Vector & Calculus Solutions

1. $$10A^2 = 4A^2 + 2A^2 + 2 \times 2A \times \sqrt{2}\,A \times \cos\theta$$

or $4A^2 = 4\sqrt{2}\,A^2 \cos\theta$

or $\cos\theta = \dfrac{1}{\sqrt{2}} \Rightarrow \theta = 45^o$.

2. $13^2 = 12^2 + 5^2$

$169 = 169$

So, $\vec{A} \perp \vec{B}$. $\therefore \theta = \dfrac{\pi}{2}$ radian.

3. $$(5\hat{i} + 2\hat{j} - \ell\hat{k}) \cdot (3\hat{i} + \hat{j} + 2\hat{k}) = 0$$

4. The length of the vector is not changed by the rotation of the co-ordinate axes.

$\therefore \sqrt{(n+1)^2 + 1^2} = \sqrt{n^2 + 3^2}$

or $n^2 + 2n + 2 = n^2 + 9$

or $2n = 7$ or $n = \dfrac{7}{2} = 3.5$.

5. $$\vec{\tau} = \vec{r} \times \vec{F}$$

$$= \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 3 & 2 & 3 \\ 2 & -3 & 4 \end{vmatrix}$$

$$= i[8+9] - \hat{j}[12-6] + \hat{k}[-9-4] = (17\hat{i} - 6\hat{j} - 13\hat{k}).$$

6. $$\vec{a} \times (\vec{b} \times \vec{a})$$

since $\vec{b} \times \vec{a}$ will be perpendicular to both $\vec{a}$ and $\vec{b}$. Let $\vec{a} \times \vec{b} = \vec{c}$

Then $\vec{c} \times \vec{a}$ will be a vector perpendicular to $\vec{a}$.

7. $AB\sin\theta = \sqrt{3}\,AB\cos\theta$ or $\tan\theta = \sqrt{3}$ or $\theta = 60^o = \dfrac{\pi}{3}$ radian.

11. $$\left|2\vec{a} - \vec{b}\right|^2 = 25$$

12. Conceptual.

13. Conceptual.

14. Since $\vec{a} = 2i + 4j - 4k$

$$\therefore |\vec{a}| = \sqrt{4 + 16 + (-4)^2} = 6$$

$$\therefore \hat{a} = \frac{\hat{a}}{|\vec{a}|} = \frac{2i + 4j - 4k}{6} = \frac{\vec{i} + 2\vec{j} - 2\vec{k}}{3}$$

$$\therefore \vec{F}.\hat{a} = \left(\vec{i} + 2\vec{j} - 4\vec{k}\right).\left(\frac{\vec{i} + 2\vec{j} - 2\vec{k}}{3}\right)$$

$$= \frac{1}{3}(1 + 4 + 8) = \frac{13}{3}$$

$\therefore$ resolved part of $\vec{F}$ along $\vec{a} = \left(\vec{F}.\hat{a}\right)\hat{a}$

$$= \left(\frac{13}{3}\right)\frac{\vec{i} + 2\vec{j} - 2k}{3}$$

$$= \left(\frac{13}{9}\right)\left(\vec{i} + 2\vec{j} - 2k\right)$$

15. We have $\left|\vec{a}.\vec{b}\right| = \left|\vec{a} \times \vec{b}\right|$

$\Rightarrow |\vec{a}|\left|\vec{b}\right||\cos q| = |\vec{a}|\left|\vec{b}\right|\sin q \quad \Rightarrow |\cos q| = \sin q \Rightarrow q = 45^0, 135^0$

18. (a) 19. (a)

20. 3

$\overline{A}.\overline{B} = 0$

$8 - 2\lambda - 2 = 0$

$\lambda = 3$

21. 8

Power $P = \vec{F}.\vec{v}$

$= (3\hat{i} + 2\hat{j} + \hat{k}).(2\hat{i} + \hat{j})$

$= (3 \times 2 + 2 \times 1 + 1 \times 0)$

= 8 N-m/s = 8 W