[SOLUTION]

1. Distance between the given parallel lines

\[ \frac{15}{2} - 5 = \frac{\sqrt{5}}{2} \]

Thus, radius of circle = \( \frac{\sqrt{5}}{4} \) units.

2. \[ 3a - 2b + 5c = 0 \]
\[ \Rightarrow \quad \frac{3}{5}a - \frac{2}{5}b + c = 0 \]

Thus, the line \( ax + by + c = 0 \) passes through the point \( \left( \frac{3}{5}, -\frac{2}{5} \right) \).

3. \[ m_{AC} = \frac{6 - 2}{5 - 3} = 2 \]
\[ \Rightarrow \quad m_{BD} = -\frac{1}{2} \]

Thus, equation of BD is

\[ (y - 6) = -\frac{1}{2}(x - 5) \]

i.e., \[ 2y + x - 17 = 0 \]

4. \( A_1 \equiv (3, 4) \)
\( B_1 \equiv (2, \sqrt{21}) \),
\( C_1 \equiv (5, 0) \)

We have

\[ A_1B_1 = \sqrt{1 + (\sqrt{21} - 4)^2} = \sqrt{38 - 8\sqrt{21}} \]
\[ B_1C_1 = \sqrt{9 + 21} = \sqrt{30} \]
\[ A_1C_1 = \sqrt{4 + 16} = \sqrt{20} \]

We have

\[ B_1C_1 > A_1C_1 > A_1B_1 \]

Also, \[ A_1C_1^2 + A_1B_1^2 = 58 - 8\sqrt{21} \neq 30 \]

Thus, triangle \( A_1B_1C_1 \) is neither right-angled nor equilateral or isosceles. Same thing holds true for triangle \( ABC \), because \( \Delta_{ABC} \) and \( \Delta_{A_1B_1C_1} \) are similar.
5. \( P = \left( \frac{c}{a}, 0 \right), Q = \left( 0, \frac{c}{b} \right) \)
\[
\Delta_{OPQ} = \frac{1}{2} (OP)(OQ)
\]
\[
= \frac{1}{2} \frac{c^2}{ab}
\]
Clearly, \( \Delta_{OPQ} \) will not depend upon \( a, b \) and \( c \) if \( c^2 = ab \)

6. For any point \( P(x, y) \) that is equidistant from given line, we have
\[
x + y - \sqrt{2} = -(x + y - 2\sqrt{2})
\]
\[
\Rightarrow 2x + 2y - 3\sqrt{2} = 0
\]

7. Shifting the origin at \((1,3)\) we get
\[
|x'| + |y'| = 1
\]
Which represent a square of area 2 sq. units.

8. \[
\frac{x}{a} + \frac{y}{b} = 1, \frac{x}{b} + \frac{y}{a} = 1
\]
\[
\Rightarrow x \left( \frac{1}{a} - \frac{1}{b} \right) + y \left( \frac{1}{b} - \frac{1}{a} \right) = 0
\]
\[
\Rightarrow x = y
\]
[as \( a \neq b \)]

9. Given figure represents the given isosceles triangle
Clearly the equation of other equal side is
\( y = 2 \)
10. If the given lines are concurrent, then

\[
\begin{vmatrix}
1 & 2 & 3 \\
-2 & 3 & 5 \\
1 & -5 & a \\
\end{vmatrix} = 0
\]

\[7a + 56 = 0\]
\[\Rightarrow a = -8\]

11. If ‘D’ be the foot of altitude, drawn from origin to the given line, then ‘D’ is the required point.

Let \(\angle OBA = 0\)

\[\Rightarrow \tan \theta = \frac{4}{3}\]
\[\Rightarrow \angle DOA = 0\]

![Diagram of triangle OAB with point D](image)

We have,
\[OD = \frac{12}{15}\]

If D is \((h, k)\), then
\[h = OD \cos \theta, \quad k = OD \sin \theta\]
\[\Rightarrow h = \frac{36}{25}, k = \frac{48}{25}\]

12. Equation of any line drawn through the intersection point of the given line is,

\[3x + 4y - 12 + \lambda(x + 2y - 5) = 0\]
\[\Rightarrow A \equiv \left(\frac{5\lambda + 12}{3 + \lambda}, 0\right), B \equiv \left(0, \frac{5\lambda + 12}{2\lambda + 4}\right)\]

If midpoint of AB is \(C(h, k)\), then
\[2h = \frac{5\lambda + 12}{3 + \lambda}, 2k = \frac{5\lambda + 12}{2\lambda + 4}\]
\[\Rightarrow h = \frac{2(\lambda + 2)}{3 + \lambda}, k = \frac{3 + \lambda}{4\lambda + 3}\]
\[\Rightarrow \lambda = \frac{4k - 3h}{(h - 2k)}\]

Thus,
\[2h = \frac{5(4k - 3h)}{(h - 2k)}\]
\[\Rightarrow 12 + \frac{5(4k - 3h)}{h - 2k}\]

Thus locus is, \(3x + 4y = 4xy\)
13. If \( Q(h, k) \) is the image, then
\[
\frac{h - 1}{4/5} = \frac{k - 5}{3/5} = -10
\]
\[\Rightarrow h = -7, k = -1\]

14. If the line meets the \( x \) and \( y \) – axis at \( A \) and \( B \).
Then \( A = \left(-\frac{c}{a}, 0\right), B = \left(0, -\frac{c}{b}\right) \)

Line will pass through the first quadrant, if
\[-\frac{c}{a} > 0 \text{ and } -\frac{c}{b} > 0\]
\[\Rightarrow ac < 0 \text{ and } bc < 0\]

15. \( AB = 5, D = \left(2, \frac{3}{2}\right) \)
\( CD = 5, \frac{\sqrt{3}}{2} = \frac{5\sqrt{3}}{2} \)

If \( C \equiv (h, k) \) then
\[
\frac{h - 2}{3/5} = \frac{k - 3/2}{4/5} = \pm \frac{5\sqrt{3}}{2}
\]
\[\Rightarrow h = 2\left(1 - \frac{3\sqrt{3}}{4}\right), h = \frac{3}{2}\left(1 - \frac{4}{\sqrt{3}}\right)\]

Or \[h = 2\left(1 + \frac{3\sqrt{3}}{4}\right), h = \frac{3}{2}\left(1 + \frac{4}{\sqrt{3}}\right)\]

16. Triangle is clearly equilateral. That means incenter will be same as it’s centroid i.e. the point \( \left(1, \frac{1}{\sqrt{3}}\right) \)
17. \( A_i = \left( \frac{13}{2}, 1 \right) \)

\[ m_{AA_i} = \frac{-1 - \frac{13}{2}}{2 - \frac{13}{2}} = -\frac{2}{9} \]

Thus equation of required line is

\[ (y + 1) = -\frac{2}{9} (x - 1) \]

\[ \Rightarrow 2x + 9y + 7 = 0 \]

18. Diagonals are perpendicular to each other.
Thus parallelogram is definitely a rhombus.

19. Clearly,
\[ a + 4 = 1 + 5, b + 6 = 7 + 2 \]

\[ \Rightarrow \quad a = 2, b = 3 \]

20. If vertices are rational points then, incenter is not necessarily a rational point.

21. We have
\[
\begin{vmatrix}
1 & 1 & -a \\
1 & 4 & 4 \\
-a & 4 & c
\end{vmatrix} = 0\]

\[ 1.1 + 1 - \frac{5}{4} - a = 0, 1.1 + 4 - \frac{5}{4} + 4 = 0 \]

\[ \Rightarrow \quad a = -\frac{1}{4} (4c - 16) - 1(c + 4a) - a(4 + 4a) = 0 \]

\[ \Rightarrow \quad c = \frac{19}{4} \]
22. Equations of angle bisectors of the given lines is
\[ \frac{x^2 - y^2}{a - b} = \frac{xy}{h} \]
It should be same as \( xy = 0 \)
\[ \Rightarrow \quad h = 0 \]

23. Given figure indicates that there are exactly six good points inside the quadrilateral ABCD

24. We have, \( a - \frac{a^2}{4} > 0 \) and \( a - \frac{a^2}{2} < 0 \)
\[ \Rightarrow \quad 0 < a < 4, a \in (-\infty, 0) \cup (2, \infty) \]
\[ \Rightarrow \quad a \in (2, 4) \]

25. We have
\[ \frac{1 - 2m - c}{\sqrt{1 + m^2}} + \frac{2 - 3m - c}{\sqrt{1 + m^2}} + \frac{7 + 4m - c}{\sqrt{1 + m^2}} = 0 \]
\[ \Rightarrow \quad 10 - m - 3c = 0 \]
\[ \Rightarrow \quad \frac{10}{3} = \frac{m}{3} + c \]
Hence the line \( y = mx + c \) will always pass through the point \( \left( \frac{1}{3}, \frac{10}{3} \right) \)