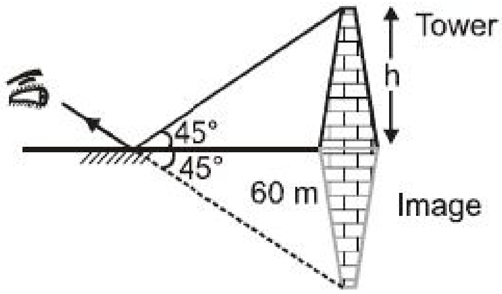
IIT – JEE: 2017 **TW TEST (XL-A1 & NAPJC – 1, 2, 3)** **DATE: 11/03/16**

TOPIC: RAY OPTICS

SOLUTION

1. **(B)**

$\tan 45^o = \frac{h}{60} \Rightarrow h = 60\,m$



2. **(D)**

If mirror is turned, about an axis perpendicular to plane of mirror, then there will be no change in incident angle and reflected angle so angle between incident & reflected rays after rotation will be same as before.

Ans. 45º

3. **(C)**

4. **(A)**

$\vec{V}_I m = \vec{V}_0 m =$ (normal to plane mirror)

$\Rightarrow \quad \vec{V}_I m - \vec{V}_m = -\left(\vec{V}_0 - \vec{V}_m\right)$

$\vec{V}_I - V\sin\theta = -\left(0 - V\sin\theta\right)$

$V_I = 2V\sin\theta$

5. **(C)**

For total internal reflection angle of incidence should be greater than critical angle. For total internal reflection to take place, angle of incidence > critical angle

$i > C$

or $\theta > C$

or $\sin\theta > \sin C$

but $\sin C = \frac{1}{\mu}$

and from figure, $\theta = 90^o - r$

so $\sin\left(90^o - r\right) > \frac{1}{\mu}$

i.e. $\mu = \frac{1}{\cos r}$ ...(i)

from Snell's law

$$\frac{\sin 45^o}{\sin r} = \mu \Rightarrow \sin r = \frac{1}{\sqrt{2}\mu}$$

$$\therefore \quad \cos r = \sqrt{1 - \sin^2 r} = \sqrt{1 - \frac{1}{2\mu^2}}$$

Thus, equation (i) becomes

$$\mu > \frac{1}{\sqrt{1 - \frac{1}{2\mu^2}}} \quad \therefore \quad \mu^2 = \frac{1}{1 - \frac{1}{2\mu^2}}$$

or $\mu^2 - \sqrt{\frac{1}{2}} = 1$ or $\mu = \sqrt{\frac{3}{2}}$

6. **(C)**

As the object moves from infinity to centre of curvature, the distance between object and image reduces from infinity to zero.

As the object moves from centre of curvature to focus, the distance between object and image increases from zero to infinity.

As the object moves from focus to pole, the distance between object and its image reduces from infinity to zero. Hence the distance between object and its image shall be 40 cm three times.

7. **(D)**

When object moves normal to the mirror, image velocity will be opposite to it. When object moves parallel to the mirror, image velocity will be in the same direction.

8. **(A)**

Using newt on's formula

$xy = f^2$

$\Rightarrow 20\ y = (10)^2 \Rightarrow y = 5$ cm.

9. **(B)**

When a light ray enters from air to glass, its frequency remains unchanged, velocity (in denser medium) decreases; so wavelength $\lambda = \frac{\upsilon}{n}$ decreases.

10. **(B)**

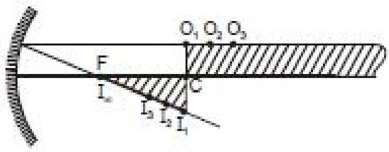
$$i + r = 70^o$$

$$\delta = 360^o - 2(i + r)$$

$$= 360^o - 2 \times 70^o = 360^o - 140^o = 220^o$$

11. **(BC)**

Draw an incident ray along the top side of rectangular strip, which happens to be parallel to the principal axis. After reflection this ray passes through focus. Hence image of all points (for e.g.



O_1, O_2, O_3, ...) on top side of the strip lie on this reflected ray (at I_1, I_2, I_3, ...) in between focus and centre of curvature. Thus the image of this strip is a triangle as shown in figure

12. **(BD)**
A concave or convex mirror is to be placed left of the object. The object and the image both will be real for concave mirror and virtual for convex mirror.

13. **(AC)**

14. **(A)**

15. **(B)** 16. **(B)**

$\vec{v}_A = \hat{i} + \vec{a}t = \hat{i} + \left(2\hat{i} + \hat{j}\right)(2) = 5\hat{i} + 2\hat{j}$

$\vec{v}_{A'} = -5\hat{i} + 2\hat{j}\ \vec{v}_{A'A} = \vec{v}_{A'} - \vec{v}_A = -10\hat{i}$

$\vec{v}_B = \left(-\hat{i} + 3\hat{j}\right), \vec{v}_{B'} = \hat{i} + 3\hat{j}$ so $\vec{v}_{B',B} = 2\hat{i}$

For particel C $\frac{dv_y}{dt} = 2t \quad \Rightarrow \quad v_y - 6 = t^2 \Rightarrow \quad v_y = 6 + 4 = 10$

$\vec{v}_C = 5\hat{i} + 10\hat{j}, v_{C'} = -5\hat{i} + 10\hat{j}$ so $\vec{v}_{C',C} = -10\hat{i}$

$\vec{v}_D = 3\hat{i} - \hat{j},\ \vec{v}_{D'} = -3\hat{i} - \hat{j}, \vec{v}_{D',D} = -6\hat{i}$

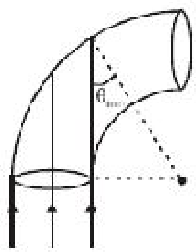
17. **(C)**

$\theta_{min} > C$

$\sin\theta_{min} > \sin C$

$\frac{R-d}{R} > \frac{1}{n}$

$\Rightarrow \quad Rn - dn > R$



$\Rightarrow \quad R > \frac{nd}{n-1}$

$R > \frac{2.4\,mm}{2-1}$

$R > 8\,mm$

18. **(ABC)**

19. **(AB)**

20. **(AC)**

$$\frac{1}{f_{film}} = (n_1 - 1)\left(\frac{1}{R} - \frac{1}{R}\right) \Rightarrow f_{film} = \infty \text{ (infinite)}$$

No effect of presence of film.

From Air to Glass

Using single spherical refraction

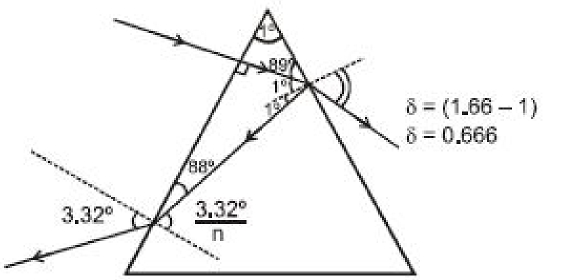
$$\frac{n_2}{v} - \frac{1}{u} = \frac{n_2 - 1}{R}, \quad \frac{1.5}{v} - \frac{1}{\propto} = \frac{15.1}{R} \Rightarrow v = 3R \quad f_1 = 63R$$

From Glass to Air

21. **(ABCD)**

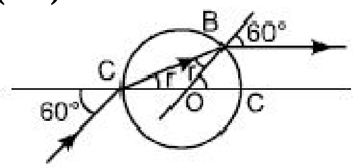
Apparent depth = Actual Depth /(μ_{rel})

22. **(B)**



$$\frac{3.32°}{n} = 2° \quad \Rightarrow \quad n = 1.66$$

23. **(AB)**



In diagram angle of emergence = 60°

$\therefore \quad \angle BOC = 60^o$

$\therefore \quad r + r = 60^o \quad \Rightarrow \quad r = 30^o$

$\therefore \quad \mu = \frac{\sin 60^o}{\sin 30^o} \quad \Rightarrow \quad \mu = \sqrt{3}$

24. **(AC)**

Given: Focal length of the objective, f_o = 60 cm

Focal length of the eye piece, f_e = 5 cm

angle subtended $\theta = 2^o$

From the formula of magnification

$$m = \frac{f_o}{f_e} = \frac{60}{5} = 12$$

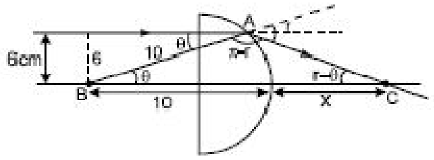
Hence, angular width of the image is given by

$\beta = a.m$

$= 2 \times 12 = 24^o$

25. **(AC)**

R = 10 cm



Applying snell's law $\frac{\sin\theta}{\sin r} = \frac{3}{4} \Rightarrow \; r = 53^{o}$

By sine law in $\Delta ABC \frac{\sin(r-\theta)}{10} = \frac{\sin(\pi - r)}{(10+x)}; = \frac{10+x}{10} = \frac{4}{5(\sin r cis\theta - \cos r \sin\theta)}$

$= \frac{4}{5\left(\frac{4}{5}\times\frac{4}{5}-\frac{3}{5}\times\frac{3}{5}\right)}; 10 + x = \frac{200}{7} \Rightarrow x = \frac{200-70}{7} = \frac{130}{7}$