CIRCLE (ANSWER KEY) NAPJC – 4, 5, 6

1. (a) 2. (d) 3. (a) 4. (c) 5. (b) 6. (a) 7. (a)

8. (a) 9. (b) 10. (c) 11. (b,c) 12. (a,b,c,d) 13. (a,d)

14. (a,c,d) 15. (a,b,c) 16. (a,b) 17. (a,d) 18. (a), 19-(d)

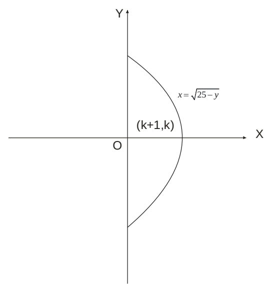
Integer type
Q.20) 9
Q.21) 1

SOLUTION

1. (a) Since the point (k + 1, k) lies inside the region bounded by $x = \sqrt{25 - y^2}$ and y – axis,

$$\therefore \quad (k+1)^2 + k^2 - 25 < 0$$

and $\quad k + 1 > 0$



$\Rightarrow \quad 2k^2 + 2k - 24 < 0 \; and \; k > -1$

$\Rightarrow \quad k^2 + 2k - 12 < 0 \; and \; k > -1$

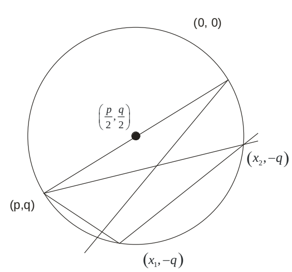
$\Rightarrow \quad (k+4)(k-3) < 0 \; and \; k > -1$

$\Rightarrow \quad -4 < k < 3 \; and \; k > -1$

$\Rightarrow \quad -1 < k < 3$

2. (d) Given circle is $x^2 + y^2 = px + qy$.

Since the centre of the circle is $\left(\frac{p}{2}, \frac{q}{2}\right)$, so (p, q) and (0, 0) are the end points of a diameter. As the two chords are bisected by x-axis, the chords will cut the circle at the points $(x_1, -q)$ and $(x_2, -q)$, where x_1, x_2 are real.



The equation of the line joining these points is y = - q.
Solving y = - q and $x^2 + y^2 = px + qy$, we get $x^2 - px + 2q^2 = 0$
The roots of this equation are x_1 and x_2. Since the roots are real and distinct, $\therefore$ discriminant > 0
i.e., $\quad p^2 - 8q^2 > 0 \; or \; p^2 > 8q^2$

3. (a) Let equation of circle be $x^2+y^2+2gx+2fy+c=0$

Since it passes through (2a, 0), so

$$4a^2+4ag+c=0 \qquad \ldots(i)$$

Also its radical axis with $x^2+y^2=a^2$ is

$$2gx+2fy+c+a^2=0$$

But the radical axis is $x=\frac{a}{2}$, so we get

$$\frac{2g}{1}=\frac{c+a}{-a/2} \text{ and } f=0$$

or ag + c + a = 0 ..(ii)

From (i) and (ii), we get g and c.

Hence equation is $x^2+y^2-2ax=0$

4. (c) Let $P\equiv(x_1, y_1)$ and $Q\equiv(x_{2,}\, y_2)$

Let the equation of given circle be $x^2+y^2=a^2$

The equation of chord of contact of tangents drawn from the point $P(x_1, y_1)$ to the given circle is

$$xx_1+yy_1=a^2$$

Since it passes through $Q(x_{2,}\, y_2)$

$\therefore \quad x_1x_2+y_1y_2=a^2 \qquad \ldots(I)$

Now $l_1=\sqrt{x_1^{\,2}+y_1^{\,2}-a^2},\ l_2=\sqrt{x_2^{\,2}+y_2^{\,2}-a^2}$

and PQ = $\sqrt{(x_1-x_2)^2+(y_1-y_2)^2}$

$$=\sqrt{(x_1^{\,2}+y_1^{\,2})+\left(x_2^2+y_2^2\right)-2\left(x_1x_2+y_1y_2\right)}$$

$$=\sqrt{(x_1^{\,2}+y_1^{\,2})+\left(x_2^2+y_2^2\right)-2a^2} \qquad \text{[Using (i)]}$$

$$=\sqrt{(x_1^{\,2}+y_1^{\,2}-a^2)+\left(x_2^2+y_2^2-a^2\right)}=\sqrt{l_1^2+l_2^2}$$

5. (b)

6. (a)

7. (a)

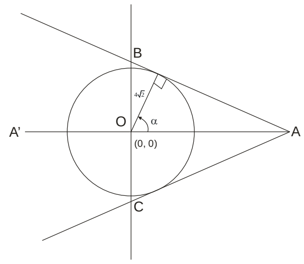
8. (a)

9. (d)

10. (c)

11. $OA = 4\sqrt{2}\sec\alpha$

$BC = 2OB = 8\sqrt{2}\operatorname{cosec}\alpha$



Area of $\triangle ABC = \frac{1}{2}\times OA\times BC$

$= \frac{1}{2}\times 4\sqrt{2}\sec\alpha\times 8\sqrt{2}\operatorname{cosec}\alpha$

$= \frac{64}{\sin 2\alpha}$

Which is minimum when sin 2α is maximum ie, when $\alpha = \frac{\pi}{4}$

$\therefore OA = 4\sqrt{2}\times\sqrt{2} = 8$

$\therefore A \equiv (8,0)$ and symmetrically on the other side

$A' \equiv (-8,0)$

12. Let $S \equiv x^2 + y^2 + 2gx + 2fy + c = 0$

$S_1 \equiv x^2 + y^2 - 4 = 0$

$S_2 \equiv x^2 + y^2 - 6 - 8Y + 10 = 0$

$S_3 \equiv x^2 + y^2 + 2x - 4y - 2 = 0$

$\therefore$ Common chords are

$S - S_1 \equiv 2gx + 2fy + c + 4 = 0$ …(i)

$S - S_2 \equiv (2g+6)x + (2f+8)y + c - 10 = 0$ …(ii)

$S - S_3 \equiv (2g-2)x + (2f+4)y + c + 2 = 0$ …(iii)

For cutting the extremities of diameter, chords (i), (ii) and (iii) pass through the centres of S_1, S_2 and S_3

respectively, then

$\therefore c + 4 = 0, (2g+6)3 + (2f+8)4 + c - 10 = 0$

and $(2g-2)(-1) + (2gf+4)(2) + c + 2 = 0$

after solving $c = -4, g = -2, f = -3$

13. (a,d)

14. $C_1 \equiv x^2 + y^2 - 2x - 4y - 4 = 0$

$and\ C_2 \equiv x^2 + y^2 + 2x + 4y + 4 = 0$

$\therefore$ Radical axis is $C_1 - C_2 = 0$

$\Rightarrow \quad -4x - 8y = 0$

or $x + 2y + 2 = 0$

(a) option is correct.

Centre and radius of $C_1 = 0$ are (1,2) and 3.

is $\frac{|1+4+2|}{\sqrt{1+4}} = \frac{7}{\sqrt{5}} \neq$ radius

$\therefore$ (b) option is wrong.

L is also the common chord of C_1 and C_2

$\therefore$ (c) option is correct

$\because$ Centres of $C_1 = 0$ and $C_2 = 0$ are (1, 2) and (-1, -2)

$\therefore$ Slope of Line joining centres of circles $C_1 = 0$ & $C_2 = 0$ is

$\frac{-2-2}{-1-1} = \frac{4}{2} = 2 = m_1$ (say)

And slope of L = 0 is $-\frac{1}{2} m_2$ (say)

$\therefore \quad m_1 m_2 = -1$

Hence L is perpendicular to the line joining centres of C_1 and C_2.

$\therefore$ (d) option is correct.

15. Given circle is $x^2 + y^2 = a^2$

clearly (0, 0) will belong the interior of circle (i) Also other points interior to circle (i) will have the coordinates of the form

$(\pm\lambda, 0,), (0, \pm\lambda)$, where $\lambda^2 < a^2$

and $(\pm\lambda, \pm\mu)\ and\ (\pm\mu, \pm\lambda)$, where $\lambda^2 + \mu^2 < a^2 and\ \lambda, \mu \in \mathrm{I}$

$\therefore$ Number of lattice points in the interior of the circle will be of the form 1 + 4 r + 8t, where r, t = 0, 1, 2, …..

$\therefore$ Number of such points must be of the form 4 n + 1, where n = 0, 1, 2, …..

16. The given points lie on the circle $x^2 + y^2 = a^2$ for all (i), since the points are the vertices of an equilateral triangle, the circumcentre and the centroid of the triangle are same,

$\Rightarrow \frac{1}{3} a\left(\cos\theta_1 + \cos\theta_2 + \cos\theta_3\right) = 0$

and $\frac{1}{3} a\left(\sin\theta_1 + \sin\theta_2 + \sin\theta_3\right) = 0$

$\Rightarrow \cos\theta_1 + \cos\theta_2 + \cos\theta_3 = \sin\theta_1 + \sin\theta_2 + \sin\theta_3 = 0$

17. Since the given line touches the given circle, the length of the perpendicular from the centre (2, 4) of the circle from the line $3x - 4y - \lambda = 0$ is equal to the radius $\sqrt{(4+16+5)} = 5$ of the circle.

$$\Rightarrow \frac{(3\times 2-4\times 4-\lambda)}{\sqrt{(9+16)}}=\pm 5 \quad \Rightarrow \lambda = 15 \ or \ -35$$

Now, equation of the tangent at (a, b) to the given circle is

$$xa+yb-2(x+a)-4(y+b)-5=0$$

$$\Rightarrow (a-2)x+(b-4)y-(2a+4b)=0$$

If it represents the given line $3x-4y-\lambda=0$, then let

$$\frac{a-2}{3}=\frac{b-4}{-4}=\frac{2a+4b+5}{\lambda}=l$$

Then $a=3l+2, b=4-4l \ and \ 2a+4b+5=\lambda l$ …(i)

$\Rightarrow \quad 2(3l+2)+4(4-4l)+5=15l$ (if $\lambda=15$)

$\Rightarrow \quad l=l \Rightarrow a=5, b=0 \ and \ \lambda+a+b=20$

Again, if $\lambda=-35$ (from i)

$$25-10l=-35l \Rightarrow l=-1 \Rightarrow a=-1, b=8$$

and $\quad \lambda+a+b=-35-1+8=-28$

Passage

∵ Length of tangents from a point to circle are equal.

PQ = PR

Then Parallelogram PQRS is rhombus.

∴ Mid point of QR = mid point of PS

and QR ⊥ PS

∴ S is the mirror image of P w.r.t. QR

18. ∵ $L \equiv Y = 4$

Let $P \equiv (h, 4)$

∴ Circumcentre of $\triangle PQR$ is $\left(\frac{h}{2}, 2\right)$,

then $\quad y=2 \Rightarrow y-2=0$

19. ∵ $P \equiv (6,8)$

∴ Equation of QR is 6x + 8y = 4

$\Rightarrow \quad 3x+4y-2=0$

$$\therefore \quad PM = \frac{|3\times 6+4\times 8-2|}{\sqrt{(3^2+4^2)}}=\frac{48}{5}$$

and $\quad PQ=\sqrt{(6^2+8^2-4)}=\sqrt{96}$,

then $\quad QM=\sqrt{(PQ)^2-(PM)^2}=\sqrt{96-\frac{(48)^2}{25}}$

$$=\sqrt{48\left(2-\frac{48}{25}\right)}$$

$$= \sqrt{\frac{96}{25}}$$

$\therefore$ Area of $\triangle PQR = \frac{1}{2} \cdot PM \cdot QR$

$$= \frac{1}{2} \cdot \frac{48}{5} \cdot \sqrt{\frac{96}{25}}$$

$$= \frac{48\sqrt{24}}{25} = \frac{96\sqrt{6}}{25}$$

$\because$ PQRS is a rhombus

$\therefore$ Area of $\triangle QRS$ = Area of $\triangle PQR$

$$= \frac{96\sqrt{6}}{25} \text{ sq unit}$$