IIT – JEE: 2018 **TW TEST (3 YRS.)** **DATE: 26/08/16**

TOPIC: EQUATION & INEQUATION

SOLUTION

41. **(C)**

$$\frac{1-\frac{1}{\sqrt{a}}}{1+\sqrt{a}}-\frac{\sqrt{a}-\frac{1}{\sqrt{a}}}{a-1}=\frac{\sqrt{5}-5}{10}$$

$$=\frac{-2}{\sqrt{a}\left(1+\sqrt{a}\right)}=\frac{\sqrt{5}-5}{10}$$

$$-20=\sqrt{5}\sqrt{a}\left(1-\sqrt{5}\right)\left(1+\sqrt{a}\right)$$

$$\frac{-20}{\sqrt{5}\left(1-\sqrt{5}\right)}=\sqrt{a}\left(1+\sqrt{a}\right)$$

$$\frac{-20\left(1+\sqrt{5}\right)}{\sqrt{5}(1-5)}=\sqrt{a}\left(1+\sqrt{a}\right)$$

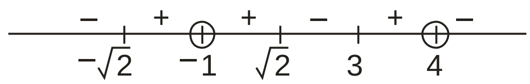
$$\sqrt{5}\left(1+\sqrt{5}\right)=\sqrt{a}\left(1+\sqrt{a}\right)$$

$$\Rightarrow a=5$$

42. **(D)**

$$\frac{\left(2-x^2\right)(x-3)^3}{(x+1)\left(x^2-3x-4\right)}\ge 0$$

$$\frac{\left(\sqrt{2}+x\right)\left(\sqrt{2}-x\right)(x-3)^3}{(x+1)^2(x-4)}\ge 0$$



$$\Rightarrow x\in\left[-\sqrt{2},1\right)\cup\left(-1,\sqrt{2}\right]\cup[3,4)$$

43. **(A)**

$$\log_{1.5}\frac{2x-8}{x-2}<0$$

Since $1.5>1$

$$\therefore\ 0<\frac{2x-8}{x-2}<1\ \Rightarrow\ x\in(4,6)$$

44. **(D)**

$$1+xyz=1+\frac{\log 7}{\log 14}\times\frac{\log 14}{\log 21}\times\frac{\log 21}{\log 28}$$

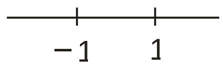
$$=1+\frac{\log 7}{\log 28}=\frac{\log 28}{\log 28}+\frac{\log 7}{\log 28}$$

$$=\frac{\log(28\times 7)}{\log 28}=\log_{28}(28\times 7)$$

45. **(A)**

46. **(ABC)**

$(1+x)^2 \ge |1-x^2|$



Case I

$x<-1$ .....(i)

$(1+x)^2 \ge -1+x^2$

$\Rightarrow$ $x\in[-1,\infty)$ ....(ii)

From (i) and (ii), $x=-1$ ...(iii)

Case II

$-1\le x<1$ ...(iv)

$(1+x)^2 \ge 1-x^2 \Rightarrow x\in(-\infty,-1]\cup(0,\infty)$ ...(v)

From (iv) and (v),

$x\in(0,\infty)\cup\{-1\}$ ...(vi)

Case III $x\ge 1$ ...(vii)

$(1+x)^2 \ge -1+x^2 \Rightarrow x\in[-1,\infty)$ ...(viii)

From (vii) and (viii), $x\in[1,\infty)$ ....(xi)

From (iii), (vi) and (xi), $x\in(0,\infty)\cup\{-1\}$

47. **(AC)**

$\Rightarrow 18^{4x-3}=\left(54\sqrt{2}\right)^{3x-4}$

$\Rightarrow 18^{4x-3}=(18)^{3x-4}\left(3\sqrt{2}\right)^{3x-4}$

$\Rightarrow 18^{x+1}=\left(3\sqrt{2}\right)^{3x-4}$

$\Rightarrow \left[\left(3\sqrt{2}\right)^{2}\right]^{x+1}=\left(3\sqrt{2}\right)^{3x-4}$

$\Rightarrow \left(3\sqrt{2}\right)^{2x+2}\times\left(3\sqrt{2}\right)^{-3x+4}=1$

$\Rightarrow \left(3\sqrt{2}\right)^{-x+6}=1$

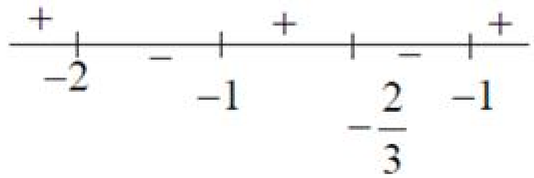
So $-x=6 \Rightarrow x=6$

48. **(BC)**

$$\Rightarrow \frac{2x}{2x^2+5x+2} > \frac{1}{x+1} \Rightarrow \frac{2x}{2x^2+5x+2} - \frac{1}{x+1} > 0$$

$$\Rightarrow \frac{2x^2+2x-2x^2-5x+2}{(2x^2+5x+2)(x+1)} > 0$$

$$\Rightarrow \frac{3x+2}{(2x+1)(x+2)(x+1)} < 0$$



So, $x \in (-2,-1) \cup \left(-\frac{2}{3},-1\right)$

49. **(AC)**

$\Rightarrow \log_2 3 > 1$

$\Rightarrow \log_{12} 5 < 1$

So, $\log_2 3 > \log_{12} 5$

Similarly, $\log_6 5 < 1$

$\Rightarrow \log_7 11 > 1$

$\Rightarrow \log 82 > \log_3 3^4$

$\Rightarrow \log_3 81 > 4$

$\Rightarrow \log_2 15 < \log_2 16$

$\Rightarrow \log_2 15 < \log_2 2^4$

$\Rightarrow \log_2 15 < 4$

$\Rightarrow \log_{16} 15 < 1$

$\Rightarrow \log_{10} 11 > 1$

$\Rightarrow \log_7 6 < 1$

50. **(BC)**

$$\Rightarrow 2x^2 + 6\sqrt{2}x + 1 = 0$$

$$\Rightarrow x = \frac{-6\sqrt{2} \pm \sqrt{72-8}}{4}$$

$$\Rightarrow x = \frac{-3\sqrt{2}}{2} + 2, \frac{-3\sqrt{2}}{2} - 2$$

51. **(BD)**

$$\Rightarrow \log_{x+1}(x-0.5) = \log_{x-0.5}(x+1)$$
$$\Rightarrow x-5>0;\ x-0.5>0;\ x-0.5\neq 1$$
$$\Rightarrow x>0.5;\ x>0.5;\ x\neq 1.5$$
$$\Rightarrow x+1>0;\ x+1>0$$
$$\Rightarrow x>-1;\ x>-1$$

So, $x\in(0.5,1.5)\cup(1.5,\infty)$ is feasible reason

$$\Rightarrow \frac{\log(x-0.5)}{\log(x+1)} = \frac{\log(x+1)}{\log(x-0.5)}$$
$$\Rightarrow \log^2(x-0.5) - \log^2(x+1) = 0$$

$$\Rightarrow \log\left(\frac{x-0.5}{x+1}\right)\log\{(x-0.5)(x+1)\} = 0$$

So, $\frac{x-0.5}{x+1} = 1$ or $(x-0.5)(x+1) = 1$

$\Rightarrow x - 0.5 = x + 1$ (no solution)

Or $x^2 + 0.5x - 1.5 = 0$
$$\Rightarrow 2x^2 + x - 3 = 0$$
$$\Rightarrow 2x^2 + 3x - 2x - 3 = 0$$
$$\Rightarrow x = 1$$

As $x \neq \frac{-3}{2}$

So $x = 1$ is the only solution.

52. **(A)**

$$\Rightarrow x^{1-og_5 x} = 0.04$$

$$\Rightarrow x > 0$$

$$\Rightarrow x.x^{\log\frac{1}{x}} = 0.04$$

$$\text{Only } x = 25, \frac{1}{5} \text{ satisfy the equation}$$

53. **(A,B)**

$$\Rightarrow 10^{\frac{2}{x}} + 25^{\frac{1}{x}} = \frac{17}{4}(50)^{\frac{1}{x}}$$

$$\Rightarrow 100^{\frac{1}{x}} + 25^{\frac{1}{x}} = \frac{17}{4}(50)^{\frac{1}{x}}$$

$$\Rightarrow 4^{\frac{1}{x}} + 1 = \frac{17}{4}2^{\frac{1}{x}}$$

$$\Rightarrow 2^{\frac{2}{x}} - \frac{17}{4}2^{\frac{1}{x}} + 1 = 0$$

$$\Rightarrow \left(2^{\frac{1}{x}} - 4\right)\left(2^{\frac{1}{x}} - \frac{1}{x}\right) = 0$$

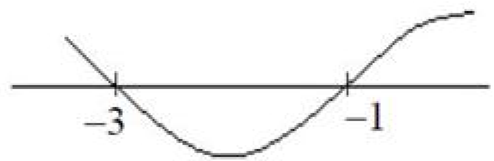
$$\Rightarrow 2^{\frac{1}{x}} = 4, \frac{1}{4}$$

$$\Rightarrow x = \frac{-1}{2}, \frac{1}{2}$$

54. (B,C)

$\Rightarrow |x^2+4x+3|+2x+5=0$

$\Rightarrow |(x+1)(x+3)|+2x+5=0$



Case 1: $x \in (-\infty, -3] \cup [-1, \infty)$

$\Rightarrow x^2+4x+3+2x+5=0$

$\Rightarrow x^2+6x+8=0$

$\Rightarrow x=-4,-2$

So $x=-4$

Case 2: $x \in (-3,-1)$

$\Rightarrow -(x^2+4x+3)+2x+5=0$

$\Rightarrow x^2+2x-2=0$

$$\Rightarrow x=\frac{-2\pm\sqrt{12}}{2}$$

$\Rightarrow x=-1+\sqrt{3}, -1-\sqrt{3}$

So, $x=-1-\sqrt{3}$

55. (A)

56. (2)

N = $3^{4\log_3 5} + 3^{3\log_{3^2} 36} + 3^{4\log_{3^2} 7}$

$5^4+(36)^{3/2}+7^2=625+216+49=890$

57. (3)

$$\frac{1}{\log_{ab} abcd}+\frac{1}{\log_{ac} abcd}+\frac{1}{\log_{ad} abcd}+\frac{1}{\log_{bc} abcd}+\frac{1}{\log_{bd} abcd}+\frac{1}{\log_{cd} abcd}$$

$$=\frac{\log_e ab}{\log_e abcd}+\frac{\log_e ac}{\log_e abcd}+\frac{\log_e ad}{\log_e abcd}+\frac{\log_e bc}{\log_e abcd}+\frac{\log_e bd}{\log_e abcd}+\frac{\log_e cd}{\log_e abcd}$$

$$=\frac{\log_e ab+\log_e ac+\log_e ad+\log_e bc+\log_e bd+\log_e cd}{\log_e abcd}$$

$$=\frac{\log_e (abcd)^3}{\log_e abcd}=\frac{3\log_e abcd}{\log_e abcd}=3$$

58. (2)

$$\sqrt[3]{5^{\frac{1}{\log_7 5}}+\frac{1}{\sqrt{-\log_{10}(10)^{-1}}}}=\sqrt[3]{5^{\log_5 7}+\frac{1}{\sqrt{\log_{10}10}}}=\sqrt[3]{7+1}=\sqrt[3]{8}=2$$

59. **(5)**

Case – I $x < 3$

$\Rightarrow x^2 + 2x + 6 - 10 \le 0$

$\Rightarrow x^2 - 2x - 4 \le 0$

$\Rightarrow x \in \left[1-\sqrt{5}, 1+\sqrt{5}\right]$

So, $x \in [1-\sqrt{5}, 3)$ …(i)

Case – II $x \ge 3$

$x^2 + 2x - 6 - 10 \le 0$

$x^2 + 2x - 16 \le 0$

$x \in \left[-1-\sqrt{17}, -1+\sqrt{17}\right]$

So, $x \in [3, -1+\sqrt{17}]$ …(ii)

from (i) and (ii), $x \in \left[1-\sqrt{5}, -1+\sqrt{17}\right]$

60. **(1)**

$|x| + x^3 = 0$

Case I If $x < 0$

$-x + x^3 = 0$

$-x(1 - x^2) = 0$

$x(x - 1)(x + 1) = 0$

But only $x = -1$ is possible

Case II if $x \ge 0$

$x + x^3 = 0$

$x(1 + x^2) = 0$

This is possible only when $x = 0$

$\therefore x = -1, 0$