C_9$

$= {}^9C_1 + {}^9C_2 + ... + {}^9C_9$

$= 2^9 - 1$

11. **(B)**



Total = 7^n

Any element has 7 regions

6^n : AnB = ϕ

6^n : BnC = ϕ

5^n : If AnB & BnC = ϕ

$= 7^n - 2.6^n + 5^n$

12. **(A)**

No of elements where
1 is minimum is $(n-1)$

No of element where
2 is minimum is (n – 2)

No of elements where $(n-1)$ is minimum is 1

$$\sum_{r=1}^{n-1} r(n-r)$$

$$= n\left(\frac{n(n-1)}{2}\right) - \frac{(n-1)(n)(2n-1)}{6}$$

$$= \frac{n(n-1)}{2}\left\{n - \left(\frac{2n-1}{3}\right)\right\}^{6} = {}^{n+1}C_3$$

13. **(C)**

$$\sum_{i=1}^{k} x_i = 93$$

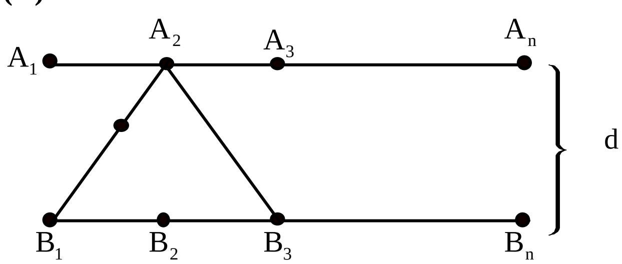
$$\sum_{i-1}^{k} \frac{1}{x_i} = \frac{\Sigma x_i}{N}$$

$\because$ N = 50 = $5^2 2^1$

So # of division = $(2+1)(1+1)$ = 6

K = 6

14. **(A)**



# of A's = ${}^nC_1\ {}^nC_2 \times 2$

$= nn(n-1)$

$= n^2(n-1)$

15. **(D)**

Case - 1

Right ∡d Δs

Select 1 column A_kB_k

${}^nC_1$ ways

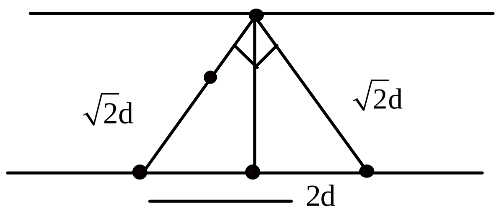
Select next vertex in ways

$n(2n-2)$

Case - 2

$A_1A_3B_2$

$A_2A_4B_3$



$(n-2)2 = 2n-4$ ways

$A_{n-2}A_nB_{n-1}$

Total $2n^2-2n+2n-4$

$2n^2-4$

16. **(D)**

Isosceles triangles

Case – 1

Select 2 odd/even vertices on A_1, s'

$A_i, A_j \ \& B_{\frac{i+j}{2}}$

$\frac{n}{{}^2C_2}(2)$

Case : 2

Select 2 odd/ even vertices on B_1,s'

$\frac{n}{{}^2C_2}(2)$

Case 3:

Right $\measuredangle$d isosceles Δ with AiBi as column

$\left.\begin{matrix} A_1B_1B_2 \\ A_1B_1A_2 \end{matrix}\right\} 2w$

3 rd vertex

$A_2B_2A_1A_3B, B_3 \quad 4w$

$2+2+4(n-2)$

$= 4+4n-8$

$= 4n-4$

3rd vertex

$A_{n-1}B_{n-1}A_nB_nA_{n-2}B_{n-2} \ 4w$

3rd vertex

$A_nB_nA_{n-1}B_{n-1} \ 2w$

$$4\left(\frac{\left(\frac{n}{2}\right)\left(\frac{n}{2}-1\right)}{2}\right)+4n-y$$

$$= n\left(\frac{n-2}{2}\right)+4n-y$$

$=\frac{n^2-2n+8n-8}{2}$

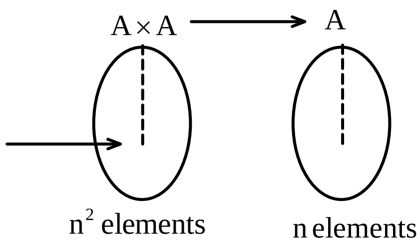
$=\frac{n^2+6n-8}{2}$

17. **(A)**

$n(A\times A)=n^2$

$n(A*A)={}^nC_2+{}^nC_1$

$=\frac{n(n+1)}{2}$



n^{n^2} ways F"s

$A*A\to A$

18. **(C)**

# of F"s = $n^{\frac{n(n+1)}{2}}$

19. **(A)**

$f(\{a_i,a_j\})=(a_i,a_j)$ or (a_j,a_i) 2 ways

If $i=j$

$f(\{a_i,a_j\})=(a_i,a_j)$ If $i\neq j$ then fore

Each $\{a_{i,}a_j\}$ we have

2w . So

$2^{{}^nC_2}$ ways

If i = j we have 1 way

so $1\,{}^nC_1w$

$2^{{}^nC_2}$

20. **(D)**

f(1) : k ways

$f(2):(k-1)$ ways

$f(n):(k-1)w$

Total = $k(k-1)^{n-1}$

21. **(A)**

$$k(k-1)^n$$

If $f(n)=f(1)$	$f(n)\neq f(1)$
$f(n-1)\neq f(1)$	$c(n,k)$
$c(n-1,k)$	

$c(n-1,k)+c(n,k)=k(k-1)^{n-1}$

$c(n,k)=k(k-1)^{n-1}\,c(n-1,k)$

22. **(C)**

$k(k-1)^{n-1}-k(k-1)^{n-2}+k(k-1)^{n-3}\,\ldots\ldots\ldots+k(k-1)^{1}$

$=c(n_1k)-c(1,k)$

$$k(k-1)^{n-1}\left(\frac{1-\left(-\frac{1}{k-1}\right)^{n-1}}{1-\left(\frac{-1}{k-1}\right)}\right)=c(n,k)-0$$

$$(k-1)^n\left(1+\frac{(-1)^n}{(k-1)^{n-1}}\right)=c(n_1k)$$

$c(n_1k)=(k-1)^n+(-1)^n(k-1)$

23. **(B)**

$3375=3\times1125$

$=3\times9\times125$ $\qquad$ $1125=3^2 5^3$

$=5^3 3^3$

$x=3^{\alpha_1}5^{\beta_1}$ $\qquad$ $\max(\alpha_1,\alpha_2)=3$

$y=3^{\alpha_2}5^{\beta_2}$ $\qquad$ $\max(\alpha_2\alpha_3)=2$

$z=3^{\alpha_3}5^{\beta_3}$ $\qquad$ $\max(\alpha_1\alpha_3)=3$

so $\alpha_1=3$

$\alpha_1,\alpha_2,\alpha_3$ 5 ways

α_2	α_3	
2	0	5 ways
2	1	
2	2	
0	2	
1	2	

$\max(\beta_i,\beta_j)=3$

So 2 of them must be 3

	β_1	β_2	β_3
16 cases	3	3	0
	3	3	1
	3	3	2
	3	3	3

3 cases ${}^3C_2$ = 9 + 1 = 10 cases

So total = $5\times10=40$

$k-47=3$

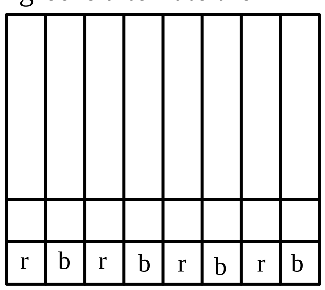
24. **(C)**

First we paint bottom now

Case 1:

If red's & greens alternate then

2^8 ways



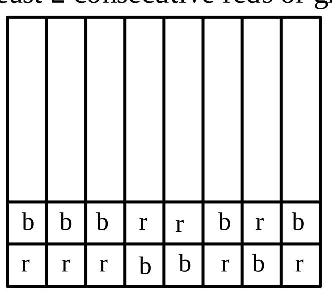
$\rightarrow$ r br br br b or

br br br br

2^{nd} row can be painted in 2w each all rows in 2 ways

If row has atleast 2 consecutive reds or greens

$\left(2^8-2\right)\times1\times1$



Row 2 in 1 way

Row 3 in 1 way

.

.

.

Row 8 in 1 way

$=2^8+2^8-2$

$=512-2=510$

Sum of design = 6

25. **(CD)**

f_n doesnot contains consecutive elements

# of elements is not fixed

$$k\left(k-1\right)^{n}$$

elements 'n' exists ↙ ↘ elements 'n' doesnot exist

We can't take n – 1 So we can select non consecutive from $\{1,2,....n-1\}$

So select non consecutive f_{n-1} ways

From $\{1,2,.....n-2\}$

i.e f_{n-2}w $f_n = f_n + f_{n-1}$

$f_1 = 2 \quad \{13, \phi$

$f_2 = 3 \quad \{13, \{2\}, \phi$

$f_3 = 2 + 3 = 5$

$f_4 = f_3 + f_2 = 8$

26. **(AB)**

$x_1 \quad x_2 \quad x_3$

$\downarrow \ \checkmark \ \downarrow \ \checkmark \ \downarrow$

$x_1 + x_2 + x_3 = 198$

$x_2 \le 19$

Total $-\{x_2 \ge 203\}$

$=\left({}^{198+3-1}C_{3-1}\right) - \left({}^{178+3-1}C_{3-1}\right)$

$={}^{200}C_2 - {}^{180}C_2$

$=\dfrac{200\times199}{2} - \dfrac{180\times179}{2}$

$=10\left(1990 - 9\times179\right)$

$=10\left(1990 - 1611\right)$

$=10\left(379\right) = 3790$

${}^{180}C_2 \times 20 + 190$

$=3600 + 190$

$=3790$

${}^{180}C_2 = \dfrac{180}{2}\times179$

27. **(ABCD)**

(A) ${}^{6}C_3\,{}^{4}C_2\,5!\times5!$

$=\left(5\times4\times3\times2\right)5!^2$

$(5!)^3$

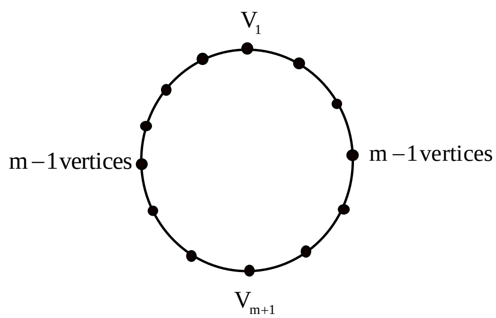
(B) ${}^{6}C_1 \times 9!$

(C) 7! 4! String method

(D) ${}^{10}C_6 \times 4! = {}^{10}C_4 \times 4! = {}^{10}P_4$

28. **(AC)**

If n = even = 2m



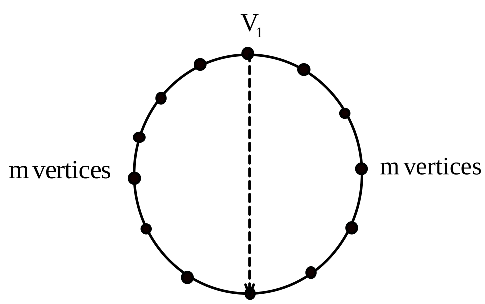
${}^{n}C_1 \times {}^{m-1}C_2$

First vertex

$= {}^{n}C_1 \; {}^{\frac{n}{2}-1}C_2 = \dfrac{n\left(\frac{n}{2}-1\right)\left(\frac{n}{2}-2\right)}{2}$

$= \dfrac{n(n-2)(n-4)}{8}$

If n = odd = 2m + 1



${}^{n}C_1 \times {}^{m}C_2$

$= \dfrac{nm(m-1)}{2}$

$= \dfrac{n\left(\frac{n-1}{2}\right)\left(\frac{n-3}{2}\right)}{2}$