IIT – JEE - 2017 **TW TEST (BATCH – 2 & 6)**

TOPIC: STRAIGHT LINES AND PAIR OF LINES **DATE:19/10/15**

[SOLUTION]

1. $\Delta_{POA} = \frac{1}{2}(OA)|x| = 2|x|$,

$\Delta_{POB} = \frac{1}{2}(OB)|y| = 3|y|$

Where $P \equiv (x, y)$

$\Rightarrow 2|x| = 6|y|$

$\Rightarrow |x| = 3|y|$

$\Rightarrow 3y - x = 0$ or $3y + x = 0$

2. Distance between $x + 2y + 3 = 0$ and

$x + 2y - 7 = 0$ is $\frac{10}{\sqrt{5}}$

Let the remain side be $2x - y + \lambda = 0$

We have, $\frac{|\lambda + 4|}{\sqrt{5}} = \frac{10}{\sqrt{5}}$

$\Rightarrow \lambda = 5, -14$

Thus remaining side is

$2x - y + 6 = 0$ or $2x - y - 14 = 0$

$m_{AB} = \frac{-4-2}{3-1} = -3$

3. Thus equation of CD is

$y - 8 = -3(x - 3)$

i.e., $y + 3x = 17$

Equation of right bisector of AB is,

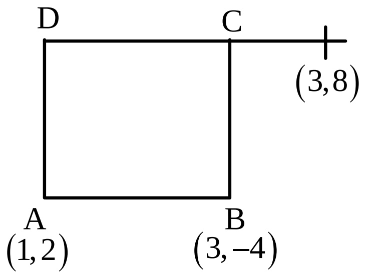
$y + 1 = \frac{1}{3}(x - 2)$

i.e. $3y = x - 5$

Solving it with line CD we get

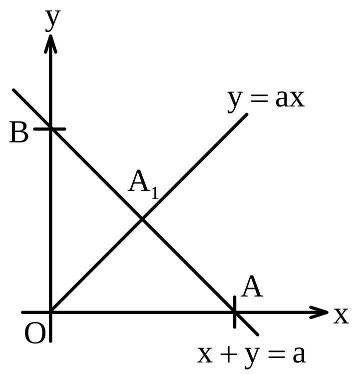
$x = \frac{28}{5}, y = \frac{1}{5}$.

Thus midpoint of CD is $\left(\frac{28}{5}, \frac{1}{5}\right)$



4. $B \equiv (0, a), A_1 \equiv \left(\frac{a}{1+a}, \frac{a^2}{1+a}\right),$

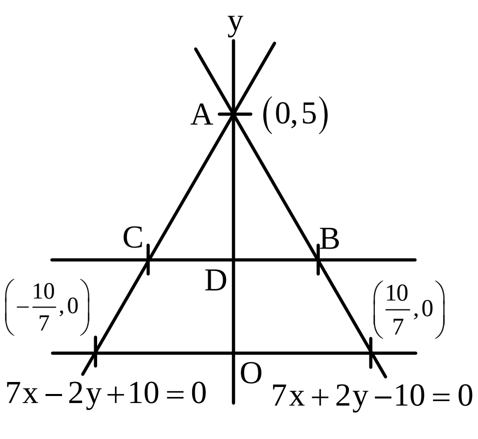
$\Delta_{OA_1B} = \frac{1}{2}.(OB).\left|\frac{a}{1+a}\right| = \frac{1}{2}\frac{a^2}{|1+a|}$



5. We have, $B \equiv \left(\frac{6}{7}, 2\right), C \equiv \left(-\frac{6}{7}, 2\right)$

$\Rightarrow BC = \frac{12}{7}, AD = 3$

$\Rightarrow \Delta_{ABC} = \frac{1}{2}.\frac{12}{7}.3 = \frac{18}{7}$ sq.units



6. Equation of bisectors is

$$\frac{3x - 4y + 7}{5} = \pm\frac{(12x + 5y - 2)}{13}$$

i.e. $21x + 77y - 101 = 0, 11x - 3y + 9 = 0$

Now, slope of $3x - 4y + 7 = 0$ is, $m_1 = \frac{3}{4}$

Slope of $11x-3y+9=0$ is, $m_2=\frac{11}{3}$

If θ be the angle between lines, then

$$\tan\theta\left|\frac{\frac{11}{3}-\frac{3}{4}}{1+\frac{11}{4}}\right|=\frac{35}{45}<1$$

Thus equation of obtuse angle bisector is

$$21x+77y-101=0$$

7. Given lines are mutually perpendicular and intersect at $\left(\frac{6}{5},\frac{13}{5}\right)$

Equation of angle bisector of the given lines are

$$x-2y+4=\pm(2x+y-5)$$

i.e., $x+3y=0, 3x-y=1$

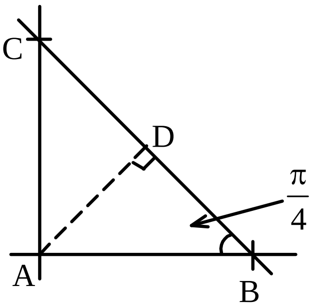
Side BC will be parallel to these bisectors.

Let $AD=a$ $\Rightarrow AB=a\sqrt{2}$

And, area of $\Delta_{ABC}=\frac{1}{2}\left(a\sqrt{2}\right)^2$

$\Rightarrow a^2=10$

$\Rightarrow a=\sqrt{10}$



Let equation of BC is

$$x+3y=\lambda$$

$$\Rightarrow \sqrt{10}=\frac{\left|\frac{6}{5}+\frac{39}{5}-\lambda\right|}{\sqrt{10}}$$

$\Rightarrow \lambda=-1,19$

Thus equation of BC is

$$x+3y=-10, \text{ or } x+3y=19$$

If equation of BC is, $3x-y=\lambda$

$$\Rightarrow \sqrt{10}=\frac{\left|\frac{18}{5}-\frac{13}{5}-\lambda\right|}{\sqrt{10}}$$

$\Rightarrow \lambda=-9,11$

Hence equation of BC is

$$3x-y=-9 \text{ or } 3x-y=11$$

8. Let the line be $\frac{x}{a}+\frac{y}{b}=1$

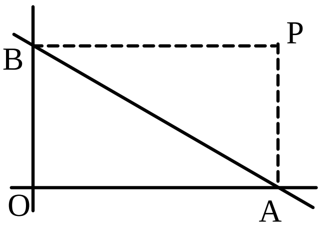
Then, $\frac{1}{a}+\frac{3}{b}=1$

$A \equiv (a,0), B \equiv (0,b)$

$\Rightarrow \quad P \equiv (a,b)$

Thus locus of 'P' is

$\frac{1}{x}+\frac{3}{y}=1$

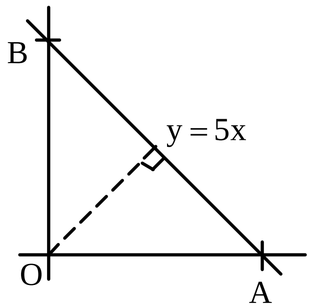


9. Let the equation of line be

$\frac{x}{a}+\frac{y}{b}=1$

$\Rightarrow \quad -\frac{b}{a}.5=-1$

$\Rightarrow \quad 5b=a$



Area of $\Delta_{OAB}=\frac{1}{2}|ab|$

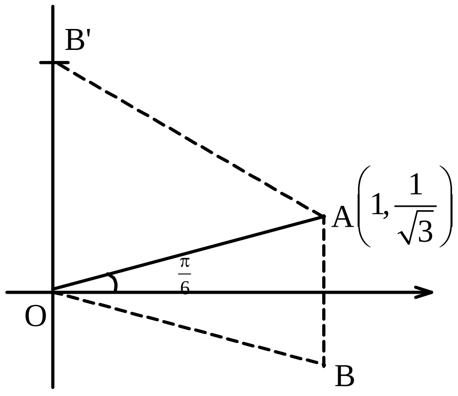
$\Rightarrow \quad 10=\frac{1}{2}|5b^2|$

$\Rightarrow \quad b^2=4$

$\Rightarrow \quad b=\pm 2, a=\pm 10$

The line can be $\frac{x}{10}+\frac{y}{2}=1$ or $\frac{x}{10}+\frac{y}{2}=-1$

10. $OA = \sqrt{1+\frac{1}{3}} = \frac{2}{\sqrt{3}}$



Line OA makes an angle $\frac{\pi}{6}$ with x – axis

Thus B can be $\left(\frac{2}{\sqrt{3}}.\cos 90^0, \frac{2}{\sqrt{3}}.\sin 90^0\right)$

i.e., $\left(0, \frac{2}{\sqrt{3}}\right)$ or $\left(\frac{2}{\sqrt{3}}\cos\left(-30^0\right), \frac{2}{\sqrt{3}}\left(-30^0\right)\right)$

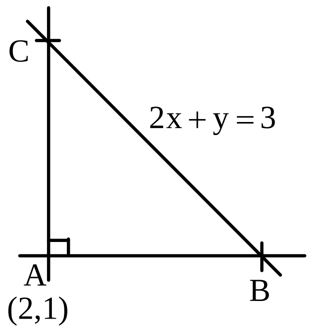
i.e., $\left(1, -\frac{1}{\sqrt{3}}\right)$

11. We have, $\angle B = \angle C = \frac{\pi}{4}$

Let ‘m’ be the slope of side ABV. Then

$$1 = \left|\frac{m-2}{1+2m}\right|$$

$\Rightarrow$ $m = \frac{1}{3}, m = -3$



Thus equation of equal sides are

$$x - 3y + 1 = 0 \text{ and } 3x + y - 7 = 0$$

Their combined equation is

$$\left(x - 3y + 1\right)\left(3x + y - 7\right) = 0$$

i.e., $3x^2 - 3y^2 - 8xy - 4x + 22y - 7 = 0$

12. Let the equation of side AB be

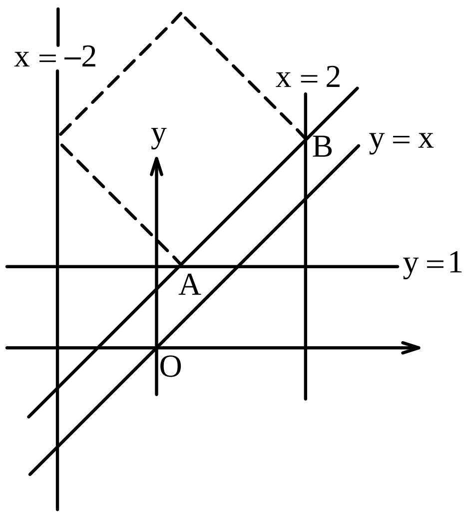
$y = x + a$

$\Rightarrow \quad A \equiv (1-a,1), B \equiv (2, 2+a)$

Equation of side AD is

$y - 1 = -(x - (1-a))$

$\Rightarrow \quad D \equiv (-2, 4-a)$



Let $C \equiv (h,k)$

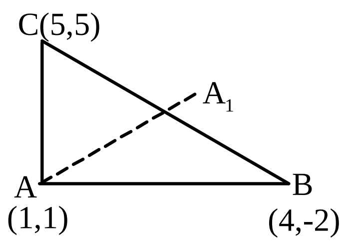
$\Rightarrow h + 1 - a = 2 - 2$

$\Rightarrow h = a - 1, \text{and } k + 1 = 2 + a + 4 - a$

$\Rightarrow k = 5$

Thus locus of C is, $y = 5$

13. $m_{AB} = \dfrac{-2-1}{4-1} = -1$



$m_{AC} = \dfrac{5-1}{5-1} = 1$

$\Rightarrow \quad m_{AA_1} = 0$

Hence , equation of required bisector is,

14. Distance between parallel sides must be equal

$\Rightarrow \quad \dfrac{|c_1 - c_2|}{\sqrt{a_1^2 + b_1^2}} = \dfrac{|d_1 - d_2|}{\sqrt{a_2^2 + b_2^2}}$

$\Rightarrow \quad (a_2^2 + b_2^2)(c_1 - c_2)^2 = (a_1^2 + b_1^2)(d_1 - d_2)^2$

15. Lines $5x+3y-2+\lambda_1(3x-y-4)=0$ are

Concurrent at $(1,-1)$

And lines $x-y+1+\lambda_2(2x-y-2)=0$ are concurrent at $(3,4)$

Thus equation of line common to both family is,

$$y-4=\frac{-1-4}{1-3}(x-3)$$

i.e., $5x-2y-7=0$

16. Angle bisectors will make the angles $\frac{\theta_1+\theta_2}{2}$

And $\left(\frac{\pi}{2}+\frac{\theta_1+\theta_2}{2}\right)$ with the x – axis

Hence their equations are,

$$\frac{x-x_1}{\cos\left(\frac{\theta_1+\theta_2}{2}\right)}=\frac{y-y_1}{\sin\left(\frac{\theta_1+\theta_2}{2}\right)}$$

Or $$\frac{x-x_1}{-\sin\left(\frac{\theta_1+\theta_2}{2}\right)}=\frac{y-y_1}{\cos\left(\frac{\theta_1+\theta_2}{2}\right)}$$

17. If the lines intersect at $(x_1,0)$, then

$$ax_1+h.0+g=0, hx_1+0.b+f=0$$

$\Rightarrow x_1=-\frac{g}{a}=-\frac{f}{h}$

$\Rightarrow af=gh$

18. $ax+by=1$ will be one of the bisectors of the given line

Equation of bisectors of the given lines are,

$$\frac{3x+4y-5}{5}=\pm\left(\frac{5x-12y-10}{13}\right)$$

$\Rightarrow$ $64x-8y=115$

Or $14x+112y=15$

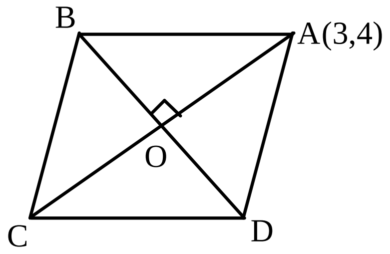
$\Rightarrow$ $a=\frac{64}{115}, b=-\frac{8}{115}$

Or $a=\frac{14}{15}, b=\frac{112}{15}$

19. $OA = 5$,

Area of rhombus $= 2.(OA.OB) = 10$

$\Rightarrow OB = 1$



Clearly, $C \equiv (-3, -4)$

Let $B \equiv (h, k)$

$\Rightarrow \quad \dfrac{h}{-\frac{4}{5}} = \dfrac{k}{\frac{3}{5}} = 1$

$\Rightarrow \quad h = -\dfrac{4}{5}, k = \dfrac{5}{3}$

Thus, $D \equiv \left(\dfrac{4}{5}, -\dfrac{3}{5}\right)$

20. Let the vertices 'B' and 'C' lie on the given line.

$$OD = \frac{2\sqrt{2}}{\sqrt{2}} = 2$$

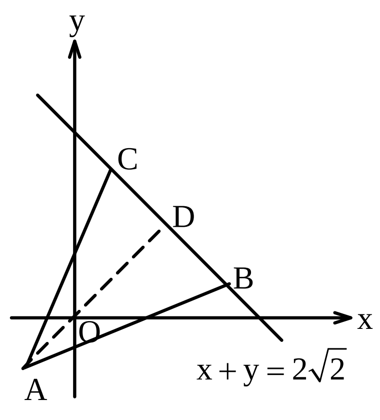
Equation of OD is $y = x$

$\Rightarrow \quad x = y = \sqrt{2}$ (for point D)

Also $BD = OD.\ \tan 60^0 = 2\sqrt{3}$

For the co – ordinates of B and C, using parametric equation of line, we get

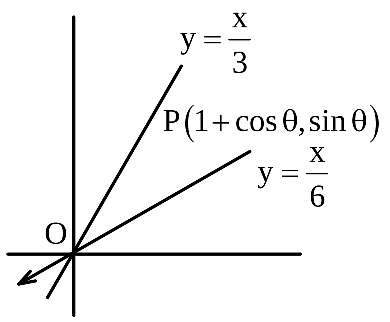
$$\frac{x-\sqrt{2}}{-\frac{1}{\sqrt{2}}} = \frac{y-\sqrt{2}}{\frac{1}{\sqrt{2}}} = \pm 2\sqrt{3}$$



$\Rightarrow \quad C \equiv \left(\sqrt{2}+\sqrt{6}, \sqrt{2}-\sqrt{6}\right)$

And $\quad D \equiv \left(\sqrt{2}-\sqrt{6}, \sqrt{2}+\sqrt{6}\right)$

21. If the 'P' lies between the actute region of the given lines, then



$$\left(6\sin\theta-1-\cos\theta\right)\left(3\sin\theta-1-\cos\theta\right)<0$$

$$\Rightarrow\left(12\sin\frac{\theta}{2}.\cos\frac{\theta}{2}-2\cos^2\frac{\theta}{2}\right)$$

$$\left(6\sin\frac{\theta}{2}.\cos\frac{\theta}{2}-2\cos^2\frac{\theta}{2}\right)<0$$

$$\Rightarrow 4\cos^2\frac{\theta}{2}\left(6\sin\frac{\theta}{2}-\cos\frac{\theta}{2}\right)\left(3\sin\frac{\theta}{2}-\cos\frac{\theta}{2}\right)<0$$

$$\Rightarrow 72\cos^4.\frac{\theta}{2}\left(\tan\frac{\theta}{2}-\frac{1}{2}\right)\left(\tan\frac{\theta}{2}-\frac{1}{3}\right)<0$$

$$\Rightarrow\frac{1}{6}<\tan\frac{\theta}{2}<\frac{1}{3}$$

$$\Rightarrow\left(n\pi+\tan^{-1}\left(\frac{1}{6}\right)<\frac{\theta}{2}<\tan^{-1}\left(\frac{1}{3}\right)+n\pi\right)$$

$$\Rightarrow 2n\pi+2\tan^{-1}\left(\frac{1}{6}\right)<\theta<2n\pi+2\tan^{-1}\left(\frac{1}{3}\right)$$

22. Clearly, A will remain as $(0,0)$, 'f_1' will make B as $(0,4)$. 'f_2' will make it $(12,4)$ and 'f_3' will make it (4, 8)

'f_1' will make 'C' as (2, 4). 'f_2' will make it (14, 4). 'f_3' will make it (5, 9)

Finally 'f_1' will make 'D' as (2, 0). 'f_2' will make it $(2,0)$. 'f_3' will make it (1, 1)

So we finally get

$A\equiv(0,0),B\equiv(4,8),C\equiv(5,9),D\equiv(1,1)$

$$m_{AB}=\frac{8}{4},m_{BC}=\frac{9-8}{5-4}=1,m_{CD}=\frac{9-1}{5-1}=\frac{8}{4}$$

$$m_{AD}=1,m_{AC}=\frac{9}{5},m_{BD}=\frac{8-1}{4-1}=\frac{7}{3}$$

Hence final figure will be a parallelogram

23. Given family is concurrent at 'P' $\left(-\frac{1}{3},\frac{1}{3}\right)$

If $Q\equiv(1,-3)$, then

$$m_{PQ}=\frac{-3\frac{1}{3}}{1+\frac{1}{3}}=-\frac{5}{2}$$

Now, member of the family that is farthest from 'Q' will have it's slope as $\frac{2}{5}$

$\Rightarrow \quad \frac{2}{5} = -\frac{(1+2\lambda)}{(1-\lambda)}$

$\Rightarrow \quad \lambda = -\frac{7}{8}$

Thus equation of required line is

$(x+y) - \frac{7}{8}(2x-y+1) = 0$

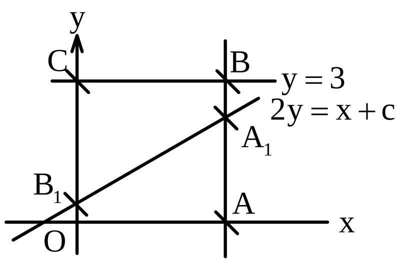
i.e., $15y - 6x - 7 = 0$

24. Let the required line be

$$2y = x + c$$

We have, $A_1 \equiv \left(4, \frac{4+C}{2}\right), B_1 \equiv \left(0, \frac{c}{2}\right)$

Area of rectangle ABCD is 12 sq. units,



Thus area of trapezium OAA_1B is

$$6 = \frac{1}{2}(4)\left(\frac{c}{2} + \frac{4+C}{2}\right)$$

$\Rightarrow \quad c = 1$

Thus required line is

$$2y = x + 1$$

25. Equation of incident ray is

$$y = -\frac{1}{\sqrt{3}}(x-2)$$

Equation of refracted ray is,

$$y = -\sqrt{3}(x-2)$$

Thus the combined equation of these lines is

$$\left(x - 2 + y\sqrt{3}\right)\left(x - 2 + \frac{y}{\sqrt{3}}\right) = 0$$

i.e., $\quad (x-2)^2 + y^2 + \frac{y}{\sqrt{3}}(x-2)4 = 0$