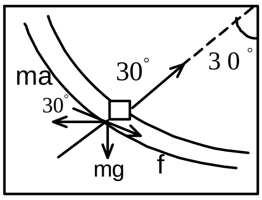
## FULL MECHANICS SOLUTION

1.




For along the tangential direction

$$m\left(\sqrt{3g}\right)\cos 30^\circ = mg\sin 30^\circ + f$$

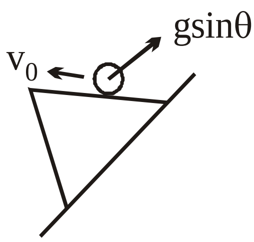
$$N = m\sqrt{3g}\sin 30^\circ + mg\cos 30^\circ$$

from solving

$$\mu \ge \frac{1}{\sqrt{3}}$$

2. $$\frac{(N-4)a}{(N-8)a} = 2$$

$$\Rightarrow N = 12$$




3. $x = ut + \frac{1}{2}at^2$

$$0 = ut - \frac{1}{2} g \sin\theta \cos\theta\, t^2$$

$$\therefore\ t = \frac{2u}{g\sin\theta\cos\theta} = \frac{2\times 24\times 5\times 5}{10\times 3\times 4} = 10 \text{ s]}$$

4. $$\vec{v_1} = u\cos 60^o\,\hat{i} + (u\sin 60^o - gt)\,\hat{j}$$

$$\vec{v_2} = u\cos 60^o\,\vec{j} + (u\sin 60^o - gt)\,\hat{k}$$

$$\cos\theta = \frac{\vec{v_1}.\vec{v_2}}{\left|\vec{v_1}\right|.\left|\vec{v_2}\right|} = \frac{(u\sin 60^o - gt)u\cos 60}{(u\cos 60^0)^2 + (u\sin 60^o - gt)^2} = \frac{1}{2}$$

5. In fig. (a) $1000 - T_1 = 50a$

$T_1 - 800 = 80a$

$T_1 = \frac{12,000}{13}$

In fig.(b)

$1000 + 500 - T_2 = 50a$

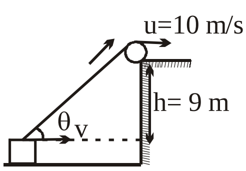
$T_2 - 800 = 80a$

$T_2 = \frac{16,000}{13}$

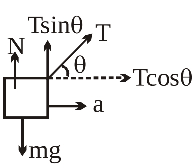
6. By constraint motion

v cos θ = u

v = u sec θ




or $a = \frac{dv}{dt} = u\sec\theta\tan\theta\left(\frac{d\theta}{dt}\right)$



When contact leaves, N=0

i.e. T sin  = mg and T cos θ = ma

$\Rightarrow$ tan θ = $\frac{g}{a}$ or a = g cotθ

T cos θ = ma and T sin θ + N = mg

also = tanθ

$\Rightarrow$ hcotθ = x

So $\boxed{v = \frac{dx}{dt} = h\,\text{cosec}^2\theta\,\frac{d\theta}{dt}}$

and $\boxed{a = u\sec\theta\tan\theta\frac{d\theta}{dt}}$

or $\frac{v}{a} = \frac{h\,\text{cosec}^2\theta}{u\sec\theta\tan\theta}$  or $\frac{u\sec\theta}{g\cot\theta} = \frac{h\,\text{cosec}^2\theta}{u\sec\theta\tan\theta}$

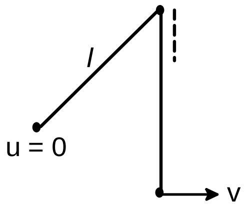
or tan²θ = $\sqrt{\frac{gh}{u}}$  or tanθ = $\sqrt{3}$

or θ = 60°

7. (b)

Friction force = $\sqrt{(\mu_s mg\cos\theta)^2 + (mg\sin\theta)^2}$

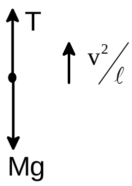
8. (b)



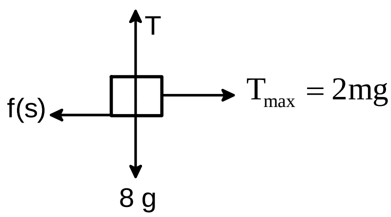

$\frac{1}{2}mu^2 + mgl(1-\cos 60^\circ) = \frac{1}{2}mv^2 + 0$

$\frac{mv^2}{l} = 2\ mg(1-\cos 60^\circ) = mg$

Tension will be maximum at the bottom (lower most point)




$T - mg = \frac{mv^2}{l}$  $T_{(max)} = 2mg$




For

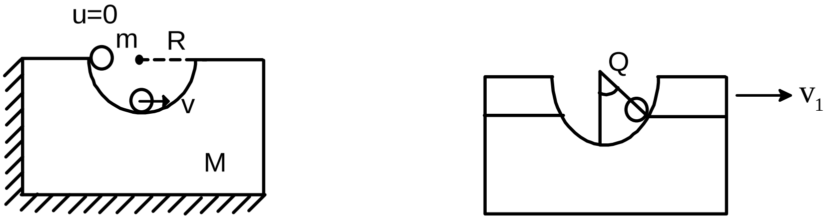
$T_{max} = 2mg$  $N = 8g$  $f_s = 2\ mg$

To ensure no slipping,

$f_s \le \mu N$  $2mg \le \mu 8g$

$\mu \geq m/4$ $\mu \geq 1/2$ $\mu_{min} = 1/2$

9. (d)




$$mgR + 0 = \frac{1}{2}mv^2 + 0$$

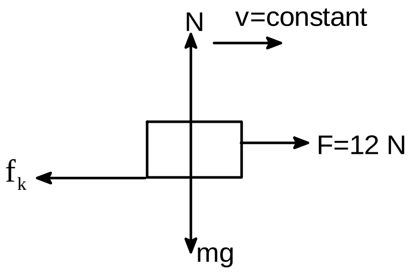
$$v = \sqrt{2gR}$$

Till then the ball reaches the lowest point, linear momentum will not be conserved. Post that, linear momentum shall also be conserved.

$$m\sqrt{2gR} = (M+m)v_1$$

$$v_1 = \frac{m\sqrt{2gR}}{(M+m)}$$

10. (c)




$$f_k = F = 12\,N$$

$$N = mg = 0.5g = 5N$$

Total contact force $= \sqrt{(12)^2 + 5^2} = 13\,N$

11. (a)

$40(4\hat{i}) + 40(3\hat{j}) = 80\vec{V}$ perfectly inelastic collision

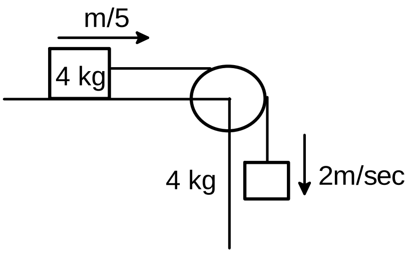
$$\vec{v} = \frac{40(34\hat{i} + 3\hat{j})}{80} = \left(2\hat{i} + \frac{3}{2}\hat{j}\right)$$

Loss of mechanical energy $\Rightarrow \Delta KE = \left[\frac{1}{2}40(4)^2 + \frac{1}{2}40(3)^2\right] - \left[\frac{1}{2}80\left(4 + \frac{9}{4}\right)\right]$

$$\Rightarrow \Delta KE = \frac{1}{2}40(16+9) - 40\left(\frac{25}{4}\right)$$

$$= 20(25) - 40\left(\frac{25}{4}\right) = 250\,J$$

12. (c)

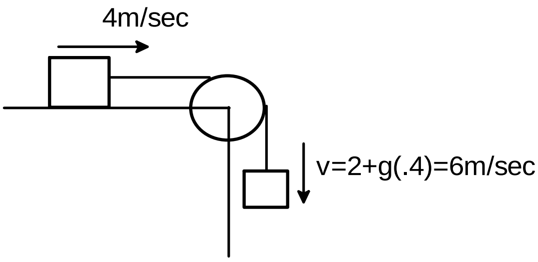



4t the string will be taut again after tencet. The hanging mass will be under free fall.

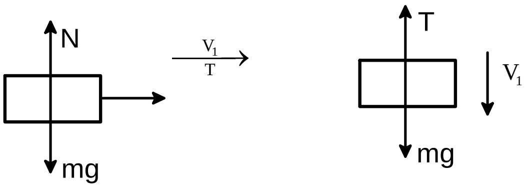
$4t = 2t + \frac{1}{2}gt^2$ $\qquad 2t = \frac{1}{2}10t^2$

$t = 4/10 \Rightarrow t = 4\sec$

At this instant the velocities of each mass. 4m/sec




After the jirk




$\int Tdt = 4(v_1 - 4)$

$-\int Tdt = 4(v_1 - 6) \Rightarrow 4(2v_1 - 10) = 0 \Rightarrow v_1 = 5m/\sec$

$\Rightarrow \qquad \int Tdt = 4(5-4) = 4N\sec$

13. (c)

Given $a \propto -v^3$

$\therefore a = -kv^3$ where $k$ is a constant of proportionality

$-v\frac{dv}{dx} = kv^3$

Integrating $\int v^{-2}dv = -\int k\,dx$

$\therefore -\frac{1}{2} = -kx + C$ [C is constant of integration]

When $x = 0, v = v_0$

$\therefore \frac{-1}{v_0} = C$ $\qquad \therefore \frac{1}{v} = -kx - \frac{1}{v_0}$

$\therefore \frac{1}{v} - \frac{1}{v_0} = kx$ $\qquad$ …………(1)

Also when $x = x_0, v = \frac{v_0}{2}$ $\qquad \therefore \frac{2}{v_0} - \frac{1}{v_0} = kx_0$

$\therefore k = \frac{1}{v_0 x_0}$

From (1), we get $v = \dfrac{v_0}{1 + \dfrac{x}{x_0}}$

When $x = 3x_0 \quad v = \dfrac{v_0}{4}$

14. (a)

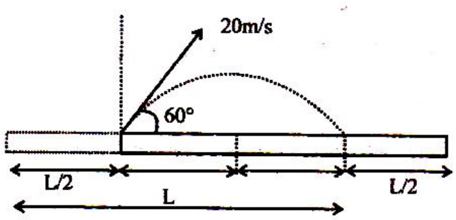
In acceleration motion slope $\dfrac{ds}{dt}$ increases whereas for retarded motion it decreases.

15. (c)

Using vector form of kinematic equation $-h = uT - \dfrac{1}{2}gT^2$

$$T = \frac{v + \sqrt{2gh}}{g} = \frac{v}{g}\left[1 + \sqrt{1 + \frac{2gh}{v^2}}\right]$$

16. (a)




Since the mass is same therefore the length of the plank should be twice the range

$$\ell = 2R = 2 \times \frac{u^2 \sin 2\theta}{g} = 40\sqrt{3}\text{m}$$

17. (a)

$$-1500 = -100t - \frac{1}{2} \times 10t^2$$

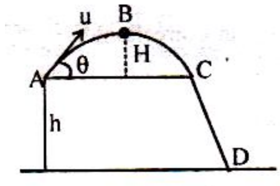
$$\Rightarrow \frac{t^2}{2} + 10t - 150 = 0 \Rightarrow t = \frac{-20 \pm 40}{2}$$

So, $t = 10\,\text{sec}$. i.e., horizontal distance $= \dfrac{500}{3} \times \dfrac{4}{5} \times 10 = \dfrac{4000}{3}\text{m}$

18. (a)

$T_{AC} = 2\,\text{sec}$.

So, $\dfrac{2u\sin\theta}{g} = 2 \Rightarrow u\sin\theta = g = 10\text{m/s}$




Now $y = u_y t + \dfrac{1}{2}a_y t^2 \Rightarrow -h = u\sin\theta \times 3 - \dfrac{1}{2}g \times 3^2$

$$\Rightarrow -h = 10 \times 3 - \frac{1}{2} \times 10 \times 9 \Rightarrow -h = 30 - 45$$

$\Rightarrow h = 45 - 30 = 15m$

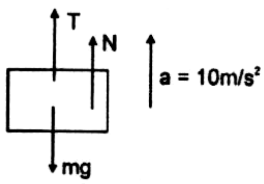
19. (b)

$m = $ mass of the block

Applying Newton's Law on the block in vertical direction

$T + N - mg = ma$ …….. (1)

$T = 300\,N$ (given reading of spring balance)



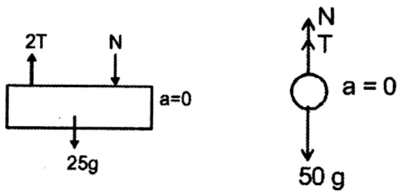

$N = 500\,N$ (given reading of W.M.)

$a = 10\,m/s^2$

$$m = \frac{T+N}{g+a} = \frac{500+300}{20} m = 40kg$$

20. (b)

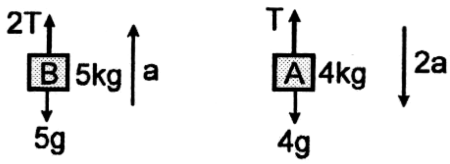
F.B.D. of man and plateform




$2T + N = 25g \quad N + T = 50g \; 3T = 750 \quad T = 250N$

21. (a)

The F.B.D. of block A and B are




From constraint, the acceleration of A & B are '2a' and 'a' respectively. Applying Newton's second law to blocks A and B, we we get

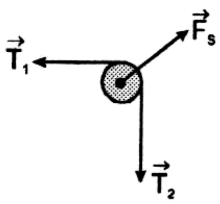
$4g - T = 4(2a)$ ……. (i)

$2T - 5g = 5(a)$ …….. (ii)

Solving we get acceleration of A and B as $\frac{2g}{7}$

Downward, $\frac{g}{7}$ upward respectively

22. (c)



The F.B.D. of pulley is as shown

Let $\vec{T_1}, \vec{T_2}$ and $\vec{F_S}$ be the forces

Exerted by the horizontal string, vertical

String by the support on the massless pulley

Respectively. Then

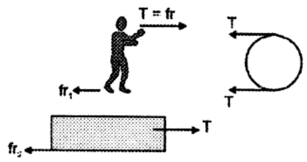
$\vec{T_1} + \vec{T_2} + \vec{F_S} = 0$ or $\left|\vec{F_S}\right| = \left|\vec{T_1} + \vec{T_2}\right| = 2\sqrt{2}\,\text{mg}$

$\left(\because \text{Tension in each string is} \left|\vec{T_1}\right| = \left|\vec{T_2}\right| = 2\text{mg}\right)$

$2\sqrt{2}\,\text{mg}$

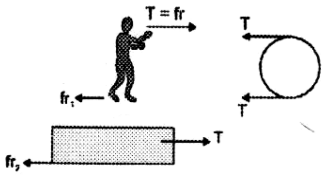
23. (a)

24. (d)




$\text{fr}_{1\max} = \text{fr}_{\text{available at man's feet}} = 0.1(20)\,\text{N}$

$\text{fr}_2 = \text{fr}_{\text{available at ground}} = 0.1(40)(10) = 40\,\text{N}$

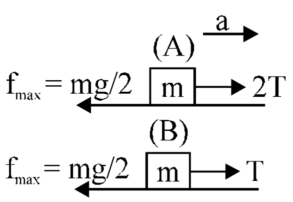



So net friction on man is $= 15 - 15 = 0\,\text{N}$.

25. (A,C)

26. Time depends only on y-direction ( perpendicular to river flow) velocity which is same for all the four

27.




Suppose only block (A) and (C) move

$2T - \dfrac{mg}{2} = ma$

$2mg - T = 2m \,.\, 2a$

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$3.5mg = 9\,ma$

$a = \dfrac{7}{18}g$ & $2a = \dfrac{7}{9}\,g$

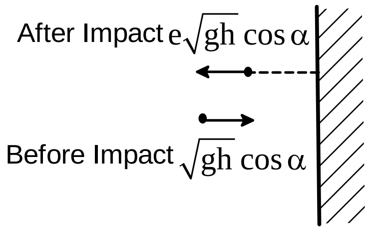
$T = 2mg - 2m \cdot \frac{7g}{9} = \frac{4}{9}\ mg < \frac{mg}{2}$ ]

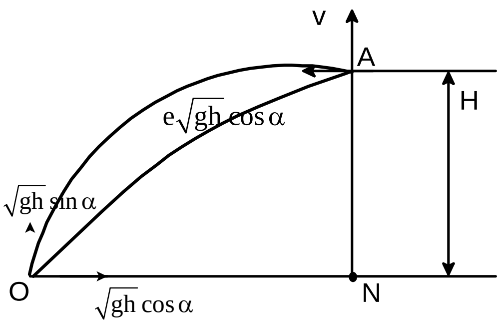
28. (A,C,D)

29. (C,D)
direction of friction does not change
friction is constant.

30. (a, c, d)
As there is no friction between the ball and the groove so there will be no centripetal force i.e., No net force on the ball towards the center hence the ball moves in radially outward direction w.r.t disc

31. (b, c, d)




$$t_{(OA)} = \frac{c}{\sqrt{gh}\cos\alpha}$$

$$v = \sqrt{gh}\sin\alpha - \frac{gc}{\sqrt{gh}\cos\alpha}$$

$$t_{(AO)} = \frac{c}{e\sqrt{gh}\cos\alpha}$$

Due to the impact only horizontal component of velocity gets affected so total time of flight remains the same

$$t_{(OA)} + t_{(AO)} = T$$

$$\frac{c}{\sqrt{gh}\cos\alpha} + \frac{c}{e\sqrt{gh}\cos\alpha} = \frac{2\sqrt{gh}\sin\alpha}{g}$$

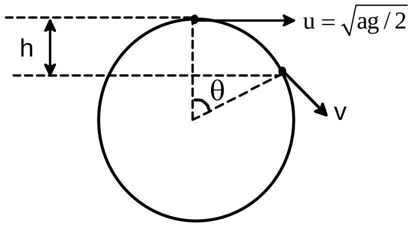
$$\frac{c}{\sqrt{gh}\cos\alpha}\left(1+\frac{1}{e}\right) = \frac{2\sqrt{gh}\sin\alpha}{g}$$

$$1+\frac{1}{e} = \frac{h\sin 2\alpha}{c}$$

$$\frac{1}{e} = \frac{h\sin 2\alpha - c}{c}$$

$$e = \frac{c}{h\sin 2\alpha - c}$$

32. (a, b, c)




$$\frac{1}{2}mu^2+mga\left(1-\cos\theta\right)=\frac{1}{2}mv^2 \qquad \ldots(1)$$

As N = 0 at angle of rotation $\theta$

So $mg\cos\theta=\dfrac{mV^2}{a}$ $\qquad \ldots(2)$

Using eq. (1)

$$\frac{1}{2}m\left(\sqrt{\frac{ag}{2}}\right)^2+mga\left(1-\cos\theta\right)=\frac{1}{2}\left(mg\,a\cos\theta\right)$$

$$\frac{mga}{4}+mga-mga\cos\theta=\frac{mga\cos\theta}{2}$$

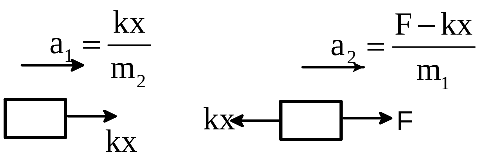
$$\frac{5mga}{4}=\frac{3}{2}mga\cos\theta$$

$$\cos\theta=\frac{5}{6}$$

$$\theta=\cos^{-1}\left(5/6\right)$$

$$h=a\left(1-\cos\theta\right)=a\left(1-\frac{5}{6}\right)=\frac{a}{6}$$

33. (a, c, d)




$$a_{rel}=\left(a_2-a_1\right)=\frac{vdv}{dx}$$

$$\left(\frac{F-kx}{m_1}\right)-\left(\frac{kx}{m_2}\right)=v\frac{dv}{dx}$$

$$\frac{F-kx}{m_1}\left[\frac{1}{m_1}+\frac{1}{m_2}\right]=\frac{v\,dv}{dx}$$

$$\int_0^x\left[\frac{F}{m_1}-kx\left(\frac{1}{m_1}+\frac{1}{m_2}\right)\right]dx=\int_0^0 v\ dv$$

At maximum compression or maximum extension relative velocity should become zero.

$$\Rightarrow\frac{Fx}{m_1}-\frac{kx^2}{2}\left[\frac{1}{m_1}+\frac{1}{m_2}\right]=0$$

$$\Rightarrow \quad x=0 \quad \text{or} \quad x=\frac{F/m_1}{\frac{K}{2}\left[\frac{1}{m_1}+\frac{1}{m_2}\right]}$$

$$\Rightarrow \quad x=\frac{2m_2F}{K\left(m_1+m_2\right)}$$

When $m_1 = m_2 = m$ then $x_{max} = F/K$
Compression the spring will be zero.

34. (a, d)
$x = 2t^3 - 33t^2 + 180t + 2$
$v = \frac{dx}{dt} = 6t^2 - 66t + 180$
$a = \frac{d^2x}{dt^2} = 12t - 66$
Direction changes when $v = 0$
$\Rightarrow t = 5,\ t = 6$
Minimum speed is when it is zero
i.e. $t = 5$ & $t = 6$
Retarded means velocity is opposite of acceleration
i.e. $v \times a < 0 \Rightarrow 0 < t < 5$ & $5.5 < t < 6$

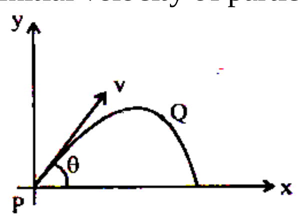
35. (a, c, d)

| time | distance left |
|---|---|
| $t = 0$ | $x_0$ |
| $t = T$ | $x_0/2$ |

$t = 2T \rightarrow x_0/2^2$
$t = nT \rightarrow \frac{x_0}{(2)^n} = \frac{x_0}{(2)^{t/T}} = x_0(2)^{-t/T} = x_0(2)^{-t}$
$(\because T = 1s)$
$\therefore$ Distance travelled in time
$t = x = x_0 - x_0(2)^{-t} = x_0(1 - 2^{-t})$
$v = \frac{dx}{dt} = x_0 2^{-t} \Rightarrow a = \frac{dv}{dt} = -x_0 2^{-t}$
$|a| = x_0 2^{-t}$.

36. (a, c, d)
Slope of displacement-time curve gives velocity.
(a) During OA slope is + ve but decreasing hence velocity is positive and acceleration is negative.
(c) During BC slope is –ve and going to zero hence velocity is –ve but acceleration is +ve.
(d) During DE slope is +ve and increasing hence velocity is +ve and increasing $\therefore$ +ve acceleration

37. (b, c)
Initial velocity of particle in vector from can be written as




$\overrightarrow{v_p} = v\cos\theta\hat{i} + v\sin\theta\hat{j}$ ………….(1)
Velocity of particle at any time t will be:
$\overrightarrow{v_Q} = v\cos\theta\hat{i} + (v\sin\theta - gt)\hat{j}$ …………..(2)
Given that $\overrightarrow{v_p} \perp \overrightarrow{v_Q}$

$\therefore \overrightarrow{v_p}.\overrightarrow{v_Q} = 0$

Or $v^2\cos^2\theta + v^2\sin^2\theta - v\sin\theta gt = 0$

Or $v^2 = v\sin\theta gt$ or $t = \frac{v}{g}\operatorname{cosec}\theta$

Substituting this value of $t$ in Eq. (2) we get:

$$\overrightarrow{v_Q} = v\cos\theta\hat{i} + \left(v\sin\theta - \frac{v}{\sin\theta}\right)\hat{j}$$

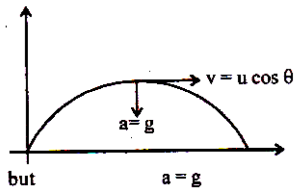
Or $\left|\overrightarrow{v_Q}\right| = \sqrt{v^2\cos^2\theta + v^2\sin^2\theta + \frac{v^2}{\sin^2\theta} - 2v^2}$

$= v\cot\theta$

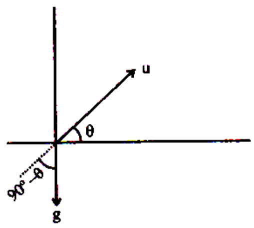
38. (a, c)

At highest point angle between $\vec{a}$ and $\vec{v}$ is $90^0$. Hence, total acceleration is only normal or radial acceleration.

$\therefore a = a_n = \frac{v^2}{R}$




$\therefore g = \frac{(u\cos\theta)^2}{R}$



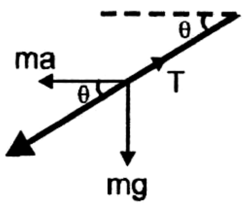

Or $R = \frac{u^2\cos^2\theta}{g}$

At point of projection component of acceleration (= g) along velocity vector is $-g\cos\left(90^0 - \theta\right)$ or $-g\sin\theta$.

39. (b,c)

40. (a,c)

Sol:




$\tan\theta = \frac{g}{a}$

$T = m\sqrt{g^2 + a^2}$

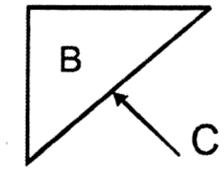
41. (b,d)

Since the apparent weight is increasing, hence acceleration of the lift should be upwards. This is possible in case of (b) and (d)

42. (a,b,c,d)

There is no horizontal force on block A, therefore it does not move in x-directing, whereas there is net downward forve $(mg - N)$ is acting on it, making its acceleration along negative y-direction.

Block B moves downward as well as in negative x-direction. Downward acceleration of A and B will be equal due to constrain, thus w.r.t. B, A moves in positive x-direction.




Due to the component of normal exerted by C on B, it moves in negative x-direction

43. (a,d)

Maximum value of friction force between 4m and inclined plane

$= \mu(4mg)\cos 45^0$

$= \frac{1}{\sqrt{2}}(4mg)\frac{1}{\sqrt{2}} = 2mg$

Here pulling force

$F_P = 4mg\cos 45 - mg$

$= (2\sqrt{2} - 1)mg < 2mg$

$\therefore$ Block will not move

$\therefore$ Acceleration of 4m block

$= 0, T = mg$

$\therefore$ frictional force on 4m block

$= (2\sqrt{2} - 1)mg$

44. (b,c)

$f_{max} = 20 \times 0.4 \times 10 = 80N$

For equilibrium

Minimum value of $m \Rightarrow m(10) = 100 - 80 \Rightarrow m = 2kg$

Maximum value of $m \Rightarrow m(10) = 100 + 80 \Rightarrow m = 18kg$

So, (B) and (c) are correct

45. (CD) 46. (BD) 47. (D) 48. (BC) 49. (AB)

50. (a), 51. (b), 52. (a)

Slope of $V_y$ versus $t$ graph is $-g$

$\therefore -g = \frac{-10}{t_1}$

As displacement along y-axis is zero $k = -V_y = -10$

$\tan\alpha = \frac{u_y}{u_x} = 1$

53. (D) 54. (D)

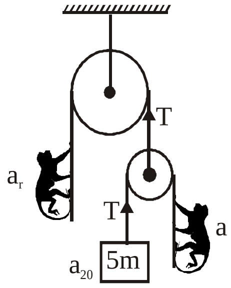
55. T = 5mg

$a = \frac{5mg - mg}{m} = 4g$

T' = 2T = 10mg

$a_r = \frac{10mg - mg}{m} = 9g$

$a_{rp} = 9g - (-2g) = 11g$




56. Let a = A – Bt

At $t = 0$, $\frac{dv}{dt} = a = \tan 37^\circ = 3/4$

$\Rightarrow A = \frac{3}{4}$

Again , t = 3 sec, a = 0

$\Rightarrow \frac{3}{4} - B\times 3 = 0 \Rightarrow B = \frac{1}{4}$

So $a = \frac{3}{4} - \frac{1}{4}t$

$\int_{10}^{0} dv = \int_{0}^{t} \frac{(3-t)}{4} dt$

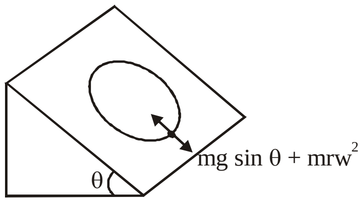
t = 10

57. [6]

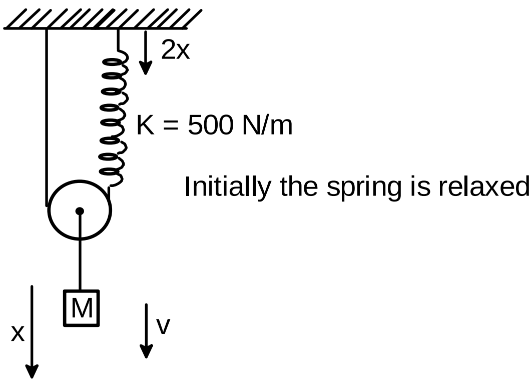
$f \geq mg\sin\theta + mr\,\omega^2$

$\mu\, mg\cos\theta \geq mg\sin\theta + mr\,\omega^2$

or $\mu \geq \tan\theta + \frac{\omega^2 r}{g\cos\theta}$




58. (2)




If the block moves down by x, then the spring is extended by 2x as the string connecting the spring is in extensible. Using energy conservation principle,

$0 + mgx + 0 = \frac{1}{2}mv^2 + \frac{1}{2}K(2x)^2 + 0$

The extension in the spring will be maximum when the block comes to rest momentarily.

i.e., v = 0

$$mgx = 0 + \frac{1}{2}k(2x)^2$$

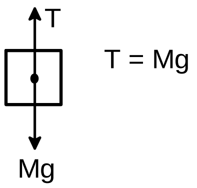
$$2mgx = k4x^2$$

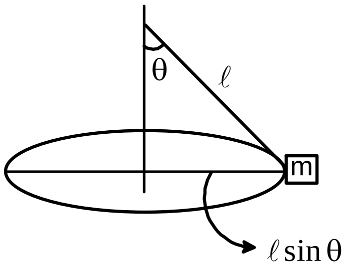
$$mg = 2kx \Rightarrow x = mg/2k$$

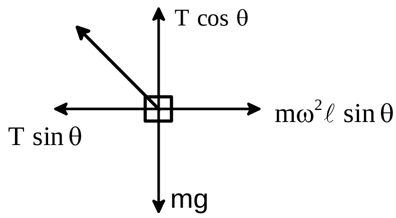
Maximum extension in spring $\Rightarrow 2x = \frac{mg}{k} = 2cm$

59. (5)

If M remains stationary




$$T\cos\theta = mg$$

$$T\sin\theta = m\omega^2\ell\sin\theta$$

$$mg = m\omega^2\ell$$

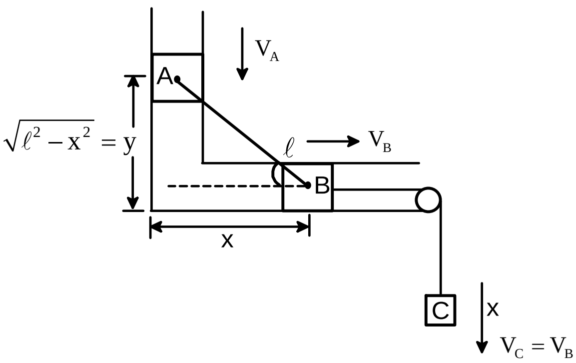
$$\omega = \sqrt{\frac{Mg}{m\ell}}$$

$$\omega = \sqrt{\frac{200}{2(.1)}} = 10\sqrt{10}$$

$$\omega = 10\pi \ \text{rad/sec}$$

$$\omega = \frac{10\pi}{2\pi}\text{rev/sec}$$

60. (5)




Using energy conservation,

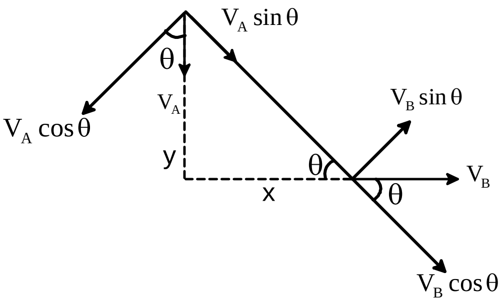
$$m_A\left(\ell - \sqrt{\ell^2 - x^2}\right) + 0 + m_C gx = \frac{1}{2}m_A v_A^2 + \frac{1}{2}m_B v_B^2 + \frac{1}{2}m_C v_C^2$$

$$200g\left(\frac{10 - \sqrt{100 - 36}}{1008}\right) + 10096 = \frac{1}{2}200(138\beta)^2 + \frac{1}{2}100\left(\frac{138\beta}{100}\right)^2$$

$$\Rightarrow 200(9.8)\times 2+600(9.8)=\left(\frac{138\beta}{100}\right)^2 100\left[\frac{36}{64}+1+0.5\right]$$

$$\Rightarrow\ 9.8(400+600)=\frac{(138\beta)^2}{100}\left(\frac{132}{64}\right)$$

$$\Rightarrow\ \beta=5$$




$$V_A \sin\theta = V_B\cos\theta$$

$$V_A = V_B \frac{x}{\sqrt{l^2 - x^2}}$$

$$V_A = 138\beta\frac{6}{8}$$