IIT – JEE - 2017 **TW TEST [BATCH – 7 & 4]**

TOPIC: STRAIGHT LINES AND PAIR OF LINES **DATE:19/10/15**

[SOLUTION]

1. Distance between the given parallel lines

$$\frac{\frac{15}{2}-5}{\sqrt{5}}=\frac{\sqrt{5}}{2}$$

Thus, radius of circle $=\frac{\sqrt{5}}{4}$ units.

2. $3a-2b+5c=0$

$\Rightarrow \quad \frac{3}{5}a-\frac{2}{5}b+c=0$

Thus the line $ax+by+c=0$ passes through the point $\left(\frac{3}{5},-\frac{2}{5}\right)$

3. $m_{AC}=\frac{6-2}{5-3}=2$

$\Rightarrow \quad m_{BD}=-\frac{1}{2}$

Thus equation of BD is

$$(y-6)=-\frac{1}{2}(x-5)$$

i.e., $2y+x-17=0$

4. $A_1\equiv(3,4)$

$B_1\equiv\left(2,\sqrt{21}\right)$,

$C_1\equiv(5,0)$

We have

$$A_1B_1=\sqrt{1+\left(\sqrt{21}-4\right)^2}=\sqrt{38-8\sqrt{21}}$$

$$B_1C_1=\sqrt{9+21}=\sqrt{30},$$

$$A_1C_1=\sqrt{4+16}=\sqrt{20}$$

We have

$$B_1C_1>A_1C_1>A_1B_1$$

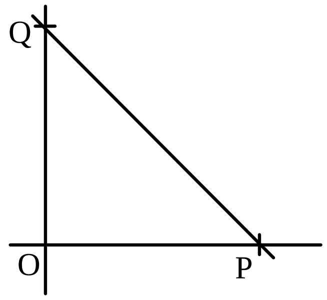
Also, $A_1C_1^2+A_1B_1^2=58-8\sqrt{21}\neq 30$

Thus triangle $A_1B_1C_1$ is neither right angled nor equilateral or isosceles. Same thing holds true for triangle ABC, because Δ_{ABC} and $\Delta_{A_1B_1C_1}$ are similar

5. $P \equiv \left(\frac{c}{a}, 0\right), Q \equiv \left(0, \frac{c}{b}\right)$

$\Delta_{OPQ} = \frac{1}{2}(OP)(OQ)$

$= \frac{1}{2}\frac{c^2}{ab}$



Clearly, Δ_{OPQ} will not depend upon a, b and c if $c^2 = ab$

6. For any point $P(x, y)$ that is equidistant from given line, we have

$x + y - \sqrt{2} = -\left(x + y - 2\sqrt{2}\right)$

$\Rightarrow \quad 2x + 2y - 3\sqrt{2} = 0$

7. Shifting the origin at $(1, 3)$ we get

$|x'| + |y'| = 1$

Which represent a square of area 2 sq. units.

8. $\frac{x}{a} + \frac{y}{b} = 1, \frac{x}{b} + \frac{y}{a} = 1$

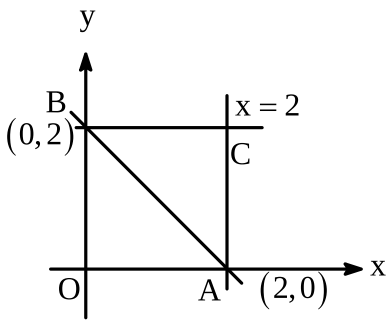
$\Rightarrow \quad x\left(\frac{1}{a} - \frac{1}{b}\right) + y\left(\frac{1}{b} - \frac{1}{a}\right) = 0$

$\Rightarrow \quad x = y$

$[\text{as } a \neq b]$

9. Given figure represents the given isosceles triangle
Clearly the equation of other equal side is

$y = 2$



10. If the given lines are concurrent, then

$$\begin{vmatrix} 1 & 2 & 3 \\ -2 & 3 & 5 \\ 1 & -5 & a \end{vmatrix} = 0$$

$7a + 56 = 0$

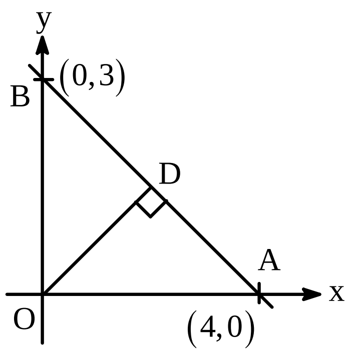
$\Rightarrow \quad a = -8$

11. If 'D' be the foot of altitude, drawn from origin to the given line, then 'D' is the required point.

Let $\angle OBA = \theta$

$\Rightarrow \quad \tan\theta = \dfrac{4}{3}$

$\Rightarrow \quad \angle DOA = \theta$



We have, $OD = \dfrac{12}{15}$

If D is (h,k), then

$h = OD\cos\theta$,

$k = OD\sin\theta$

$\Rightarrow \quad h = \dfrac{36}{25}, k = \dfrac{48}{25}$

12. Equation of any line drawn through the intersection point of the given line is,

$3x + 4y - 12 + \lambda(x + 2y - 5) = 0$

$\Rightarrow \quad A \equiv \left(\dfrac{5\lambda + 12}{3 + \lambda}, 0\right), B \equiv \left(0, \dfrac{5\lambda + 12}{2\lambda + 4}\right)$

If midpoint of AB is $C(h,k)$, then

$2h = \dfrac{5\lambda + 12}{3 + \lambda}, 2k = \dfrac{5\lambda + 12}{2\lambda + 4}$

$\Rightarrow \quad \dfrac{h}{k} = \dfrac{2(\lambda + 2)}{3 + \lambda}$

$\Rightarrow \quad \lambda = \dfrac{4k - 3h}{(h - 2k)}$

Thus, $2h = \dfrac{12 + \dfrac{5(4k - 3h)}{(h - 2k)}}{3 + \dfrac{4k - 3h}{h - 2k}}$

Thus locus is, $3x + 4y = 4xy$

13. If $Q(h, k)$ is the image, then

$$\frac{h-1}{4/5} = \frac{k-5}{3/5} = -10$$

$\Rightarrow \quad h = -7, k = -1$

14. If the line meets the x and y – axis at A and B.

Then $A \equiv \left(-\frac{c}{a}, 0\right), B \equiv \left(0, -\frac{c}{b}\right)$

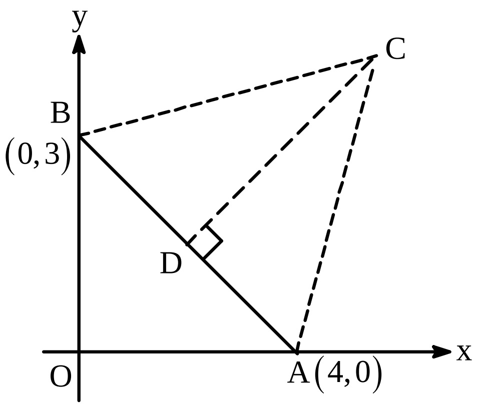
Line will pass through the first quadrant, if

$$-\frac{c}{a} > 0 \text{ and / or } -\frac{c}{b} > 0$$

$\Rightarrow \quad ac < 0 \text{ and / or } bc < 0$

15. $AB = 5, D \equiv \left(2, \frac{3}{2}\right)$

$CD = 5.\frac{\sqrt{3}}{2} = \frac{5\sqrt{3}}{2}$



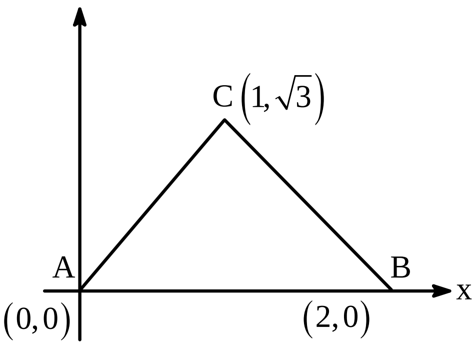
If $C \equiv (h, k)$ then

$$\frac{h-2}{3/5} = \frac{k-3/2}{4/5} = \pm\frac{5\sqrt{3}}{2}$$

$\Rightarrow \quad h = 2\left(1 - \frac{3\sqrt{3}}{4}\right), h = \frac{3}{2}\left(1 - \frac{4}{\sqrt{3}}\right)$

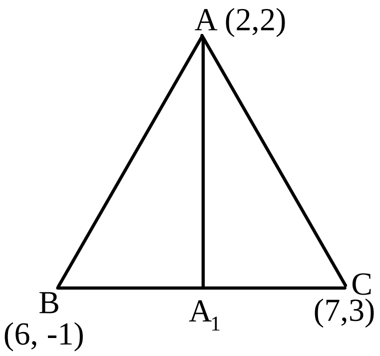
Or $\quad h = 2\left(1 + \frac{3\sqrt{3}}{4}\right), h = \frac{3}{2}\left(1 + \frac{4}{\sqrt{3}}\right)$

16. Triangle is clearly equilateral. That means incenter will be same as it's centroid i. e. the point $\left(1, \frac{1}{\sqrt{3}}\right)$



17. $A_1 \equiv \left(\frac{13}{2},1\right)$

$m_{AA_1} = \frac{2-1}{2-\frac{13}{2}} = -\frac{2}{9}$



Thus equation of required line is

$$(y+1) = -\frac{2}{9}(x-1)$$

$\Rightarrow$ $2x+9y+7=0$

18. Diagonals are perpendicular to each other.
Thus parallelogram is definitely a rhombus.

19. Clearly,

$$a+4=1+5, b+6=7+2$$

$\Rightarrow$ $a=2, b=3$

20. If vertices are rational points then, incenter is not necessarily a rational point.

21. We have $\begin{vmatrix} 1 & 1 & -a \\ 1 & 4 & 4 \\ -a & 4 & c \end{vmatrix} = 0$ and

$1.1+1.-\frac{5}{4}-a=0, 1.1+4.-\frac{5}{4}+4=0$

$\Rightarrow$ $a=-\frac{1}{4}, (4c-16)-1(c+4a)-a(4+4a)=0$

$\Rightarrow$ $c=\frac{19}{4}$

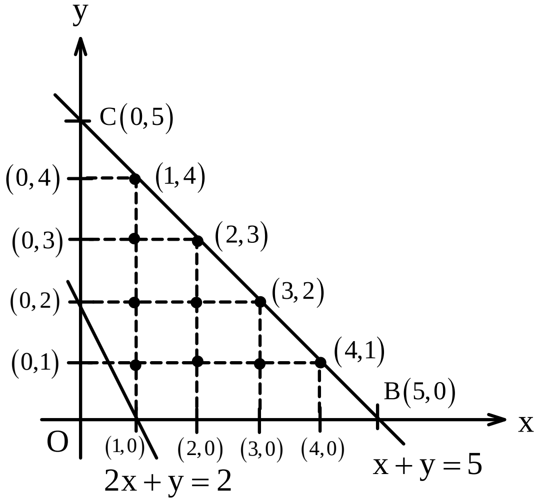
22. Equations of angle bisectors of the given lines is

$$\frac{x^2-y^2}{a-b}=\frac{xy}{h}$$

It should be same as $xy=0$

$\Rightarrow \quad h=0$

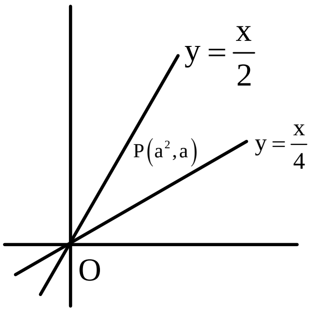
23. Given figure indicates that there are exactly six good points inside the quadrilateral ABCD



24. We have, $a-\frac{a^2}{4}>0$ and $a-\frac{a^2}{2}<0$

$\Rightarrow \quad 0<a<4, a\in(-\infty,0)\cup(2,\infty)$

$\Rightarrow \quad a\in(2,4)$



25. We have

$$\frac{1-2m-c}{\sqrt{1+m^2}}+\frac{2-3m-c}{\sqrt{1+m^2}}+\frac{7+4m-c}{\sqrt{1+m^2}}=0$$

$\Rightarrow \quad 10-m-3c=0$

$\Rightarrow \quad \frac{10}{3}=\frac{m}{3}+c$

Hence the line $y=mx+c$ will always pass through the point $\left(\frac{1}{3},\frac{10}{3}\right)$