

FULL MECHANICS SOLUTION

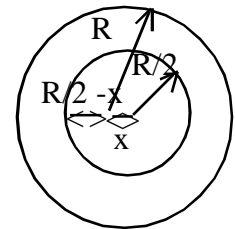
SINGLE CHOICE QUESTIONS

1. Minimum force to move the body = $\frac{\mu mg}{\sqrt{1+\mu^2}}$; $\Rightarrow \mu = \frac{3}{4}$

2. $\tau = mg \int v \cos \theta dt$; $\Rightarrow \frac{dL}{dt} = mgv \cos \theta$; $\Rightarrow L = \frac{mgv \cos \theta t^2}{2}$

3. $v^2 = \omega^2 (A^2 - x^2)$ & $a = -\omega^2 x \Rightarrow v^2 = -\frac{a}{x} (A^2 - x^2)$
 $\Rightarrow A = \sqrt{x^2 - \frac{xv^2}{a}}$

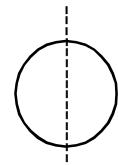
4. $\frac{\frac{4}{3}\pi R^3 \rho g - \frac{4}{3}\pi \frac{R^3}{8} x}{\frac{4}{3}\pi R^3 - \frac{R^3}{8}}$; $\Rightarrow \frac{\frac{4}{3}\pi R^3 \rho g - \frac{4}{3}\pi \frac{R^3}{8} x}{\frac{4}{3}\pi R^3 - \frac{R^3}{8}} = -\frac{x}{7} \Rightarrow x = \frac{7R}{16}$



5. Using perpendicular axis theorem,

$$2I_1 = M'R^2; I_1 = M'R^2/2; I_2 = \frac{M'R^2}{2} + M'R^2 = \frac{3}{2}M'R^2$$

$$I = \frac{I_2}{2} = \frac{3}{2} \frac{M'R^2}{2} = \frac{3}{2}MR^2$$



6. Upward force due to tension on 3 kg block is F/2 which should be greater than 3g and upward force on 2 kg block is F/4 which should be greater than 2g

7. Avg. speed $3 = \frac{\sqrt{v_r^2 t^2 + v_{mr}^2 t^2}}{t}$

$$\Rightarrow v_r^2 + 5 = 9; \quad \Rightarrow v_r = 2 \text{ m/s}$$

8. $v_{\text{avg}} = \frac{\frac{1}{2}x \frac{t}{2} xv + \frac{1}{2}xv}{t} = \frac{3v}{4}$

$$9. \quad \frac{m}{2}g - T = \frac{m}{2}a \quad \dots (i)$$

$$T \cos 60^\circ = \frac{ma}{\cos 60^\circ} ; \dots (ii) \text{ Solving (i) and (ii) accelerating o ring } \frac{2g}{9}$$

10. Work done by all the forces on the block equal to change in kinetic energy.

11. No effect of 'a' and 'g' on time period of spring pendulum.

$$12. \quad \text{Work done by friction} = \int \vec{F} \cdot d\vec{s} = \int_0^x \mu mg \cos \theta \frac{dx}{\cos \theta}$$

$$= \mu mgx = 20J$$

$$13. \quad \text{Conservation of energy } \frac{1}{2}mv^2 - mg\frac{\ell}{2} = mg\frac{\ell}{2} + m\sqrt{3}\left(2\ell\frac{\sqrt{3}}{2}\right)g; \quad v = \sqrt{8g\ell}$$

14. Conservation of momentum

$$M\sqrt{2gh} - m\sqrt{2gh} = MV_1 + mV_2 \quad (1)$$

$$-1 = \frac{V_1 - V_2}{2\sqrt{2gh}} \quad \dots (2)$$

$$V_2 = 3\sqrt{2gh}$$

$$h' = \frac{V_2^2}{2g} = 9h$$

15. FBD of 'B' and 'C'

$$\Rightarrow T - 2g = 2a \quad \dots (1)$$

$$\text{and } 3g - T = 3a \quad \dots (2)$$

$$T = \frac{12g}{5}$$

$$\text{For A } \Rightarrow T = mg$$

$$2x \frac{12g}{5} = mg \Rightarrow m = 4.8kg$$

$$16. \quad W = \int_0^{\pi/2} f \cdot R d\theta = \frac{\mu mg R \pi}{2} = 1 \text{ joule}$$

17. Let us assume cylinder is no moving then

$$T + f_s = mg \sin \theta$$

$$T \cdot R - f_s R = 0$$

$$\Rightarrow f_s = \frac{mg\sqrt{3}}{4}$$

$$\text{but } (f_s)_{\max} = \mu N = \mu mg \cos \theta$$

$$= 0.4 \cdot mg \times \frac{1}{2} = \frac{mg}{5}$$

$\therefore (f_s) < (f_s)_{\max}$, our assumption is wrong. So, friction existing must be kinetic

$$f_k = \mu mg \cos \theta = 0.4 \times mg \times \frac{1}{2} = \frac{mg}{5}$$

18. For full square about an axis passing through 'O' = $\frac{Ml^2}{6}$

by symmetry for remaining portion it must be $\frac{3}{4} \left(\frac{Ml^2}{6} \right) = \frac{Ml^2}{8}$ 29.B

19. Apply work-energy theorem

$$mgz - \frac{1}{2} K \left(\frac{mg}{K} + z \right)^2 - \frac{1}{2} K \left(\frac{mg}{K} \right)^2 + Fz = 0 ; \Rightarrow z = 2F/K$$

20. $a = 2 + |t - 2|$; for $t \leq 2$; $a = 2 - t + 2$; $a = 4 - t$

$$dv = (4 - t)dt ; v = 4t - t^2/2 \quad \text{at } t = 2, v = 6 \text{ m/s.}$$

for $t > 2$; $a = 2 + t - 2 = t$; $\int_6^v dv = \int_2^t t dv ; v - 6 = [t^2/2]_2^t$

$$v = \frac{t^2}{2} + 4 \quad \text{at } t = 4, v = 12 \text{ m/s.}$$

MULTIPLE CHOICE QUESTIONS

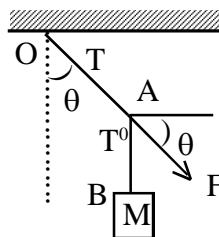
21. If at any instant elongation in spring is x and velocity of block is v from equilibrium position then

$$E_T \Rightarrow \frac{1}{2} kx^2 + \frac{1}{2} Mv^2 + \frac{1}{2} \int_0^L \frac{m}{L} dy \left(\frac{v}{L} y \right)^2$$

$$= \frac{1}{2} kx^2 + \frac{1}{2} Mv^2 + \frac{1}{2} \frac{m}{L} \frac{v^2}{L} \frac{L^3}{3};$$

$$E_T \Rightarrow \frac{1}{2} kx^2 + \frac{1}{2} \left(M + \frac{m}{3} \right) v^2$$

22. $T \sin \theta = F \cos \theta$
 $T \cos \theta = F \sin \theta + T'$
 $T' - F \sin \theta = Mg$



$$23. \quad mg - \frac{2kq^2}{R^2} + T = \frac{mv^2}{R} \quad \dots (i)$$

$$-mgR + \frac{1}{2}mu^2 = mgR + \frac{1}{2}mv^2 \quad \dots (ii)$$

$$\text{if } T = 0 \Rightarrow u_{\min} = \sqrt{5gR - \frac{2Kq^2}{mR}}$$

$$24. \quad F = -\frac{dU}{dx} = -5(2x - 4)$$

At mean position $F = 0 \Rightarrow x = 2m$

$$U_{\min} = -20 \text{ J}; \quad a = -50 \times 2(x-2)$$

$$\omega = 10 \text{ rad/sec}$$

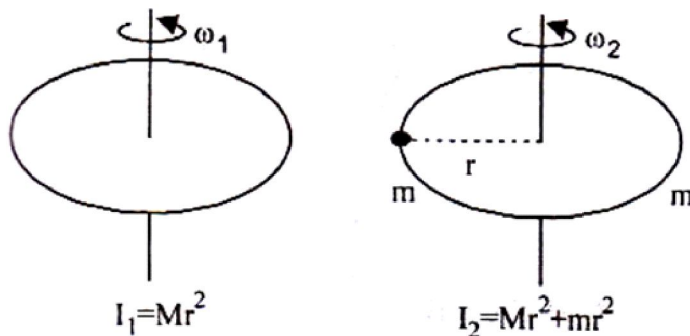
$$T = \pi/5 \text{ sec}$$

$$25. \quad \text{If the volume immersed initially is } (V/3). \text{ Then } \frac{V}{3}\rho g = mg \quad \dots (i)$$

If the volume immersed when the system accelerates is V' then

$$V'\rho\left(\frac{g}{2}\right) - mg = \frac{mg}{2} \Rightarrow V' = \frac{V}{3}$$

26. Since the objects are placed gently, therefore no external torque is acting on the system. Therefore angular momentum is constant.



$$\text{i.e., } I_1 \omega_1 = I_2 \omega_2$$

$$Mr^2 \times \omega_1 = (Mr^2 + 2mr^2)\omega_2$$

$$\therefore \omega_2 = \frac{M\omega}{M + 2m}$$

27. (a, b, c)

$$\vec{\tau} = \frac{d\vec{L}}{dt}$$

Given that

$$\vec{\tau} = \vec{A} \times \vec{L} \Rightarrow \frac{d\vec{L}}{dt} = \vec{A} \times \vec{L}$$

From cross-product rule, $\frac{d\vec{L}}{dt}$ is always perpendicular to the plane containing \vec{A} and \vec{L} .

By the dot product definition

$$\vec{L} \cdot \vec{L} = L^2$$

Differentiating with respect to time

$$\vec{L} \cdot \frac{d\vec{L}}{dt} + \vec{L} \cdot \frac{d\vec{L}}{dt} = 2L \frac{dL}{dt}$$

$$\Rightarrow 2\vec{L} \cdot \frac{d\vec{L}}{dt} = 2L \frac{dL}{dt}$$

Since $\frac{d\vec{L}}{dt}$ is perpendicular to \vec{L}

$$\Rightarrow \frac{dL}{dt} = 0$$

$$\Rightarrow L = \text{contt.}$$

28. (b) $v = v_0 - \mu g t$

$$\omega = \omega_0 + \frac{2\mu g}{R} t$$

At pure rolling, $v = R\omega$

$$\Rightarrow v_0 - \mu g t = R \left(\frac{v_0}{2R} + \frac{2\mu g}{R} t \right) \Rightarrow t = \frac{v_0}{6\mu g}$$

29. (b, c)

Let v and ω be the initial velocity of bullet and angular velocity of cylinder respectively. Applying conservation of angular momentum about the point of contact of cylinder with floor

$$mu \cdot 2R = mv \cdot 2R + (I_{CM} + MR^2) \omega$$

$$v = 2R\omega \quad \text{and} \quad I_{CM} = MR^2$$

Substituting and solving, gives,

$$v = \frac{8m u}{8m + 3M}, \text{ and}$$

COMPREHENSION

30.

31.

32.

33.

34. Velocity of cylinder as it just comes out of water.

$$v = \sqrt{2g\ell} \text{ by work energy theorem}$$

hence height above the surface of liquid = l

$$\text{total height} = \ell + \frac{\ell}{2} = \frac{3\ell}{2}$$

35. Time for $\frac{3\ell}{4} \Rightarrow \frac{1}{4} \frac{2\pi}{2} \sqrt{\frac{\ell}{g}} = \frac{\pi}{4} \sqrt{\frac{\ell}{g}}$; time for $\frac{\ell}{4} \Rightarrow \frac{1}{2} \sqrt{\frac{\ell}{g}} \sin^{-1} \frac{1}{3}$

Hence total time $\Rightarrow \frac{\pi}{4} \sqrt{\frac{\ell}{g}} + \frac{1}{2} \sqrt{\frac{\ell}{g}} \sin^{-1} \frac{1}{3}$

$$\Rightarrow \sqrt{\frac{\ell}{g}} \left[\frac{\pi}{4} + \frac{1}{2} \sin^{-1} \frac{1}{3} \right]$$

SUBJECTIVE TYPE QUESTIONS

36. $Mg - T = Ma$

$$T - mg = ma$$

$$T = \frac{2Mmg}{M+m}$$

$$\Rightarrow \frac{T}{A} = \frac{2Mmg}{A(M+m)} = \frac{2mg}{A \left(1 + \frac{m}{M} \right)} = 2 \times 10^9$$

$$\Rightarrow M = 1.86 \text{Kg}$$

37. Relative motion between block and table will start when

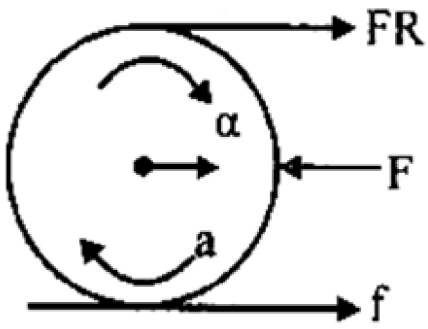
$$m\omega^2 r \sin \theta = \mu(mg + m\omega^2 r \cos \theta) \quad \dots (i)$$

$$\omega = \alpha t \quad \dots (ii)$$

solving (i) and (ii) $t = \sqrt{500} = 22.4 \text{ sec.}$

38. 2

$$f = ma, FR - fr = mR^2 \frac{a}{R}$$



$$f = \frac{F}{2}, a = \frac{F}{2m}$$

$$f = 2N$$