Kinematics - II Solution

1. (a)

$\upsilon^2 = 108 - 9x^2$

$$a = \frac{d\upsilon}{dt} = \frac{d\upsilon}{dx}.\frac{dx}{dt} = \frac{d\left(\sqrt{108-9x^2}\right)}{dx}.\frac{dx}{dt}$$

$$a = \frac{1(-18x)}{\sqrt{108-9x^2}}.\sqrt{108-9x^2} = -9x\,m/s^2$$

2. (a)

Given that $u = 0$ (the electron starts from rest)

At any time $t : \upsilon = kt = 2t.a = \frac{d\upsilon}{dt} = 2\,m/s^2$ (constant acceleration)

Now $s = ut + \frac{1}{2}at^2 = 0 \times 3 + \frac{1}{2} \times 2 \times (3)^2 = 9m$

3. (a)

$t = \sqrt{x} + 3$, differentiating with represent to t, we get, $1 = \frac{1}{2\sqrt{x}}\frac{dx}{dt} + 0$ or $\frac{dx}{dt} = 2\sqrt{x}$

When velocity is zero, then $2\sqrt{x} = 0$ or $x = 0$

4. (b)

$a_x = \frac{d^2x}{dt^2} = 8$ and $a_y = \frac{d^2y}{dt^2} = 0$

Hence, net acceleration $= \sqrt{a_x^2 + a_y^2} = 8\,m/s^2$

5. (c)

$x^2 = 1 + t^2$ or $x = \left(1+t^2\right)^{\frac{1}{2}}$

$$\frac{dx}{dt} = \frac{1}{2}\left(1+t^2\right)^{-1/2} 2t = t\left(1+t^2\right)^{-1/2}$$

$$a = \frac{d^2x}{dt^2} = \left(1+t^2\right)^{-1/2} + t\left(-\frac{1}{2}\right)\left(1+t^2\right)^{-3/2} 2t$$

$$= \frac{1}{x} - \frac{t^2}{x^3}$$

6. (b)

Because in option (a), (c) and (d) we can find from the graphs that at a single time there can be more than one velocities, which is not possible practically.

7. (a)

Area from 0 to 10 $s = \frac{1}{2} \times 2 \times [10+4] \times 5 = 35m$

Area from 10 to 12 $s = \frac{1}{2} \times 2 \times (-2.5) = -2.5\text{m}$

8. (a)

Maximum acceleration will be from 30 to 40 s, because slope in this interval is maximum

$$a = \frac{\upsilon_2 - \upsilon_1}{t_2 - t_2} = \frac{60-20}{40-30} = 4\,\text{m/s}^2$$

9. (c)

During OA, acceleration $= \tan 30^o = \frac{1}{\sqrt{3}}\,\text{m/s}^2$

During AB, acceleration $= -\tan 60^o = -\sqrt{3}\,\text{m/s}^2$

$$\text{Required ratio} = \frac{1/\sqrt{3}}{\sqrt{3}} = \frac{1}{3}$$

10. (a)

For 0 to 5 s, acceleration is positive; for 5 to 15 s acceleration is negative; for 15 to 20 s, acceleration is positive.

11. (a, d)

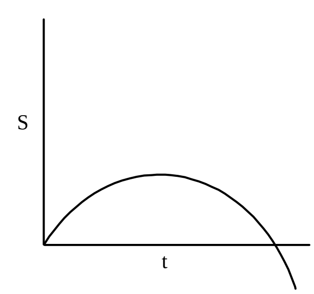
The body will speed up if angle between velocity and acceleration is acute.

12. (a, d)

Since the graph is a straight line, its slope is constant, it means acceleration of the particle is constant.

13. (b, c)

$y = \frac{x}{2}$ implies that the particle is moving a straight line passing straight origin.



$\upsilon_x = 4 - 2t$, $\quad \upsilon_x = u_x + a_x t$, $\quad \upsilon_x = 4$,

$a_x = -2$

Now, $y = \frac{x}{2} \quad \Rightarrow \frac{dy}{dt} = \frac{1}{2}\frac{dx}{dt}$

$\upsilon_y = \frac{1}{2}\upsilon_x 2 - t$, $\quad \upsilon_y = u_y + a_y t$,

$u_y = 2$ and $a_y = -1$

14. (b, c, d)

If the particle is projected with velocity u at angle θ, then equation of its trajectory will be:

$$y = x\tan\theta - \frac{gx^2}{2u^2\cos^2\theta}$$

We know slope is given by $m = \tan\theta - \frac{gx}{u^2\cos^2\theta}$

Therefore, slope $m = \tan\theta - \frac{gx}{u^2\cos^2\theta}$

It implies, that the graph between slope and x will be straight line having negative slope and a non-zero positive intercept on y-axis.

But x is directly proportional to the time t, therefore, the shape of graph between slope and time will be same as that of the graph between slope x. hence, only options (a) is correct, i.e., option (b), (c), and (d) are incorrect.

15. (b, c, d)

Point of steepest slope corresponds to the maximum speed. Particle will speed up when direction of the acceleration and velocity is the same. For region AB, both the acceleration and velocity are positive while for CD both are negative, so particle is speeding up in these regions.

Average velocity = Slope of chord on $x - t$ graph.

Which is maximum for AB.

16. (b, c)

The particle's velocity is getting zero at $t = 3s$, where it changes its direction of motion

17. (b, d)

A body moving on a circular path with uniform speed must have varying velocity as well as acceleration.

18. (c)

$$t = \frac{d}{\upsilon\sin\theta} = \frac{0.5\text{km}}{3\sin 120^o\ \text{km/h}} = \frac{1}{3\sqrt{3}}\text{h}$$

19. (a)

$$x = (u + \upsilon\cos\theta)t = (2 + 3\cos 120^o)\frac{1}{3\sqrt{3}}$$

$$= \frac{1}{6\sqrt{3}}\text{km}$$

20. 1.

Horizontal component of velocity, $u_H = u\cos 60^o = \frac{u}{2}$

$AC = u_H \times t = \frac{ut}{2}$ and

$$AB = AC\sec 30^o = \left(\frac{ut}{2}\right)\left(\frac{2}{\sqrt{3}}\right) = \frac{ut}{\sqrt{3}}$$

21. 3

Apply equation of trajectory:

$$0.5 = \frac{\sqrt{3}}{2}\tan 30^o - \frac{g\left(\frac{\sqrt{3}}{2}\right)}{2\upsilon_0^2\cos^2 30^o} \Rightarrow \upsilon_0 = \infty$$