[SOLUTION]

1. Since \( x_1, x_2, x_3 \) and \( y_1, y_2, y_3 \) are in G. P. with same common ratio, therefore

\[
\frac{y_2-y_1}{x_2-x_1} = \frac{y_3-y_1}{x_3-x_1} = \frac{y_2-y_3}{x_2-x_3}
\]

Hence, Points \( A_1, A_2, A_3 \) are collinear.

2. Clearly, \( a \in \mathbb{R}^+ \)

Also, \( a^2 + a - 2 < 0 \)

\[\Rightarrow (a+2)(a-1) < 0\]

\[\Rightarrow -2 < a < 1\]

\[\Rightarrow a \in (0,1)\]

3. If \( P_1 \) be the reflection of \( P \) in \( y \)-axis, then

\[P_1 = (-2,3)\]

Equation of line \( P_1 R \) is

\[y - 3 = \frac{10-3}{5+2}(x + 2)\]

\[y = x + 5\]

It meets \( y \)-axis at \((0,5)\)

\[\Rightarrow Q = (0,5)\]
4. Lines \( x \cos \alpha + y \sin \alpha = p \) and 
\( x \sin \alpha - y \cos \alpha = 0 \)
Are mutually perpendicular.
Thus, \( ax + by + p = 0 \) will be equally inclined to these lines and would be the angle bisector of these lines.
Now equation of angle bisectors is,
\( x \sin \alpha - y \cos \alpha = \pm (x \cos \alpha + y \sin \alpha - p) \)
\( \Rightarrow x(\cos \alpha - \sin \alpha) + y(\sin \alpha + \cos \alpha) = p \)
Or \( x(\sin \alpha + \cos \alpha) - y(\cos \alpha - \sin \alpha) = p \)
Comparing these lines with \( ax + by + p = 0 \)
We get
\[
\frac{a}{\cos \alpha - \sin \alpha} = \frac{b}{\sin \alpha + \cos \alpha} = 1
\]
\( \Rightarrow a^2 + b^2 = 2 \)
Or \( \frac{a}{\sin \alpha + \cos \alpha} = \frac{b}{\sin \alpha - \cos \alpha} = 1 \)
\( \Rightarrow a^2 + b^2 = 2 \)

5. Line \( AB \) will be farther from origin if \( OP \) is right angle to the drawn line.
\( m_{OP} = \frac{1}{3} \)
\( \Rightarrow m_{AB} = -3 \)

Thus equation of \( AB \) is
\( (y - 1) = -3(x - 3) \)
\( \Rightarrow A \equiv \left( \frac{10}{3}, 0 \right) \)
\( B \equiv (0, 10) \)
\( \Rightarrow \Delta_{OAB} = \frac{1}{2} (OA)(OB) \)
\( = \frac{1}{2} \cdot \frac{10}{3} \cdot 10 = \frac{100}{6} \) sq.units

6. \( A \equiv (2, 2), B \equiv \left( \frac{4}{3}, 4 \right) \)
Equation of bisector \( OO_1 \) is
\[
\frac{y - 3x}{\sqrt{10}} = -\frac{(y - x)}{\sqrt{2}}
\]
\[ y \left( \frac{1}{\sqrt{5}} + 1 \right) - x \left( 1 + \frac{1}{\sqrt{5}} \right) = 0 \]

Equation of \( BB_1 \) is
\[ \frac{y + 3x - 8}{\sqrt{10}} = \frac{y - 3x}{\sqrt{10}} \]
\[ \Rightarrow \quad x = \frac{4}{3} \]
\[ \Rightarrow \quad \text{Incenter} = \left( \frac{4}{3}, \frac{3 + \sqrt{5}}{3} \right) \]
i.e., \( \left( \frac{4}{3}, \frac{2 + 2\sqrt{5}}{3} \right) \)

7.

For any integral point \((x, y)\) inside the triangle
\[ 2 \leq x + y \leq 20 \]
\((1,1)(1,2)\ldots(1,19)\rightarrow 19 \text{ points} \)
\((2,1)(2,2)\ldots(2,18)\rightarrow 18 \text{ points} \)
\((3,1)(3,2)\ldots(3,17)\rightarrow 17 \text{ points} \)
\ldots
\((19,1)\rightarrow 1 \text{ points} \)
\[ \therefore \quad \text{Total number of Integral points in the interior of triangle are.} \]
\[ 1 + 2 + \ldots + 19 = \frac{19 \times 20}{2} = 190 \]
8. Let the parametric equation of drawn line is
\[
\frac{x}{\cos \theta} = \frac{y}{\sin \theta} = r
\]
\[\Rightarrow x = r \cos \theta, y = r \sin \theta\]

![Diagram of lines and points](image)

Putting it in \(L_1\) we get
\[r \sin \theta = r \cos \theta + 10\]
\[\Rightarrow 1 \frac{\sin \theta - \cos \theta}{oa} = \frac{10}{10}\]

Similarly putting the general point of drawn line in the equation of \(L_2\), we get
\[1 \frac{\sin \theta - \cos \theta}{ob} = \frac{20}{20}\]

Let \(P = (h, k)\) and \(OP = r\)
\[\Rightarrow r \cos \theta = h, r \sin \theta = k\]

We have
\[\frac{2}{r} = \frac{\sin \theta - \cos \theta}{10} + \frac{\sin \theta - \cos \theta}{20}\]
\[\Rightarrow 40 = 3r \sin \theta - 3r \cos \theta\]
\[\Rightarrow 3y - 3x = 40\]

9. \[r^2 = \frac{10.20}{(\sin \theta - \cos \theta)^2}\]
\[\Rightarrow (r \sin \theta - r \cos \theta)^2 = 200\]
Thus locus is \((y - x)^2 = 200\)

10. \[\frac{1}{r^2} = \frac{(\sin \theta - \cos \theta)^2}{100} + \frac{(\sin \theta - \cos \theta)^2}{400}\]
\[\Rightarrow 400 = 5(4 \sin \theta - r \cos \theta)^2\]
Thus locus is
\[400 = 5(x - y)^2\]
i.e., \((x - y)^2 = 80\)
11. \[ A_1 \equiv \left(-\frac{c_1}{2},0\right), A_2 \equiv \left(-\frac{c_2}{2},0\right), B_1 \equiv \left(0, c_1\right), B_2 \equiv \left(0, c_2\right) \]

Equation of \(A_1B_2\) is
\[-\frac{2x}{c_1} + \frac{y}{c_2} = 1\]

And equation of \(A_2B_1\) is
\[-\frac{2x}{c_2} + \frac{y}{c_1} = 1\]

For the point of intersection of these lines, we have
\[y\left(\frac{1}{c_2} - \frac{1}{c_1}\right) + 2x\left(\frac{1}{c_2} - \frac{1}{c_1}\right) = 0\]
\[\Rightarrow \quad y + 2x = 0\]

12. Let \(A \equiv (h, k)\)
\[m_{AH} = \frac{k-4}{h-1}, m_{OB} = \frac{4}{3}\]
\[\Rightarrow \quad \frac{(k-4)4}{(h-1)3} = -1\]
\[\Rightarrow \quad 4k + 3h = 19 \quad \text{......... (i)}\]

\[m_{OA} = \frac{k}{h}, m_{BH} = \frac{4-4}{3-1} = 0\]

Since \(OA\) and \(BH\) are mutually perpendicular, it implies that \(h = 0\)

Putting \(h = 0\) in (i), we get
\[k = \frac{19}{4}\]

Thus coordinate of \(A\) is \(\left(0, \frac{19}{4}\right)\)

13. Let \(A_i\) is the reflection of \(A\) in \(y = x\)
\[\Rightarrow \quad A_1 = (4, 3)\]

Now, \(PA + PB = A_1P + PB\)

Which is minimum if \(A_1, P\) and \(B\) are collinear.

Equation of \(A_1B\) is
\[\frac{(y-3)}{13-3} = \frac{x-4}{7-4}\]
\[3y = 10x - 31\]

Solving it with \(y = x\), we get
\[P \equiv \left(\frac{31}{7}, \frac{31}{7}\right)\],
14. We have $|PA - PB| \leq AB$

Thus for $|PA - PB|$ to be maximum, points $A$, $B$ and $P$ must be collinear

Equation of $AB$ is $x + 2y = 2$

Solving it with given line, we get $P \equiv \left( -\frac{84}{5}, \frac{13}{5} \right)$

15. Minimum value of $|PA - PB|$ is zero.

It can be attained if $PA = PB$

That means $P$ must lie on the right bisector of $AB$

Equation of right bisector of $AB$ is $y - \frac{1}{2} = 2(x - 1)$

i.e., $y = 2x - \frac{3}{2}$

Solving with given line, we get $P \equiv \left( -\frac{9}{20}, -\frac{12}{5} \right)$

16. ‘$B$’ and ‘$C$’ will be the reflection of $A$ in $y + x = 0$ and $y - x = 0$ respectively.

Thus $B \equiv (-7, -5), C \equiv (7, 5)$

Hence equation of $BC$ is $y - 5 = -\frac{5}{7}(x - 7)$

i.e., $14y = 10x$
17. Let the equation of chord be 
\[ y = mx + c \]
Combined equation of line joining the point of intersection with origin is 
\[ 3x^2 - y^2 - 2(x - 2y) \left( \frac{y - mx}{c} \right) = 0 \]
i.e., \( x^2 (3c + 2m) - y^2 (c - 4) - 2xy(1 + 2m) = 0 \)
These lines will be mutually perpendicular if, 
\[ 3c + 2m - c + 4 = 0 \]
\[ \Rightarrow \quad 2m + 2c = -4 \]
\[ \Rightarrow \quad m + c = -2 \]
That means the chord \( y = mx + c \) will always pass through the point \((1, -2)\)

18. \( y + 2 = 0 \) and \( x - a - b = 0 \) intersect at \((a + b, 2)\)
Thus lines will be concurrent if 
\[ 2 = b(a + b) \]

19. \( m_{AA_i} = \frac{O - 1}{b - 0} = \frac{2a}{b} \), \( m_{BB_i} = \frac{a - 0}{0 - b} = \frac{-a}{2b} \)
Medians \( AA_i \) and \( BB_i \) will be mutually perpendicular if 
\[ -\frac{2a}{b} - \frac{a}{2b} = 1 \]
\[ \Rightarrow \quad a^2 + b^2 = 0 \], which is not possible.

20. If the remaining vertex is \((h, k)\) then 
\[ h + 6 = 3 + 2 \]
\[ h = -1 \]
\[ k + 4 = 5 - 1 \]
\[ k = 0 \]
\((-1, 0)\)
\[ h - 3 = -2 + 6 \equiv h = 7 \]
\[ h - 5 = 1 + 4 \equiv k = 10 \]
\[ h + 6 = -2 - 3 \equiv h = -11 \]
\[ k + 4 = 1 - 5 = k = -8 \]

21. Distance of all the points from \((0, 0)\) are 5 units. That means circumcenter of the triangle formed by the given point is \((0, 0)\)
If \( G = (h, k) \) be the centroid of triangle, then 
\[ 3h = 3 + 5(\cos \theta + \sin \theta) \]
\[ 3k = 4 + 5(\sin \theta - \cos \theta) \]
3k = 4 + 5(sin θ − cos θ)

If \(H(\alpha, \beta)\) be the orthocenter, then
\[\text{OG : GH} = 1 : 2\]
\[\Rightarrow \quad \alpha = 3h, \beta = 3k\]

\[
\cos \theta + \sin \theta = \frac{\alpha - 3}{5}, \sin \theta - \cos \theta = \frac{\beta - 4}{5}
\]
\[\Rightarrow \quad \sin \theta = \frac{\alpha + \beta - 7}{10}, \cos \theta = \frac{\alpha - \beta + 1}{10}
\]

Thus, required locus of \((\alpha, \beta)\) is
\[(x + y - 7)^2 + (x - y + 1)^2 = 100\]

22. Let the lines represented by
\[ax^2 + 2hxy + by^2 = 0\] be
\[y = m_1x \text{ and } y = m_2x\]

Then, \(m_1 + m_2 = -\frac{2h}{b}, m_1m_2 = \frac{a}{b}\)

If theses lines are reflected about the x-axis, their equation becomes
\[y = m_1x = 0, y + m_2x = 0\]

And their combined equation is
\[(y + m_1x)(y + m_2x) = 0\]
\[\Rightarrow \quad y^2 + xy(m_1 + m_2) + m_1m_2x^2 = 0\]

i.e., \[by^2 - 2hxy + ax^2 = 0\]

23. Combined equation of angle bisectors is
\[\frac{x^2 - y^2}{a - b} = \frac{xy}{h}\]

If one bisector is \(y = x\), then
\[x^2 - y^2 = 0\]
\[\Rightarrow \quad a - b = 0\]
24. \(AB = 4, BC = 3\)
\[\Rightarrow AC = 5\]
\[A = (5\cos \theta, 0), C = (0, 5\sin \theta)\]

If BC and AB makes the angle \(\theta_1\) and \(\theta_2\) with x-axis, then
\[\theta_1 = C - \theta,\]
\[\theta_2 = \pi - (A + \theta)\]
\[\Rightarrow \pi - \left(\frac{\pi}{2} - C + \theta\right)\]
\[\Rightarrow \frac{\pi}{2} + (C - \theta)\]

Using parametric equation of lines for BC, we get
\[\frac{h}{\cos (c - \theta)} = \frac{k - 5\sin \theta}{\sin (c - \theta)} = 3 \quad \text{......... (i)}\]

Similarly, using parametric equation of line for AB, we get
\[\frac{h - 5\cos \theta}{-\sin (c - \theta)} = \frac{k}{\sin (c - \theta)} = 4 \quad \text{......... (ii)}\]

From (i) and (ii) we get
\[
\sin (c - \theta) = \frac{h}{3} = \frac{k}{4}
\]

Thus, locus of ‘B’ is,
\[4x = 3y\]

25. \(AB = BC = CA = 2\)

Let \(\angle BAO = \theta\)
\[\Rightarrow A = (2\cos \theta, 0), B = (0, 2\sin \theta)\]

BC makes an angle \(\left(\frac{\pi}{3} - \theta\right)\) with x-axis and
AC makes an angle \( \left( \pi - \left( \frac{\pi}{3} + \theta \right) \right) \) with x-axis.

If \( C \equiv (h, k) \), then

\[
\frac{h}{\cos\left( \frac{\pi}{3} - \theta \right)} = \frac{k - 2\sin \theta}{\sin\left( \frac{\pi}{3} - \theta \right)} = 2
\]

And

\[
\frac{h - 2\cos \theta}{-\cos\left( \frac{\pi}{3} + \theta \right)} = \frac{k}{\sin\left( \frac{\pi}{3} + \theta \right)} = 2
\]

\[
\Rightarrow 2\cos\left( \frac{\pi}{3} - \theta \right) = h, 2\sin\left( \frac{\pi}{3} + \theta \right) = k
\]

\[
\Rightarrow \cos \theta + \sqrt{3}\sin \theta = h
\]

\[
\sqrt{3}\cos \theta + \sin \theta = k
\]

\[
\Rightarrow 2\cos \theta = (\sqrt{3}k - h), 2\sin \theta = (h \theta - k)
\]

Thus locus of \( (h, k) \) is

\[
1 = x^2 + y^2 - xy\sqrt{3}
\]