HEAT & THERMO (SOLUTION)

Single Choice Questions

1. (b)

Let A is cross section area of a beaker.

Then, final cross sectional area $A' = A[1+6\alpha T]$

Final volume of liquid $V' = AH[1+3\alpha T]$

$\Rightarrow$ Final height $H' = \frac{V'}{A'}$

$\Rightarrow$ $H' = H\frac{[1+3\alpha T]}{[1+6\alpha T]} \sim\sim H[1-3\alpha T]$

$\Rightarrow$ Reduction is level $H - H' = 3\alpha TH$

2. (a)

When system of masses m_1, m_2, …. specific heat capacities s_1, s_2, … and initial temperature $\theta_1, \theta_2, \ldots$ are mixed, the temperature of the mixture is

$$\theta = \frac{\sum ms\theta}{\sum ms} = \frac{m_1 s_1 \theta_1 + m_2 s_2 \theta_2 + \ldots}{m_1 s_1 + m_2 s_2 + \ldots}$$

For system of equal mass, $\theta = \frac{\sum s\theta}{\sum s}$

Let s_1, s_2 and s_3 be the specific heat capacities of A, B and C respectively.

For A + B, $15 = \frac{10s_1 + 25s_2}{s_1 + s_2}$ or $5s_1 = 10s_2$

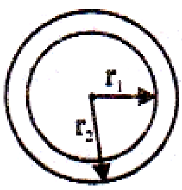
or $s_1 = s_2$

Similarly, calculate θ for A + C systems.

3. (d)

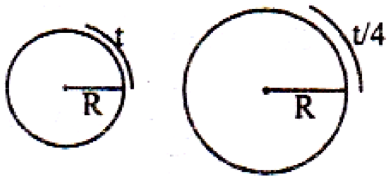
$$\int dR = \int \frac{dr}{4\pi r^2 K} = \frac{1}{4\pi K}\left[\frac{r_2 - r_1}{r_2 r_1}\right]$$ [R thermal resistance]

$$Q = \frac{4\pi k\Delta\theta}{\left(\frac{r_2 - r_1}{r_1 r_2}\right)} = \frac{4\pi K\Delta\theta}{\left(\frac{t}{r^2}\right)}$$



$$\frac{mL}{time} = \frac{4\pi K\Delta\theta}{(t/r^2)}$$ $\left[\therefore m = \rho \times \frac{4}{3}\pi r^3\right]$

$$\frac{\rho L}{time} = \left(\frac{K}{t\ r}\right) \times constant$$



$\text{time} \propto \frac{t\ r}{K}$

$$\frac{25}{16} = \frac{\frac{t}{4} 2K_s}{trK_L} = \frac{1}{2}\frac{K_s}{K_L}; \frac{K_L}{K_s} = \frac{8}{25}$$

4. (b)
The time of cooling increases as the differences between the temperature of body and surrounding is reduced. So $T_1 < T_2 < T_3$ (According to Newton's law of cooling).

5. (c)
Power radiated $P \propto T^4$ also $\lambda e\sigma\left(T^4 - T_0^4\right)$
S is same $\Rightarrow$ T is same.

6. (b)

$$Q = \int_{T_0}^{\eta T_0} CdT = a \ln \frac{\eta T_0}{T_0} = a \ln \eta$$

$$\Delta U = Cv\Delta T = \frac{R}{\gamma - 1}(\eta - 1)T_0$$

$$W = Q - \Delta U = a \ln \eta - \left[\frac{\eta - 1}{\gamma - 1}\right]RT_0$$

7. (a)
For cyclic process;
$Q_{cyclic} = W_{AB} + W_{BC} + W_{CA} = 10\text{ J} + 0 + W_{CA} = 5\text{J}$
$\Rightarrow$ $W_{CA} = -5\text{J}$

8. (d)
From Boyle's law (T = constant)
$P_1V_1 = P_2V_2$

$\therefore$ $\left(Hd_{water} + h\,d_{mercury}\right)g\left(\frac{4}{3}\pi r^3\right)$

$= hd_{mercury}g\left(\frac{4}{3}\pi(2r)^3\right)$

$\Rightarrow$ $Hd_{water} = 8hd_{mercury} - hd_{mercury}$

$\Rightarrow$ $H = 7h\frac{d_{mercury}}{d_{water}}$

$\therefore$ $H = 7h\rho$

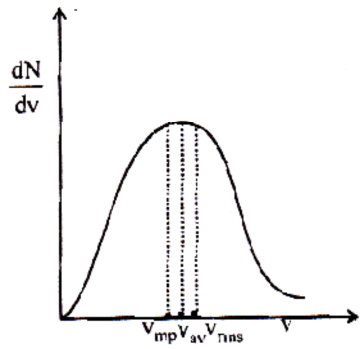
9. (b)
The r.m.s. speed of hydrogen molecule on the surface of the sun is

$$V_{rms} = \sqrt{\frac{3RT}{m}} = \sqrt{\frac{3 \times 8.3 \times 6000}{\left(2 \times 1.008 \times 10^{3}\right)}}$$

$$= \left(\sqrt{\frac{24.9 \times 6}{2.016}}\right) \times 10^{3} = 8650\,m/s$$

10. (d)

Maxwell speed distribution graph is asymmetric graph because it has long "tail" than extends to infinity. Also V_{rms} depends upon nature of gas and it's temperature.



11. (b, c, d)

W = ms or $m = \frac{W}{s} = \frac{4.5}{0.09} = 50\,gm$

The thermal capacity and the water equivalent, of a body have the same numerical value.

Also, $Q = 4.5 \times 8 = 36\,cal$

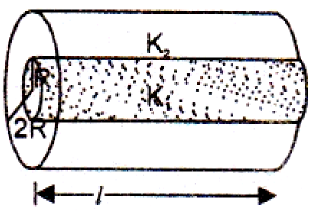
Since the temperature remains constant, during the process of melting, no heat is exchanged with the calorimeter and hence,

$$Q = 15 \times 80 = 1200\,cal$$

Hence, the correct choices are (b), (c) and (d)

12. (a, b, d)

Total transfer of heat per second through the composite = Heat transfer per second from material with thermal conductivity K_1 + Heat transfer per second form material with thermal conductivity K_2.



$$\frac{KA\Delta T}{\ell} = \frac{K_1 A_1 \Delta T}{\ell} + \frac{K_2 A_2 \Delta T}{\ell}$$

$$\Rightarrow \quad K\pi(2R)^2 = K_1\pi R^2 + K_2\pi\left[(2R)^2 - R^2\right]$$

$$\Rightarrow \quad K\pi 4R^2 = K_1\pi R^2 + K_2\pi 3R^2$$

$$\Rightarrow \quad K = \frac{K_1 + 3K_2}{4}$$

13. (d)

Rate of loss of heat $\propto$ differences in temperature with the surroundings.

At 50^0C, $\frac{dQ}{dt} = k(50-20) = 10$, where k = constant.

$\therefore$ k = 1/3

At an angle temperature of 35^0C,

$\frac{dQ}{dt} = \frac{1}{3}(35-20)\,J/s = 5J/s$

Heat lost in 1 minutes = $\frac{dQ}{dt} \times 60J = 300J = Q$

Fall in temperature = $0.2^0C = \Delta\theta$.

$Q = c\Delta\theta$.

Heat capacity $= c = \frac{Q}{d\theta} = \frac{300J}{0.2^0C} = 1500J/{}^0C$.

14. (c, d)

If container is a very good conductor (d) is correct. If container is a very bad conductor (c) is correct.

15. (a, c)

Not work is done in the process AB and CD

$W_{DA} = RT_0 \ln 2$

$W_{BC} = 2RT_0 \ln\left[\frac{v_0}{2v_0}\right] = -2RT_0 \ln 2$

Total work done by the gas in the cycle = W

$= -RT_0 \ln 2$

As $\Delta U = 0$, $Q = W = -RT_0 \ln 2$

i.e., heat is given out.

16. (a, c, d)

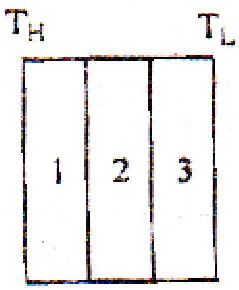
T = constant

$\therefore$ PV = constant (Boyle's law)

or $P \propto \frac{1}{V}$

Pressure of the gas is increasing, therefore, volume should be decreases. Work done by the gas is negative or work done on the gas will be positive. Further temperature of the gas is constant. Therefore, internal energy will remain constant.

17. (b)



Heat current $H = \frac{d\theta}{dt} = \frac{T_H - T_L}{(R_{th})_{eq}}$

$T_H \rightarrow$ heat temperature, $T_L \rightarrow$ low temperature

Equivalent thermal resistance $\left(R_{th}\right)_{eq} = \frac{L_1}{k_1 A} + \frac{L_2}{k_2 A} + \frac{L_3}{k_3 A}$

In each case: $H = \frac{T_H - T_L}{\left(R_{th}\right)_{eq}} = \frac{\Delta T_1}{\left(R_{th}\right)1}$; $\left(R_{th}\right)_1 = \frac{L_1}{k_1 A}$

Hence, $\left(\Delta T_1\right)_a = \left(\Delta T_1\right)_b = \left(\Delta T_1\right)_c$

18. (a)

By linear expansion of solids, we have

$\Delta \ell = \ell.\alpha.\Delta T$

So $\ell_{steel}.\alpha_{steel}.\Delta T = \ell_{copper}.\alpha_{copper}.\Delta T$

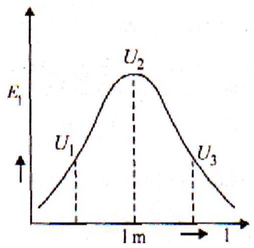
$$\ell_{steel} = \frac{\ell_{copper}.\alpha_{copper}}{\alpha_{steel}} = \frac{24 \times 18 \times 10^{-6}}{12 \times 10^{-6}} = 36\text{cm}$$

19. (d)

According to Wien's displacement law,

$\lambda_m T = 2088 \times 10^6\ nmK$

The wavelength at the peak of the spectrum becomes $\lambda_m = \frac{2.88 \times 10^6\ nmK}{2880\ K} = 10^3\ nm$



Thus, the maximum energy is radiated for 10^3 nm wavelength. It follows that the energy radiated between 499 *nm* to 500 *nm* will be less than that the emitted between 999 *nm* to 1000 *nm*, i.e. $U_1 < U_2$ or $U_2 > U_1$.

20. (d)

$Q_{AB} = \Delta U_{AB} + W_{AB}$

$W_{AB} = 0$

$\Delta U_{AB} = \frac{f}{2} nR\Delta T \Rightarrow \frac{f}{2}\left(\Delta PV\right)$

$\Delta U_{AB} = \frac{5}{2}\left(\Delta PV\right) \Rightarrow Q_{AB} = 2.5 P_0 V_0$

Process BC

$Q_{BC} = \Delta U_{BC} + W_{BC} = 0 + 2P_0 V_0 \ln 2 = 1.4 P_0 V_0$

$Q_{net} = Q_{AB} + Q_{BC} = 3.9 P_0 V_0$

COMPLEX NUMBER (SOLUTION)

1. (c)

$$E = 4 + 5(\omega)^{334} + 3(\omega)^{365} = 4 + 5\omega + 3\omega^2$$
$$= 1 + 2\omega + 3(1 + \omega + \omega^2) = 1 + \left(-1 + i\sqrt{3}\right) = i\sqrt{3}$$

2. (b)

$$\because \arg\left(\frac{z - z_1}{z_2 - z}\right) = \frac{\pi}{2} \Rightarrow \operatorname{Re}\left(\frac{z - z_1}{z_2 - z}\right) = 0$$
$$\Rightarrow \frac{z - z_1}{z_2 - z} + \frac{\bar{z} - \bar{z}_1}{z_2 - z} = 0$$
$$\Rightarrow (z - z_1)\left(\bar{z}_2 - \bar{z}\right) + (z_2 - z)\left(\bar{z} - \bar{z}_1\right) = 0$$
$$\Rightarrow z\left(\bar{z}_1 + \bar{z}_2\right) + \bar{z}(z_1 + z_2) - 2z\bar{z} - \left(z_1\bar{z}_2 + z_2\bar{z}_1\right) = 0$$
$$\Rightarrow z\bar{z} - (5 + i)z - (5 - i)\bar{z} + 21 = 0$$
$$\Rightarrow |z - (5 - i)| = \sqrt{5}$$

3. (c)

$$|iz + z_0| = |i z - i^2 + z_0 - 1| = |i(z - i) + 5 + 3i - 1| = |i(z - i) + (4 + 3i)|$$
$$\therefore |iz + z_0| \le |i(z - 2)| + |(4 + 3i)| \le 1.2 + 5 \le 7$$
$\therefore$ Maximum value of $|iz + z_0|$ is 7

4. (a)

$$\text{Given expression } = \left(\frac{z(1 + z)}{z + \bar{z}}\right)^4 + \left(\frac{z + z\bar{z}}{z(1 + z)}\right)^4$$
$$= \left(\frac{1(1 + z)}{z + |z|}\right)^4 + \left(\frac{z + |z|^2}{z(1 + z)}\right)^4$$
$$= \left(\frac{z(1 + z)}{z + 1}\right)^4 + \left(\frac{z + 1}{z(1 + z)}\right)^4$$
$$= z^4 + \frac{1}{z^4} = (\cos\theta + i\sin\theta)^4 + \frac{1}{(\cos\theta + i\sin\theta)^4}$$
$(\text{Since } z = \cos\theta + i\sin\theta)$
$$= \cos 4\theta + i\sin 4\theta + \cos 4\theta - i\sin 4\theta$$
$$= 2\cos 4\theta = 2\cos 4(\arg z)$$

5. (d)

$$\left|\frac{z_1}{|z_1|} + \frac{z_2}{|z_2|}\right| \le \left|\frac{z_1}{|z_1|}\right| + \left|\frac{z_2}{|z_2|}\right| = \frac{|z_1|}{|z_1|} + \frac{|z_2|}{|z_2|} = 2$$
$$\therefore \left(|z_1| + |z_2|\right)\left|\frac{z_1}{|z_1|} + \frac{z_2}{|z_2|}\right| \le 2\left(|z_1| + |z_2|\right)$$

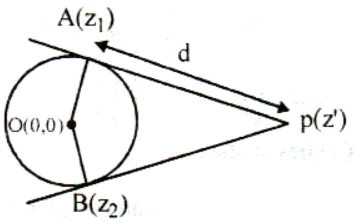
6. (c)

$AP = \frac{d}{r} AO.e^{i\pi/2}$

$z' - z_1 = \frac{d}{r}(-z_1 i)$ ....(1)

$BP = \frac{d}{r} BO.e^{-i\pi/2}$

$z' - z_2 = \frac{d}{r} z_2 i$ .....(2)



Now from eq. (1) and (2) , we get

$\frac{z' - z_1}{z' - z_2} = -\frac{z_1}{z_2} \Rightarrow z' = \frac{2z_1 z_2}{z_1 + z_2}$

7. (c)

$\frac{z_1 - z_2}{z_2 - z_3} = \frac{1 - i\sqrt{3}}{2}$

$\Rightarrow \arg\left(\frac{z_1 - z_3}{z_2 - z_3}\right) = \arg\left(\frac{1 - i\sqrt{3}}{2}\right)$

$= \arg\left(\cos(-\pi/3) + i\sin(-\pi/3)\right)$

$\Rightarrow$ Angle between $z_1 - z_3$ and $z_2 - z_3$ is 60°

and $\left|\frac{z_1 - z_2}{z_2 - z_3}\right| = \left|\frac{1 - i\sqrt{3}}{2}\right|$

$\Rightarrow \left|\frac{z_1 - z_3}{z_2 - z_3}\right| = 1 \Rightarrow z_1 - z_3 = z_2 - z_3$

$\Rightarrow$ The Δ with vertices z_1, z_2 and z_3 is isosceles with vertical angle 60°. Hence rest of the two angles should also be 60° each.

$\Rightarrow$ Req. Δ is an equilateral Δ

8. (b)

Key Concept: (D'Moivre's thm)

$(\cos\theta + i\sin\theta)^n = \cos n\theta + i\sin n\theta, n$ is any rational no.

$z = \left(\frac{\sqrt{3}}{2} + \frac{i}{2}\right)^5 + \left(\frac{\sqrt{3}}{2} - \frac{i}{2}\right)^5$

We have $\frac{\sqrt{3}}{2} + \frac{i}{2} = \cos\pi/6 + i\sin\pi/6$

and $\frac{\sqrt{3}}{2} - \frac{i}{2} = \cos\pi/6 - i\sin\pi/6$

$\therefore\ z = \left(\cos\pi/6 + i\sin\pi/6\right)^5 + \left(\cos\pi/6 - i\sin\pi/6\right)^5$

$= \left(\cos 5\pi/6 + i\sin 5\pi/6 + \cos 5\pi/6 - i\sin 5\pi/6\right)$

$= 2\cos 5\pi/ = -\sqrt{3}$

$\therefore \operatorname{Re}(z) < 0$ and $\operatorname{Im}(z) = 0$

9. (b)

$\sum_{i=1}^{13}\left(i^n + i^{n+1}\right) = \sum_{i=1}^{13} i^n\left(1+i\right) = \left(1+i\right)\sum_{i=1}^{13} i^n$

This forms a G.P.

Sum of G.P. $= i\left(1+i\right)\frac{\left(1-i^{13}\right)}{1-i} = i-1$ as $i^{13} = i$

10. (c)

We have, $1+\omega+\omega^2+....+\omega^{n-1} = \frac{1-\omega^n}{1-\omega}$

But $\omega^n = \cos\left(\frac{n\pi}{n}\right) + i\sin\left(\frac{n\pi}{n}\right) = \cos\pi + i\sin\pi = -1$

and $1-\omega = 2\sin^2\frac{\pi}{2n} - 2i\sin\frac{\pi}{2n}\cos\frac{\pi}{2n}$

$= -2i\sin\left(\frac{\pi}{2n}\right)\left[\cos\frac{\pi}{2n} + i\sin\frac{\pi}{2n}\right]$

Thus, $1+\omega+\omega^2+.....+\omega^{n-1}$

$= \frac{2\left[\cos\left(\pi/2n\right) - i\sin\left(\pi/2n\right)\right]}{-2i\sin\left(\pi/2n\right)} = 1 + i\cot\left(\pi/2n\right)$

11. (a,b,c)

$|z_1| = |z_2| = 1$

$\Rightarrow a^2 + b^2 = c^2 + d^2 = 1$ ……(1)

and $\operatorname{Re}\left(z_1\overline{z_2}\right) = 0 \Rightarrow \operatorname{Re}\left\{\left(a+ib\right)\left(c-id\right)\right\} = 0$

$\Rightarrow ac + bd = 0$ …..(2)

Now from (1) and (2), $a^2 + b^2 = 1$

$\Rightarrow a^2 + \frac{a^2c^2}{d^2} = 1 \Rightarrow a^2 = d^2$ …..(3)

Also $c^2 + d^2 = 1$

$\Rightarrow c^2 + \frac{a^2c^2}{b^2} = 1 \Rightarrow b^2 = c^2$ …..(4)

$|\omega_1| = \sqrt{a^2 + c^2} = \sqrt{a^2 + b^2} = 1$

[From (1) and (4)]

and $|\omega_2| = \sqrt{b^2 + d^2} = \sqrt{c^2 + d^2} = 1$

[From (1) and (4)]

Further $\operatorname{Re}\left(\omega_1\overline{\omega_2}\right) = \operatorname{Re}\left\{\left(a+ic\right)\left(b-id\right)\right\}$

$= ab + cd = ab + \left(-\frac{ac}{b}\right)c$ [From (2)]

$= \frac{ab^2 - ac^2}{b} = 0$ [From (4)]

Also, $\text{Im}\left(\omega_1 \overline{\omega_2}\right) = bc - ad = bc - a\left(-\frac{ac}{b}\right)$

$= \frac{\left(a^2 + b^2\right)c}{b} = \frac{c}{b} = \pm 1 \neq 0$

$\therefore |\omega_1| = 1, |\omega_2| = 1$ and $\text{Re}\left(\omega_1 \overline{\omega_2}\right) = 0$

12. (b,c,d)

Given $|z - 1| < |z + 3|$

$\Rightarrow |z - 1|^2 < |z + 3|^2$

$\Rightarrow |z|^2 + 1 - 2\,\text{Re}(z) < |z|^2 + 9 + 2\,\text{Re}(3z)$

$\Rightarrow 2\,\text{Re}(4z) > -8$

$\therefore\ 4z + 4\bar{z} > -8$

and $\omega = 2z + 3 - i$ ….(i)

$\therefore \omega + \bar{\omega} = 2z + 3 - i + 2\bar{z} + 3 + i = 2\left(z + \bar{z}\right) + 6$ …..(ii)

or $\omega + \bar{\omega} > 2$

(a) $|\omega - 5 - i| < |\omega + 3 + i|$

$\Rightarrow |2z + 3 - i - 5 - i| < |2z + 3 - i + 3 + i|$

$\Rightarrow |2z - 2 - 2i| < |2z + 6|$

$\Rightarrow |z - 1 - i| < |z + 3|$

Which is false

(b) $|\omega - 5| < |\omega + 3|$

$\Rightarrow |2z + 3 - i - 5| < |2z + 3 - i + 3|$

$\Rightarrow |z - 1 - i/2| < |z + 3 - i/2|$

or $|z - 1| < |z + 3|$ which is true

(c) $\text{Im}(i\omega) > 1$

$\Rightarrow \frac{i\omega - i\bar{\omega}}{2i} > 1 \Rightarrow \frac{i\omega + i\bar{\omega}}{2i} > 1$

$\Rightarrow \omega + \bar{\omega} > 2$ [which is true from eq. (ii)]

(d) $|\arg(\omega - 1) < \pi/2|$

$|\arg(2z + 2 - i)| < \pi/2$

$\left|\tan^{-1}\left(\frac{\text{Im}(2z + 2 - i)}{\text{Re}(2z + 2 - i)}\right)\right| < \pi/2$

$\therefore\ \text{Re}(2z + 2 - i) > 0$

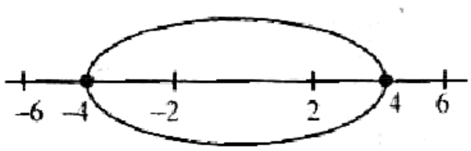
$\Rightarrow \frac{(2z + 2 - i) + \left(2\bar{z} + 2 + i\right)}{2} > 0$

$\Rightarrow z + \bar{z} > -2$ [which is true from eq. (i)]

13. (a,b)

The locus of $|z+2|+|z-2|=8$ is an ellipse having focii at $(2,0)$ and $(-2,0)$ and length of major axis is 8 units. i.e. $ae=2$ and $2a=8$ $\quad \therefore e=\frac{1}{2}, a=4$

Again, the second equation represents a straight line segment such that its locus comprise of all points inside the segment from $(-6,0)$ to $(6,0)$ as shown. Hence, it will intersect at two points, i.e., $(-4,0)$ and $(4,0)$



14. (a,b,c,d)

$\sqrt{20i-21}=\pm(2+5i)$ and $\sqrt{21+20i}=\pm(5+2i)$

$\therefore\ z=7+7i \Rightarrow \arg z=\frac{\pi}{4}$ or $z=-3+3i \Rightarrow \arg z=\frac{3\pi}{4}$

or $z=-7-7i \Rightarrow \arg z=-\frac{3\pi}{4}$ or $x=3-3i \Rightarrow \arg z=-\frac{\pi}{4}$

15. (a,b,c,d)

Let the points represented by z_1, z_2, z_3 and z_4 be A,B,C and D respectively. Since ABCD is a square mid point of AC $=$ mid point of BD

$\Rightarrow \frac{1}{2}(z_1+z_2)=\frac{1}{2}(z_2+z_4)$ or $z_1+z_3=z_2+z_4$

Also $AB=BC=CD=DA$

$\Rightarrow |z_1-z_2|=|z_2-z_3|=|z_4-z_1|$

Since diagonals of the square ABCD are equal

$\therefore AC=BD$

or $|z_1-z_3|=|z_2-z_4|$

Also, since $AC \perp BD$

$(z_1-z_3)/(z_2-z_4)$ is purely imaginary

16. (b,c,d)

$\text{Re}\left(\frac{z+1}{z+i}\right)=0 \Rightarrow \frac{\frac{z+1}{z+i}+\overline{\frac{z+1}{z+i}}}{2}=0$

$\Rightarrow \frac{z+1}{z+i}+\frac{\bar{z}+1}{\bar{z}-1}=0$

$(z+1)(\bar{z}-i)+(\bar{z}+1)(z+i)=0$

$\Rightarrow 2z\bar{z}+(1-i)z+(1+i)\bar{z}=0$

$z\bar{z}+\left(\frac{1-i}{2}\right)z+\left(\frac{1+i}{2}\right)\bar{z}=0$, which is a circle whose radius $=\sqrt{\left(\frac{1+i}{2}\right)^2}=\sqrt{\frac{i}{2}}$ and passing through $z=0$

($\because$ no constant term)

17. (a,c)

Let $z = x + iy$, then the equation is

$x^2 + y^2 - 2i(x + iy) + 2c(1 + i) = 0$

$\Rightarrow (x^2 + y^2 + 2y + 2c) + i(2c - 2x) = 0$

$\Rightarrow x^2 + y^2 + 2y + 2c = 0$ and $x = c$

$\Rightarrow c^2 + y^2 + 2y + c = 0$

$\Rightarrow y = -1 \pm \sqrt{1 - 2c - c^2}$

$\because y \in r \Rightarrow 1 - 2c - c^2 \geq 0$

$\Rightarrow c^2 + 2c - 1 \leq 0$

$\Rightarrow -1 - \sqrt{2} \leq c \leq -1 + \sqrt{2}$

$\therefore$ The equation has a solution

If $c \in \left[-1 - \sqrt{2}, 1 + \sqrt{2}\right]$ and the solution is given by

$z = c + i\left(-1 \pm \sqrt{1 - 2c - c^2}\right)$

The equation has no solution, if

$c \in \left(-\infty, -1 - \sqrt{2}\right) \cup \left(-1 + \sqrt{2}, \infty\right)$

18. (d)

$\because |z + iw| \leq |z| + iw| = |z| + |i||w| \leq 2$

$\therefore |z + iw| = 2 \quad \Rightarrow |z| = |w| = 1$

19. (d)

Let $z = x + iy$ and $w = \alpha + i\beta$

Now $|z + iw| = 2 \Rightarrow (z + iw)(\bar{z} - iw) = 4$

$\Rightarrow |z|^2 + |w|^2 + iw\bar{z} - i\bar{w}z = 4 \Rightarrow iw\bar{z} - i\bar{w}\bar{z} = 2$ …(1)

and $|z - i\bar{w}| = 2 \Rightarrow (z - i\bar{w})(\bar{z} + iw) = 4$

$\Rightarrow |z|^2 + |w|^2 + iwz - i\bar{w}\bar{z} = 4$

$\Rightarrow |z|^2 + |w|^2 + iwz - i\bar{w}\bar{z} = 4 \Rightarrow iwz - i\bar{w}\bar{z} = 2$ …..(2)

Add(1) and (2), $i(w - \bar{w})(z + \bar{z}) = 4 \Rightarrow i(2i\beta)$

$(2x) = 4 \Rightarrow \beta x = -1$ ……(3)

Subtract (1) from (2),

$\Rightarrow i(w + \bar{w})(z - \bar{z}) = 0 \Rightarrow \alpha y = 0$ …..(4)

From (4), either $\alpha = 0$ or $y = 0$

If $y = 0$, then $x^2 + y^2 = 1 \Rightarrow x = \pm 1 \Rightarrow z = 1$ or -1

If $\alpha = 0$, then $\alpha^2 + \beta^2 = 1 \Rightarrow \beta = \pm 1 \Rightarrow w = \pm i$

So, $I_m(z) = Re(w) = 0$

20. (b)

As $z = \pm 1$, so two values of z can be obtained

Volumetric Analysis (Redox Reaction)

1. (d)
Disproportionation involves oxidation reduction of same atom in a molecule

2. (c)
$A_3(BC_4)_2, 3\times 2+\left[5+4\times(-2)\right]\times 2=0$

3. (b)
$\left(Cr^{6+}\right)_2+6e\rightarrow 2Cr^{3+}$ $\therefore E_{K_2Cr_2O_7}=\frac{M}{6}$

4. (a)
Cr has $+6$ and Mn has $+7$ oxidation state.

5. (d)
The O.Ns. of S are shown along with the compounds.

S_8	$S_2O_8{}^{2-}$	$S_2O_3{}^{2-}$,	$S_4O_6{}^{2-}$
0	+ 6	+2	+2.5

Hence the order of increasing O.N. of S is
$S_8<S_2O_3 2-<S_4O_6{}^{2-}<S_2O_8{}^{2-}$

6. (d)
Equiv. of HCl taken $=60\times 2\times 10^{3-}$
Equi. of HCl present after the reaction $=20\times 1\times 10^{-3}$
$\therefore$ Equiv. of HCl utilized $=100\times 10^{-3}$
$\therefore 100\times 10^{-3}$ equiv. of metal carbonate $=6.90\,gm$
$\therefore 1$ equiv. of metal carbonate $=\frac{6.90}{10^{-1}}=69\,gm$
$\therefore$ equiv.wt of metal $=69-30=39$
[because equiv. wt. of carbonate =30]

7. (d)
'n' factor of $Cr_2O_7{}^{2-}$ ion in the given reaction is 6.
$\left[\overset{+6}{Cr_2}\,O_7^{2-}\longrightarrow Cr^{3+}\right]$
Equivalents of $Cr_2O_7{}^{2-}$ needed to oxidize
$N_2H_5{}^{+}=$ Equivalents of $N_2H_5^{+}=0.136$
$\therefore$ Moles $Cr_2O_7{}^{2-}$ needed to oxidize $N_2H_5{}^{+}=\frac{0.136}{6}=0.0227$

8. (b)

9. (c)

10. (c)

11. (d)
12. (d)

13. (a)

14. (a)

2 moles $Na_2S_2O_3$ correspond to 5 moles CO

10^{-5} moles $Na_2S_2O_3$ correspond to 2.5×10^{-5} mole CO

Mass of CO in 100g exhaust is 7×10^{-4}g = 7ppm

15. (b,d)

16. (a,b,d)

In the redox reaction O.S. of the some of the elements present in the reactants are varied

(a) $\overset{+1}{Na}\overset{-1}{Br} + \overset{+1}{H}\overset{-1}{Cl} \longrightarrow \overset{+1}{Na}\overset{-1}{Cl} + \overset{+1}{H}\overset{-1}{Br}$

No change in O.S. of the elements in reactants.

(b) $\overset{+1-1}{Br} + \overset{+1+5}{AgNO_3}\overset{-2}{} \longrightarrow \overset{+1-1}{Ag\,Br} + \overset{+1+5}{HNO_3}\overset{-2}{}$

No change in O.S. of the elements in reactants

(c) $\overset{0}{H_2} + \overset{0}{Br_2} \longrightarrow 2\overset{+1-1}{HBr}$

There is a change in O.S. of the elements in reactants. Therefore this is a redox reaction

(d) $\overset{+1}{Na_2}\overset{-2}{O} + \overset{+1+6}{H_2S}\overset{2-}{O_4} \longrightarrow \overset{+1}{Na_2}\overset{+6}{S}\overset{2-}{O_4} + \overset{+1}{H_2}\overset{-2}{O}$

No change in O.S. of the elements in reactants.

17. (a,c)

Mole ratio with Fe^{2+}

(a) $\overset{+8}{Os}O_4 \longrightarrow \overset{0}{Os}$ $x = 8$ $1:8\left(Fe^{2+}\right)$

(b) $\overset{+7}{M}nO_4^- \longrightarrow \overset{2+}{Mn}$ $x = 5$ $1:5\left(Fe^{2+}\right)$

(c) $\overset{+7}{Cl}O_4^- \longrightarrow \overset{-1}{Cl}$ $x = 8$ $1:8\left(Fe^{2+}\right)$

(d) $\overset{+6}{C}r_2O_7^{2-} \longrightarrow \overset{3+}{Cl}$ $x = 6$ $1:6\left(Fe^{2+}\right)$

Thus, 1 moles of OsO_4 reacts with 8 moles of Fe^{2+} and 1 moles of ClO_4^- reacts with 8 moles of Fe^{2+}

Therefore , the moles of Oso_4 and ClO_4^- require to oxidize Fe^{2+} solution are equal and minimum

18. (b)

19. (a)

20. (a)

Equivalent weight of $BrO_3^- = \frac{M}{6} = \frac{167}{6}$

$3N_2H_4 + 2BrO_3^- \longrightarrow 3N_2 + 2Br^- + 6H_2O$