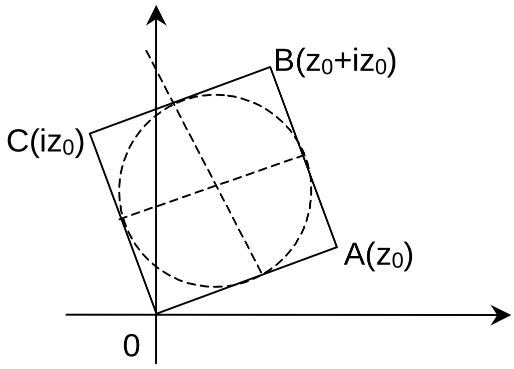
SOLUTION COMPLEX NUMBER

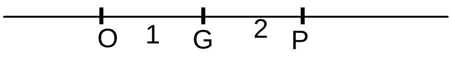
1. (B) Point corresponding to c divides the join of a and b in the ratio $\lambda : 1-\lambda$.

2. (A), (B)
Roots of $x^2 + x + 1 = 0$ are complex cube roots of unity,
so $h(w) = h(w^2) = 0$
$\Rightarrow w\,f(1) + w^2\,g(1) = 0$ and $w^2\,f(1) + wg(1) = 0$
$\Rightarrow f(1) = g(1) = 0.$

3. (B)
Clearly mid-point of OB is one centre of the circle and radius is equal $\frac{|z_0|}{2}$
$\Rightarrow$ Required equation is ;
$$\left|z - \frac{z_0}{2}(1+i)\right| = \frac{|z_0|}{2}$$



4. (B)
$z_1 = 3 + 4i$, $z_2 = 4 + 3i$, $z_3 = 2\sqrt{6} + i$
Clearly $|z_1| = |z_2| = |z_3| = 5$,
$\Rightarrow$ Points would lie on the circle centred at origin 'O' .
Now centroid of the triangle formed by these point
$$G = \left(\frac{7+2\sqrt{6}}{3} + \frac{8i}{3}\right)$$
$$OG = \sqrt{\left(\frac{7+2\sqrt{6}}{3}\right)^2 + \frac{64}{9}} = \frac{1}{3}\sqrt{137+28\sqrt{6}}$$



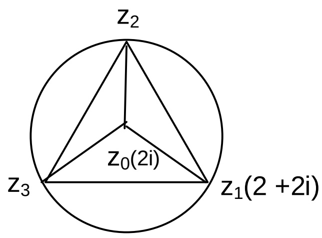
$\Rightarrow OP = 3\,OG = \sqrt{137+28\sqrt{6}}$.

5. (A), (C)

Clearly the inscried triangle is equilateral.

$\Rightarrow \dfrac{z_2 - z_0}{z_1 - z_0} = e^{i\frac{2\pi}{3}}, \ \dfrac{z_3 - z_0}{z_1 - z_0} = e^{-i\frac{2\pi}{3}}$

$\Rightarrow z_2 = -1 + i(2 + \sqrt{3})$ and $z_3 = -1 + i(2 - \sqrt{3})$



6. (C)

$|3z - 2| + |3z + 2| = 4$

$\Rightarrow \left|z - \frac{2}{3}\right| + \left|z + \frac{2}{3}\right| = \frac{4}{3}$

Sum of distances of P(z) from A(2/3, 0) and B(-2/3, 0) is 4/3

i.e. PA + PB = AB

Hence, locus of P is the line-segment AB.

7. (A)

$\bar{z}_1 = \dfrac{z_1\bar{z}_1}{z_1} = |z_1|^2 \ z_1^{-1}$

$\Rightarrow \arg(z_1^{-1}) = \arg(\bar{z}_1) = \arg(z_2) \Rightarrow z_2 = kz_1^{-1}$ $(k > 0)$

8. (A)

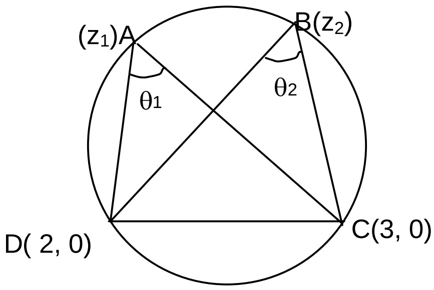
$\text{Arg}\left(\dfrac{3 - z_1}{2 - z_1}\right) + \arg\left(\dfrac{2 - z_2}{3 - z_2}\right)$

$= \arg\left(\dfrac{3 - z_1}{2 - z_1}\right)\left(\dfrac{2 - z_2}{3 - z_2}\right)$

Now if $\left(\dfrac{3 - z_1}{2 - z_1}\right)\left(\dfrac{2 - z_2}{3 - z_2}\right)$ is a +ve real number, then its argument will be zero

So, angles θ_1 and θ_2 are equal in magnitude but opposite in sign.

So chord DC subtends equal angles at A and B. So points are concylic for $k > 0$



9. (C)

$t_n = (n+1)\left(n + \frac{1}{\omega}\right)\left(n + \frac{1}{\omega^2}\right)$

$= n^3 + n^2\left(\frac{1}{\omega^2} + \frac{1}{\omega} + 1\right) + n\left(1 + \frac{1}{\omega^2} + \frac{1}{\omega}\right) + 1$

$= n^3 + n^2(\omega + \omega^2 + 1) + n(\omega + \omega^2 + 1) + 1$

$= n^3 + 1$

$\therefore S_n = \sum_{r=1}^{n} t_r = \sum_{r=1}^{n}(r^3 + 1) = \dfrac{n^2(n+1)^2}{4} + n$.

10. (B)

We are finding out sum of distances of a complex number z from origin and $(\cos\alpha, \sin\alpha)$. This sum will be minimum if z lies on the line joining the two points and minimum value of sum will be the distance between two points

i.e. 1 .

11. (A), (C), (D)

$z^2 + az + a^2 = 0$

$\Rightarrow$ $z = a\omega,\ a\omega^2$ (where 'ω' is non real root of cube unity)

$\Rightarrow$ locus of z is a pair of straight lines

and arg (z) =arg(a) + arg(ω) or arg(a) + arg(ω^2)

$\Rightarrow \arg(z) = \pm \frac{2\pi}{3}$

also, $|z| = |a||\omega|$ or $|a|\,|\omega^2| \Rightarrow |z| = |a|$.

12. (C)

Given $z^{n-1} + z^{n-2} + z^{n-3} + \ .\ .\ .\ + z + 1 = 0$

$\Rightarrow (z - 1)\,(z^{n-1} + z^{n-2} + \ .\ .\ .\ + z + 1) = 0,\quad z \neq 1$

$\Rightarrow z^n = 1 = e^{i2r\pi}\qquad (r \in N)$

$\Rightarrow z_r = e^{\frac{i2r\pi}{n}}\qquad r = 1, 2, 3, \ldots, n-1$

$\Rightarrow$ The roots are $e^{\frac{i2\pi}{n}},\ e^{\frac{i4\pi}{n}},\ e^{\frac{i6\pi}{n}}, \cdots, e^{\frac{i(2n-2)\pi}{n}}$.

Which is a G.P., with common ratio $e^{\frac{i2\pi}{n}}$.

13. (A)

Given expression = $i\left[\log\left(\frac{x-i}{x+i}\right)\right] - \pi + 2\tan^{-1}x = k$ (say)

$\log\left(\frac{x+i}{x-i}\right) = (k + \pi - 2\tan^{-1}x)i$

or $\frac{x+i}{x-i} = e^{i\theta}$ where $\theta = k + \pi - 2\tan^{-1}x$

$\Rightarrow$ $(x + i) = (x\cos\theta + \sin\theta) + i\,(x\sin\theta - \cos\theta)$

$x = x\cos\theta + \sin\theta$ and $1 = x\sin\theta - \cos\theta$

$\Rightarrow$ $x = \cot(\theta/2) \Rightarrow \theta = 2\cot^{-1}x$

or $k + \pi - 2\tan^{-1}x = 2\cot^{-1}x$

or $k + \pi = 2\,[\tan^{-1}x + \cot^{-1}x] = 2(\pi/2)$

or $k + \pi = \pi$

$k = 0$.

14. (C)

$\left|\frac{z+1}{z}\right| = 1^{1/n} = 1$

$|z + 1| = |z|$

$\Rightarrow$ z lies on the right bisector of the line joining the points (-1, 0) and (0, 0) $\Rightarrow$ roots are collinear.

15. (A)

We have $|z_1| = |z_2| + |z_1 - z_2|$

$\Rightarrow |z_1 - z_2|^2 = (\,|z_1| - |z_2|)^2$

$\Rightarrow |z_1|^2 + |z_2|^2 - 2|z_1|\,|z_2|\cos(\theta_1 - \theta_2) = |z_1|^2 + |z_2|^2 - 2|z_1|\,|z_2|$

$\Rightarrow$ $\cos(\theta_1 - \theta_2) = 1 \Rightarrow \theta_1 - \theta_2 = 0 \Rightarrow \arg(z_1) - \arg(z_2) = 0 \Rightarrow \frac{z_1}{z_2}$ is purely real

$\Rightarrow \text{Im}\left(\frac{z_1}{z_2}\right) = 0$.

16. (B) Let $z_1 = r_1(\cos\theta_1 + i\sin\theta_1)$ then $\left|\frac{z_1}{z_2}\right| = 1 \Rightarrow |z_1| = |z_2| \Rightarrow |z_1| = |z_2| = r_1$.

Now $\arg(z_1 z_2) = 0 \Rightarrow \arg(z_1) + \arg(z_2) = 0$

$\Rightarrow \arg(z_2) = -\theta_1$

Therefore, $z_2 = r_1(\cos(-\theta_1) + i\sin(-\theta_1)) = r_1(\cos\theta_1 - i\sin\theta_1) = \bar{z}_1$

$\Rightarrow \bar{z}_2 = \left(\bar{\bar{z}}_1\right) = z_1 \Rightarrow |z_2|^2 = z_1 z_2$.

17. (D) $(3+4i)^n = 25^n$

$\Rightarrow |3+4i|^n = |25|^n \Rightarrow (25)^{n/2} = 5^{2n} \Rightarrow n = 0.$

18. (D) $|iz + (3-4i)| \le |iz| + |3-4i|$

$= |z| + 5 < 4 + 5 = 9.$

19. (C) $\text{Arg}(z_1 + z_2) = 0$, $\text{Im}(z_1 z_2) = 0 \Rightarrow z_1 + z_2$ and $z_1 z_2$ are real

Let $z_1 = a_1 + ib_1$, $z_2 = a_2 + ib_2$

$z_1 + z_2$ real $\Rightarrow b_2 = -b_1$

$z_1 z_2$ real $\Rightarrow a_1 b_2 + a_2 b_1 = 0$

$\Rightarrow -a_1 b_1 + a_2 b_1 = 0$ (since $b_2 = -b_1$)

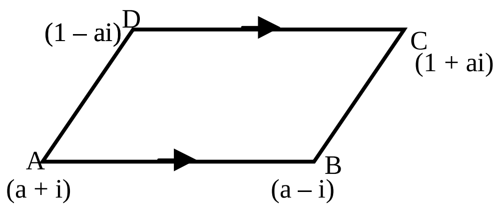
$\Rightarrow a_1 = a_2$

Therefore, $z_1 = a_1 + ib_1 = a_2 - i b_2 = \bar{z}_2$

$\Rightarrow z_1 = \bar{z}_2$.

20. (C) Let $z = x + iy$, then $z^2 + \bar{z}^2 = 2 \Rightarrow x^2 - y^2 = 1$, which represents a hyperbola.

21. (B)



$\overline{AB} = \overline{DC}$

$\Rightarrow (a - i) - (a + i) = (1 + ai) - (1 - ai)$

$\Rightarrow -2ai = 2i$

$\Rightarrow a = -1$

22. (A) $A + iB = \frac{1 - i\alpha}{1 + i\alpha} \Rightarrow A - iB = \frac{1 + i\alpha}{1 - i\alpha}$.

$\Rightarrow (A + iB)(A - iB) = \frac{(1 - i\alpha)(1 + i\alpha)}{(1 + i\alpha)(1 - i\alpha)} = 1$

$\Rightarrow A^2 + B^2 = 1$

23. (B), (C), (D) Let $|z_1| = |z_2| = r$

$\Rightarrow z_1 = r(\cos\theta + i\sin\theta)$

and $z_2 = r\left(\cos\left(\frac{\pi}{2}-\theta\right)+i\sin\left(\frac{\pi}{2}-\theta\right)\right)$

$\Rightarrow$ $z_1z_2 = r^2 i$, which is purely imaginary

$z_1+z_2 = r[(\cos\theta + \sin\theta)+ i(\cos\theta + \sin\theta)]$

$\Rightarrow (z_1+z_2)^2 = r^2(2i(\cos\theta + \sin\theta))^2$ which is purely imaginary .

Also $\arg(z_1^{-1}) + \arg(z_2^{-1}) = -\pi/2$.

24. B

$k = |z - z_1|^2 + |z - z_2|^2 = |z_1 - z_2|^2$ $\Rightarrow k = 4$

25. C

The given determinant is the product of two determinant as follows.

$$\begin{vmatrix} z & \bar{z} & 1 \\ \bar{z} & z & 1 \\ 1 & z & \bar{z} \end{vmatrix} \times \begin{vmatrix} a_1 & b_1 & 0 \\ a_2 & b_2 & 0 \\ a_3 & b_3 & 0 \end{vmatrix} = 0$$

26. (A)

$x - iy = (a-ib)^3 = a^3 - 3a^2 bi - 3ab^2 + ib^3 = a(a^2 - 3b^2) + ib(b^2 - 3a^2)$

$\Rightarrow \frac{x}{a}+\frac{y}{b} = a^2 - 3b^2 + b^2 - 3a^2 = -2(a^2 + b^2)$

27. (B)

$\sqrt{3}+i = 2(\cos\pi/6+i\sin\pi/6) \Rightarrow (\sqrt{3}+i)^n = 2^n\left(\cos\frac{n\pi}{6}+i\sin\frac{n\pi}{6}\right)$

According to the given condition $\cos\frac{n\pi}{6}+i\sin\frac{n\pi}{6}=1$

$\Rightarrow \frac{n\pi}{6}=2m\pi$, m is an integer. Thus n = 12m

28. (C)

$\frac{|z_1 - z_2|}{|z_3 - z_2|}=1$

Also $\arg\left(\frac{z_1 - z_2}{z_3 - z_2}\right) = \pm\pi/2$. Thus $\frac{z_1 - z_2}{z_3 - z_2} = \pm i$

29. **D**

$|\omega z - 1 - \omega^2| = a \Rightarrow |z + 1| = a$

$\Rightarrow |z - 1 + 2| = a \Rightarrow |z - 1| + 2 \geq a \Rightarrow 0 \leq a \leq 4$.

30. **C**

First equation represent ellipse and second a line segment joining (–1, 0) and (1, 0) totally contained inside ellipse.

31. **D**

Let $z = \cos\theta + i\sin\theta$

then $|\cos\theta + i\sin\theta + 4\cos\theta - 4i\sin\theta| = 3$

$|5\cos\theta - 3i\sin\theta| = 3 \Rightarrow 25\cos^2\theta + 9\sin^2\theta = 9$

$\Rightarrow 25\cos^2\theta = 9\cos^2\theta$

$\Rightarrow \cos^2\theta = 0 \Rightarrow \theta = \pm\frac{\pi}{2}$

$z = \pm i$, sum $= +i - i = 0$.

32. **C**

$|z_1 - z_2|^2 + |z_2 - z_3|^2 + |z_3 - z_1|^2$

$= 2(|z_1|^2 + |z_2|^2 + |z_3|^2) - (z_1\bar{z}_2 + \bar{z}_1 z_2 + z_2\bar{z}_3 + \bar{z}_2 z_3 + z_3\bar{z}_1 + \bar{z}_3 z_1)$

$= 58 - (z_1\bar{z}_2 + \bar{z}_1 z_2 + z_2\bar{z}_3 + \bar{z}_2 z_3 + z_3\bar{z}_1 + \bar{z}_3 z_1)$ ……(1)

Now $|z_1 + z_2 + z_3|^2 \ge 0$

$\Rightarrow |z_1|^2 + |z_2|^2 + |z_3|^2 + z_1\bar{z}_2 + \bar{z}_1 z_2 + z_2\bar{z}_3 + \bar{z}_2 z_3 + z_3\bar{z}_1 + \bar{z}_3 z_1 \ge 0$

$\Rightarrow -(z_1\bar{z}_2 + \bar{z}_1 z_2 + z_2\bar{z}_3 + \bar{z}_2 z_3 + z_3\bar{z}_1 + \bar{z}_3 z_1) \le 29$ ……(2)

from (1) and (2)

maximum value = 58 + 29 = 87.

33. **C**

Since Arg $\frac{z-1-i}{z}$ is the angle subtended by the chord joining the points O and 1 + i at the circumference of the circle $|z - 1| = 1$, so it is $= -\frac{\pi}{4}$.

34. **C**

$z^3 + \bar{\omega}^7 = 0 \Rightarrow z^3 = -\bar{\omega}^7 \Rightarrow z^{15} = -\bar{\omega}^{35}$

$z^5 . \omega^{11} = 1 \Rightarrow z^5 = \frac{1}{\omega^{11}} \Rightarrow z^{15} = \frac{1}{\omega^{33}}$.

$\therefore -\bar{\omega}^{35} = \frac{1}{\omega^{33}} \Rightarrow \bar{\omega}^{35} . \omega^{33} = -1$.

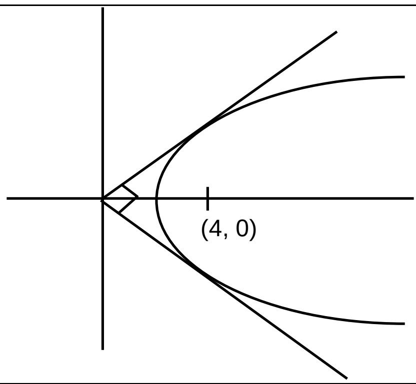
$\Rightarrow |\bar{\omega}^{35} . \omega^{33}| = 1 \Rightarrow |\omega^{33}|\,|\bar{\omega}^{35}| = 1 \Rightarrow \omega^{68} = 1 \Rightarrow |\omega| = 1$

Again $\bar{\omega}^{35} . \omega^{33} = -1$

$\Rightarrow (\bar{\omega}^{33} . \omega^{33}) . \bar{\omega}^2 = 1 \Rightarrow (|\omega|^2)^{33} . \bar{\omega}^2 = 1 \Rightarrow \omega = \pm i \Rightarrow z = \pm i$

35. **D**

The given relation represents the parabola with focus (4, 0) and the imaginary axis as the directrix. Pair of tangents from directrix are at right angle. By symmetry greatest positive argument of z is $\frac{\pi}{4}$.

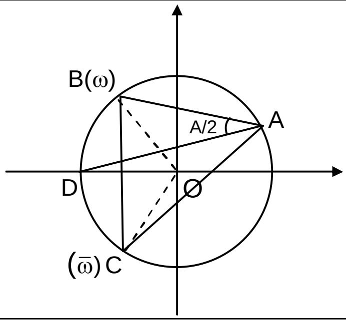


36. **D**

Clearly $\angle DOB = \angle COD = A$

$\Rightarrow z = \omega e^{iA}$ and $\bar{\omega} = ze^{iA} \Rightarrow z^2 = \omega\bar{\omega} = 1$

$\Rightarrow z = -1$ (As A and D are on opposite side of BC).



37. **C**

$|k + z^2| = |z|^2 - k = |z^2| + |k|$

$\arg(z^2) = \arg(k)$

$\Rightarrow 2\arg(z) = \pi \Rightarrow \arg(z) = \frac{\pi}{2}$.

38. **B**

$x^n - 1 = (x - \alpha_1)(x - \alpha_2)(x - \alpha_3) \ldots.. (x - \alpha_n)$

By Putting x = 2,

$2^n - 1 = (2 - \alpha_1)(2 - \alpha_2) \ldots\ldots (2 - \alpha_{n-1})$

$\Rightarrow |2 - \alpha_1|\,|2 - \alpha_2| \ldots\ldots |2 - \alpha_{n-1}| = |2^n - 1| = 2^n - 1.$

39. **A**

$z\omega = |z|^2 \Rightarrow \omega = \bar{z}$

$\Rightarrow |z + \bar{z}| + |z - \bar{z}| = 2$

$\Rightarrow |\text{Re}(z)| + |\text{Im}(z)| = 1$

Let $z = x + iy$

$\Rightarrow |x| + |y| = 1$

which is a square.

40. **C**

Let $u = \frac{z-1}{e^{\theta i}} \Rightarrow \frac{e^{\theta i}}{z-1} = \frac{1}{u}$.

Given that imaginary part of $u + \frac{1}{u}$ is zero.

$$\left(u + \frac{1}{u}\right) - \left(\bar{u} + \frac{1}{\bar{u}}\right) = 0$$

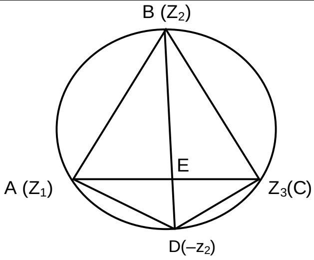
$$(u - \bar{u}) + \frac{\bar{u} - u}{u\bar{u}} = 0$$

$$(u - \bar{u})\left(1 - \frac{1}{u\bar{u}}\right) = 0$$

$$\Rightarrow 1 - \frac{1}{|u|^2} = 0 \Rightarrow |u|^2 = 1$$

$$\Rightarrow \left|\frac{z-1}{e^{i\theta}}\right|^2 = 1 \Rightarrow |z - 1| = 1, \text{ which is a circle.}$$

41. **B**

DE = BD – BE = 2 –3/2 = 1/2 Area of ΔACD = $\frac{1}{2}$ DE × AC $= \frac{1}{2} \times \frac{1}{2}\sqrt{3} = \frac{\sqrt{3}}{4}$.	

42. **B**

Here $|\bar{z}| = |iz^2| = |z|^2 \Rightarrow |z| = |z|^2 \Rightarrow |z| = 1$.

Hence $\bar{z}z = iz^3$ or $iz^3 = 1$ or $\left(\frac{z}{i}\right)^3 = 1$

$\Rightarrow \frac{z}{i} = 1, \omega, \omega^2$.

43. **C**

Given $z^{n-1} + z^{n-2} + z^{n-3} + \ . \ . \ . + z + 1 = 0$

$\Rightarrow (z-1)(z^{n-1} + z^{n-2} + \ . \ . \ . + z + 1) = 0, \quad z \neq 1$

$\Rightarrow z^n = 1 = e^{i2r\pi} \qquad (r \in N)$

$\Rightarrow z_r = e^{\frac{i2r\pi}{n}} \qquad r = 1, 2, 3, . . . , n-1$

$\Rightarrow$ The roots are $e^{\frac{i2\pi}{n}}, e^{\frac{i4\pi}{n}}, e^{\frac{i6\pi}{n}}, \cdots, e^{\frac{i(2n-2)\pi}{n}}$.

Which is a G.P., with common ratio $e^{\frac{i2\pi}{n}}$.

44 **A**

Given expression = $i\left[\log\left(\frac{x-i}{x+i}\right)\right] - \pi + 2\tan^{-1}x = k$ (say)

$\log\left(\frac{x+i}{x-i}\right) = (k + \pi - 2\tan^{-1}x)i$

or $\frac{x+i}{x-i} = e^{i\theta}$ where $\theta = k + \pi -- 2\tan^{-1}x$

$\Rightarrow (x + i) = (x\cos\theta + \sin\theta) + i\,(x\sin\theta - \cos\theta)$

$x = x\cos\theta + \sin\theta$ and $1 = x\sin\theta - \cos\theta$

$\Rightarrow x = \cot(\theta/2) \Rightarrow \theta = 2\cot^{-1}x$

or $k + \pi - 2\tan^{-1}x = 2\cot^{-1}x$

or $k + \pi = 2[\tan^{-1}x + \cot^{-1}x] = 2(\pi/2)$

or $k + \pi = \pi \Rightarrow k = 0$.

45. **B**

Obviously $\angle AOC = 2B$

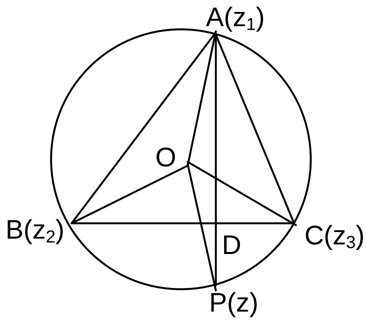
$\angle BAP = \pi/2 - B$

$\angle BOP = \pi - 2B$

$$\frac{z-0}{z_2-0} = e^{i(\pi - 2B)} \quad \ldots. (1)$$

$$\frac{z_1-0}{z_3-0} = e^{i2B} \quad \ldots. (2)$$

$$(1) \times (2) \Rightarrow z = -\frac{z_2 z_3}{z_1}$$

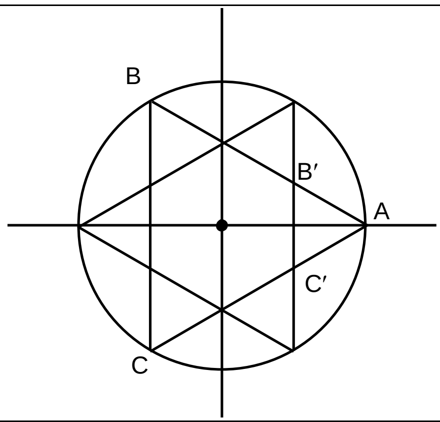


46. **A**

Area of common region, i.e., Polygon,

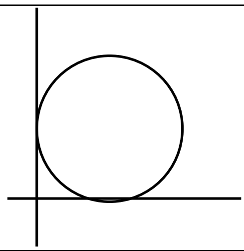
$= \Delta ABC - 3\Delta AB'C'$

$$= \frac{3\sqrt{3}}{4} - 3 \times \frac{1}{2} \times \frac{1}{2} \times \frac{1}{\sqrt{3}} = \frac{\sqrt{3}}{2}.$$



47. **C**

From the figure it is clear that $|z|$ approaches to the maximum value in the vicinity of (1, 0) or (0, 1).



48. **C**

Given $a\alpha^2 + b\alpha + 1 = 0 \quad \ldots\ldots.(1)$

Also $\bar{a}\bar{\alpha}^2 + \bar{b}\bar{\alpha} + 1 = 0 \Rightarrow \frac{\bar{a}}{\alpha^2} + \frac{\bar{b}}{\alpha} + 1 = 0$ (as $|\alpha| = 1$)

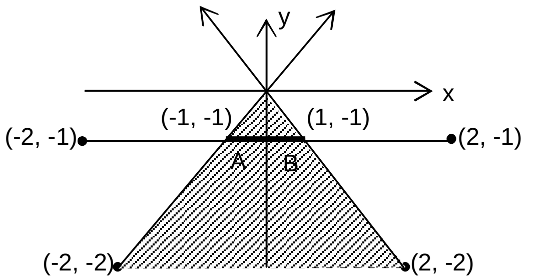
$$\alpha^2 + \bar{b}\alpha + \bar{a} = 0 \quad \ldots\ldots..(2)$$

From (1) and (2) $\frac{\alpha^2}{\bar{a}b - \bar{b}} = \frac{\alpha}{1-|a|^2} = \frac{1}{a\bar{b} - b}$

$$\Rightarrow \frac{\bar{a}b - \bar{b}}{1-|a|^2} = \frac{1-|a|^2}{a\bar{b} - b} \Rightarrow |a\bar{b} - b| = 1 - |a|^2 = \frac{3}{4}$$

49. **C**

From figure it is clear that required length of line segment is AB = 2



50. **C**

As per the definition of z' given
z' is the reflection of z on the y-axis.

51. **B**

52. **B**

Given curve is a portion of straight line through (0, –1) and on the right of the y-axis.
Value of the expression $(|z - \omega| + |z + \omega|)$ will be minimum when z, ω and –ω are collinear.
∴ Minimum distance = distance between ω and –ω = 2 units.

53. **B**

Distance of z from (3 + 2i) is same as from imaginary axis ⇒ z moves on a parabola with focus (3 + 2i) and directrix as imaginary axis.

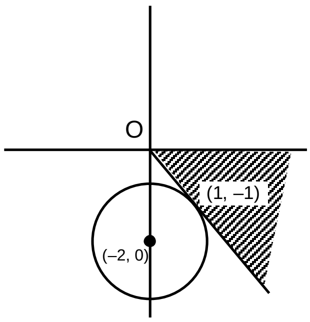
Hence z_1 and z_2 are extremities of focal chord hence $\arg\left(\dfrac{z_1 - (3+2i)}{z_2 - (3+2i)}\right) = \pm\pi$

54. **B**

Let z = x + iy, since, $\sec^{-1}\left(\dfrac{z+\sqrt{3}i+4}{5}\right)$ is acute, so $\dfrac{z+\sqrt{3}i+4}{5}$ must be real positive and more than 1. ⇒ $y = -\sqrt{3}$ and $x > 1$ so, $\arg(z) \in \left(\dfrac{-\pi}{3}, 0\right)$.

55. **C**

$\dfrac{kz}{k+1}$ represent any point lying on the line joining origin and z. So, $\dfrac{kz}{k+1}$ should lie outside the circle $|z + 2| > \sqrt{2}$.
So, z should lie in the shaded region
$\Rightarrow -\dfrac{\pi}{4} < \arg(z) < 0$.



56. (B)

Given that
$\sin\theta_1 z^4 + \sin\theta_2 z^3 + \sin\theta_3 z^2 + \sin\theta_4 z + \sin\theta_5 = 2$
or, $2 = |\sin\theta_1 z^4 + \sin\theta_2 z^3 + \sin\theta_3 z^2 + \sin\theta_4 z + \sin\theta_5|$
$\leq \dfrac{1}{2}\left[|z|^4 + |z|^3 + |z|^2 + |z| + 1\right]$
or, $3 \leq |z|^4 + |z|^3 + |z|^2 + |z|$ (1)

clearly $|z| \geq 1$ satisfied (1). If $|z| < 1$, then

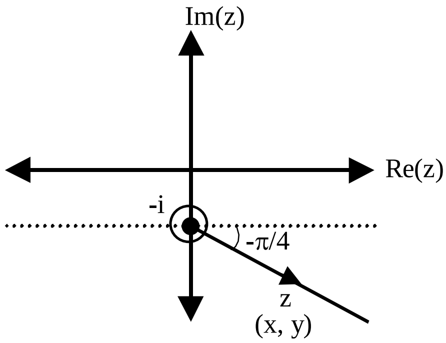
$3 < |z|^4 + |z|^3 + |z|^2 + |z| \leq |z| + |z|^2 + |z|^3 + |z|^4 \ldots \infty = \frac{|z|}{1-|z|}$

$\Rightarrow \quad 3-3|z| < |z| \quad \Rightarrow \quad |z| > \frac{3}{4}$

57. (A)
z lies on the unit circle centred at origin.
$|z-1| = AB <$ Arc AB
$\Rightarrow |z-1| < 1 \times |\theta| = |\arg z|$
$|z-1| < |\arg z|$

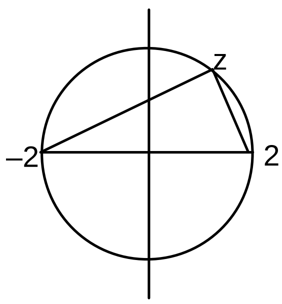
58. (C)
$z_2 = \frac{3z_1 + 2z_3}{3+2}$ $\therefore$ z_2 divides z_1 and z_3 in 2 : 3.

59. (C)
$z + i = z - (-i)$ is a ray starting from $-i$ and ending at z. This ray is inclined at an angle $-\pi/4$ w.r.t +ve side of x–axis. Hence required locus is all points satisfying $x > 0$ (or $y < -1$) and lying on the line passing through $(0, -1)$ and slope equal to -1.
i.e., $y + 1 = -1(x - 0)$
$\Rightarrow x + y + 1 = 0, x > 0, y < -1$
(The point $(0, -1)$ is not included in the locus).



60. **C**
$\gamma^2 + \alpha\gamma + \bar{\alpha} = 0$
$\gamma^2 + \bar{\alpha}\gamma + \alpha = 0$ (Taking conjugate of above equation)
$\frac{\gamma^2}{\alpha^2 - \bar{\alpha}^2} = \frac{\gamma}{\bar{\alpha} - \alpha} = \frac{1}{\bar{\alpha} - \alpha}$ (Using Cramer's rule)
$\Rightarrow \gamma = 1$

61. **C**
Clearly $\arg\left(\frac{z-2}{z+2}\right) = \pm\frac{\pi}{2}$
$\Rightarrow \arg\left(\frac{z_1 - z_2}{z_1 - z_3}\right) = \pm\frac{\pi}{2}$
$\Rightarrow z_1, z_2, z_3$ will be the vertices of a right angled triangle.



62. **D**

Let $a_i = x \Rightarrow a_2 = \frac{1}{1-x} \Rightarrow a_3 = \frac{1}{1-\frac{1}{1-x}} = \frac{1-x}{1-x-1} = \frac{x-1}{x}$

Since $a_3 = a_1$

$\Rightarrow \frac{x-1}{x} = x \Rightarrow x^2 - x + 1 = 0 \Rightarrow x = \frac{1 \pm \sqrt{1-4}}{2} = \frac{1}{2} \pm \frac{\sqrt{3}i}{2}$

$\Rightarrow$ x is a cube root of $-1 \Rightarrow x^3 = -1$

Now, $(a_9)^9 = (a_1)^9 = x^9 = (x^3)^3 = -1$

63. B

Given $1 \geq |z - (4 - 3i)| \geq \begin{cases} |z| - |4-3i| \\ |4-3i| - |z| \end{cases}$

$\Rightarrow |z| \leq 6$ and $|z| \geq 4$

$\Rightarrow 4 \leq |z| \leq 6 \Rightarrow \alpha = 4, \beta = 6$

Let $y = \frac{x^4 + x^2 + 4}{x} = x^3 + x + \frac{4}{x} = x^3 + x + \frac{1}{x} + \frac{1}{x} + \frac{1}{x} + \frac{1}{x}$

Since $x \in (0, \infty)$, therefore $x^3, x, \frac{1}{x}, \frac{1}{x}, \frac{1}{x}, \frac{1}{x}$ are positive .

Sum will be least when $x^3 = x = \frac{1}{x} \Rightarrow x = 1$.

$\Rightarrow k = 6$.

Hence $k = \beta$.

64. (D)

Let vertices be $1, \alpha, \alpha^2, \ldots\ldots\ldots\alpha^{n-1}$

given that $1 + \alpha + \alpha^2 + \ldots\ldots + \alpha^{n-1} = 0$

$\Rightarrow \alpha + \alpha^2 + \ldots\ldots + \alpha^{n-1} = -1$

$\Rightarrow \frac{\alpha(1-\alpha^{n-1})}{1-\alpha} = -1 \Rightarrow \frac{\alpha - \alpha^n}{1-\alpha} = -1$

$\Rightarrow \alpha^n = 1$

so $z_1, z_2, \ldots\ldots z_n$ are the roots of $x^n = 1$, which forms the vertices of a regular n-polygon so incentre and circumcentre will concide.

65. C

Obvious, after drawing the locus of z in the argand plane.

66. B

Given $\log_{\frac{1}{2}}\left(\frac{|z-1|+4}{3|z-1|-2}\right) > 1$

$\Rightarrow \log_{\frac{1}{2}}\left(\frac{|z-1|+4}{3|z-1|-2}\right) > \log_{\frac{1}{2}}\left(\frac{1}{2}\right)$

$\Rightarrow 2|z-1| + 8 < 3|z-1| - 2$

$\Rightarrow |z-1| > 10$

which is an exterior of a circle .