BINOMIAL THEOREM SOLUTION

1. **(D)**

$\frac{(1+x)^n}{1-x} = (C_0 + C_1x + C_2x^2 + \ . \ . \ . + C_nx^n)(1 + x + x^2 + \ldots.)$

The coefficient of $x^r = C_0 + C_1 + C_2 + C_3 + \ . \ . \ . + C_r = 2^n$ for $r = n$.

Moreover coefficient of x^r is $C_0 + C_1 + C_2 + C_3 + \ . \ . \ . + C_r$ if $r > n$. So $r \geq n$

2. **(B)**

$$\left(1 - x + \frac{x^2}{2!} - \frac{x^3}{3!} + \ldots. + \frac{(-1)^n x^n}{n!}\right)^2 = \left(e^{-x} - (-1)^{n+1}\frac{x^{n+1}}{(n+1)!} - (-1)^{n+2}\frac{x^{n+2}}{(n+2)!} + \ldots..\right)^2$$

The required coefficient = coefficient of x^n in $e^{-2x} = \frac{(-2)^n}{n!}$.

3. **(A), (B), (C)**

Putting $x = \omega$ in the equation,

$0 = a_0 + a_1\omega + a_2\omega^2 + a_3 + \ . \ . \ .$ .... (1)

Putting $x = \omega^2$ in the equation,

$0 = a_0 + a_1\omega^2 + a_2\omega + a_3 + \ . \ . \ .$ .... (2)

Putting $x = 1$ in the equation,

$3^n = a_0 + a_1 + a_2 + a_3 + \ . \ . \ .$ .... (3)

adding (1), (2) and (3),

$3^n = 3(a_0 + a_3 + a_6 + \ . \ . \ . \ .)$ .... (a)

$\Rightarrow a_0 + a_3 + a_6 + \ . \ . \ . \ . = 3^{n-1}$ (option C)

subtracting (2) from (1),

$0 = (\omega - \omega^2)(a_1 - a_2 + a_4 - a_5 + \ . \ . \ . \ )$

Since $\omega - \omega^2 \neq 0$, $a_1 + a_4 + a_7 + . \ . \ . = a_2 + a_5 + a_8 + \ . \ . \ .$ . . . (4)

Also from (3) – (a), $a_1 + a_2 + a_4 + a_5 + \ . \ . \ . \ . = 3^n - 3^{n-1} = 2.3^{n-1}$ . . . .(5)

From (4) and (5) , $a_1 + a_4 + a_7 + . \ . \ . = a_2 + a_5 + a_8 + \ . \ . \ . = 3^{n-1} = a_0 + a_3 + a_6 + \ . \ .$

4. **(C)**

In the expression $\left(\frac{x+1}{x^{2/3} - x^{1/3} + 1} - \frac{x-1}{x - x^{1/2}}\right)^{10}$

To simplify, for first term put $x = p^3$ and for second term put $x = q^2$, then it will become $(x^{1/3} - x^{-1/2})^{10}$

$T_{r+1} = {}^{10}C_r(x^{1/3})^{10-r}(x^{-1/2})^r$

For term independent of x;

$\Rightarrow \left(x^{\frac{10-r}{3}}\right)\left(x^{\frac{-r}{2}}\right) = x^0 \Rightarrow \frac{10-r}{3} - \frac{r}{2} = 0$

$\Rightarrow 5r = 20 \Rightarrow r = 4 \Rightarrow T_5 = {}^{10}C_4$

5. **B**

Obviously y can't be odd as if y is odd, then $x^y + 1$ is divisible by x + 1.

So, y = 2

Now, z must be odd $\Rightarrow$ x must be even

So, there is only one such triplet (2, 2, 5).

6. **(A)**

$(x-1)(x^2-2)(x^3-3) \ldots\ldots (x^n-n)$

Heighest power of x $=1+2+3+\ldots+n = \frac{n(n+1)}{2} = \alpha$

We are looking for the coefficient of $x^{\alpha-7}$

$\Rightarrow$ Either we should leave

(x^7-7), $(x-1)(x^6-6)$, $(x^2-2)(x^5-5)$, $(x^3-3)(x^4-4)$, $(x-1)(x^2-2)(x^4-4)$

If we leave (x^7-7), coeficient is -7

If we leave $(x-1)(x^6-6) = (x^7-x^6-6x+6)$, cofficient is 6

If we leave $(x^2-2)(x^5-5)$, coefficient is 10

If we leave $(x^3-3)(x^4-4)$, coefficient is 12

If we leave $(x-1)(x^2-2)(x^4-4)$, coefficient is -8

$\Rightarrow$ Required coefficient = 12 +10 + 6 –7 – 8 = 13.

7. **(A)**

$$\lim_{n\to\infty}\left[C_n-\left(\frac{2}{3}\right)C_{n-1}+\left(\frac{2}{3}\right)^2C_{n-2}+\ldots..+(-1)^n\left(\frac{2}{3}\right)^nC_0\right]$$

$$= \lim_{n\to\infty}\left[C_0-C_1\left(\frac{2}{3}\right)+C_2\left(\frac{2}{3}\right)^2+\ldots..+(-1)^nC_n\left(\frac{2}{3}\right)^n\right] \quad \text{[using } {}^nC_r = {}^nC_{n-r}\text{]}$$

$$= \lim_{n\to\infty}\left[1-\frac{2}{3}\right]^n = \lim_{n\to\infty}\frac{1}{3^n} = 0$$

8. **(A), (B) , (C), (D)**

In the expansion of $(a\alpha^2x^2 + 2b\alpha x + c)^n$

the sum of the coefficients = $(a\alpha^2 + 2b\alpha + c)^n$

Let $f(\alpha) = a\alpha^2 + 2b\alpha + c$

Its discriminant $= 4b^2 - 4ac = 4(b^2 - ac) < 0$

Hence, $f(\alpha) < 0$ or $f(\alpha) > 0$ for all $\alpha \in R$

If a > 0 then $f(\alpha) > 0 \Rightarrow (a\alpha^2 + 2b\alpha + c)^n > 0$

If c > 0 i.e. $f(0) > 0 \Rightarrow f(\alpha) > 0 \Rightarrow (a\alpha^2 + 2b\alpha + c)^n > 0$

If a < 0 then $f(\alpha) < 0 \Rightarrow (a\alpha^2 + 2b\alpha + c)^n < 0$ if n is odd

If c < 0 i.e. $f(0) < 0 \Rightarrow f(\alpha) < 0 \Rightarrow (a\alpha^2 + 2b\alpha + c)^n > 0$ if n is even.

9. **(C)**

Since $x^2y^3z^4$ is occurring in the expansion of $(x+y+z)^n$, so n should be 9 only.

Now A = $\frac{9!}{2!\times 3!\times 4!} = 1260$

Coefficient of x^4y^4z is $\frac{9!}{4!\times 4!} = 630 = A/2$.

10. **(B)**

Let n= 2m +1

A = $\frac{1}{C_1}+\frac{1}{C_2}+...+\frac{1}{C_m}=\frac{1}{C_{2m}}+\frac{1}{C_{2m-1}}+...+\frac{1}{C_{m+1}}$

$\Rightarrow$ 2A +2 = $\sum_{r=0}^{n}\frac{1}{C_r}$

Let S = $\sum_{r=1}^{n}\frac{r}{C_r}=\sum_{r=0}^{n}\frac{r}{C_r}=\sum_{r=0}^{n}\frac{n-r}{C_{n-r}}=\sum_{r=0}^{n}\frac{n-r}{C_r}$.

$\Rightarrow$ 2S = n $\sum_{r=0}^{n}\frac{1}{C_r}$ $\Rightarrow$ S = n(A+1).

11. **(D)**

All the terms in the expansion of $\left(x+\frac{1}{x}\right)^{11}$ will have odd powers of x .

So required sum = 0.

12. **(D)**

After expansion, no two terms will have the same powers of x or the terms are non over-lapping. Therefore, the total number of terms

= 2 × 2 × 2 × . . . (n +2) times = 2^{n+2} as a particular power of x can be chosen from each bracket in 2 ways.

13. **(B)**

$(1+x)^n = 3+\frac{8}{3}+\frac{80}{3^3}+\frac{240}{3^4}+....$ = 1 + nx + $\frac{n(n-1)x^2}{2}$ + ...

On comparison, n = -3 and x = $\frac{-2}{3}$.

14. **(D)**

We have $(2x+x^2)^n = [(1+x)^2 - 1]^n$

$\Rightarrow x^n(2+x)^n = (1+x)^{2n} - {}^nC_1(1+x)^{2n-2} + {}^nC_2(1+x)^{2n-4} - {}^nC_3(1+x)^{2n-6} + \ldots + (-1)^n\,{}^nC_n$ . .. .(1)

Comparing the coefficient of x^n on both sides, we have

${}^{2n}C_n - {}^nC_1.{}^{2n-2}C_n + {}^nC_2.{}^{2n-4}C_n - \ldots = 2^n$.

15. **(B)**

Since we are interested in coefficient of x we can ignore higher power of x in the binomial expansion i.e. $(1+x)^n = 1+nx$ shall be taken,

Let $\Delta = \begin{vmatrix} 1+a_1b_1x & 1+a_1b_2x & 1+a_1b_3x \\ 1+a_2b_1x & 1+a_2b_2x & 1+a_2b_3x \\ 1+a_3b_1x & 1+a_3b_2x & 1+a_3b_3x \end{vmatrix}$ $C_1 - C_2,\ C_2 - C_3$

$= \begin{vmatrix} a_1(b_1-b_2)x & a_1(b_2-b_3)x & 1+a_1b_3x \\ a_2(b_1-b_2)x & a_2(b_2-b_3)x & 1+a_2b_3x \\ a_3(b_1-b_2)x & a_3(b_2-b_3)x & 1+a_3b_3x \end{vmatrix}$

$$= (b_1 - b_2)(b_2 - b_3)x^2 \begin{vmatrix} a_1 & a_1 & 1+a_1b_3x \\ a_2 & a_2 & 1+a_2b_3x \\ a_3 & a_3 & 1+a_3b_3x \end{vmatrix} = 0.$$

Hence coefficient of 'x' = 0.

COMPLEX NUMBER

16. (A)

Since diagonals are perpendicular to each other $\arg \dfrac{z_1 - z_3}{z_2 - z_4} = \pm \dfrac{\pi}{2}$

$\Rightarrow z_1 - z_3 = ik(z_2 - z_4)$.

17. (D) $|z_1 - z_2|^2 = |z_1|^2 + |z_2|^2 - 2|z_1| . |z_2| \cos\theta$, where $\theta = |\arg z_1 - \arg z_2|$.
Hence for the given relation $\theta = 0$
$\Rightarrow \arg z_1 - \arg z_2 = 0$.

18. (B) z lies on the line segment joining the complex numbers –1 and 1 .

19. (B) Point corresponding to c divides the join of a and b in the ratio $\lambda : 1-\lambda$.

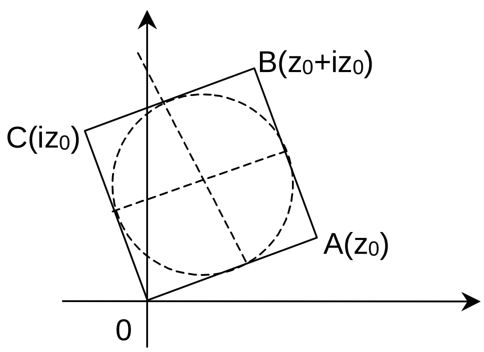
20. (A), (B)
Roots of $x^2 + x + 1 = 0$ are complex cube roots of unity,
so $h(w) = h(w^2) = 0$
$\Rightarrow w f(1) + w^2 g(1) = 0$ and $w^2 f(1) + wg(1) = 0$
$\Rightarrow f(1) = g(1) = 0$.

21. (B)

Clearly mid-point of OB is one centre of the circle and radius is equal $\dfrac{|z_0|}{2}$

$\Rightarrow$ Required equation is ;

$$\left| z - \frac{z_0}{2}(1+i) \right| = \frac{|z_0|}{2}$$



22. (B)

$z_1 = 3 + 4i$, $z_2 = 4 + 3i$, $z_3 = 2\sqrt{6} + i$

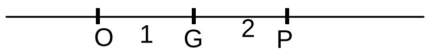
Clearly $|z_1| = |z_2| = |z_3| = 5$,

$\Rightarrow$ Points would lie on the circle centred at origin 'O'.

Now centroid of the triangle formed by these point

$$G = \left(\frac{7+2\sqrt{6}}{3} + \frac{8i}{3}\right)$$

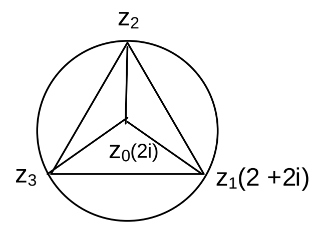
$$OG = \sqrt{\left(\frac{7+2\sqrt{6}}{3}\right)^2 + \frac{64}{9}} = \frac{1}{3}\sqrt{137+28\sqrt{6}}$$



$\Rightarrow OP = 3\,OG = \sqrt{137+28\sqrt{6}}$.

23. (A), (C)

Clearly the inscried triangle is equilateral.



$$\Rightarrow \frac{z_2 - z_0}{z_1 - z_0} = e^{i\frac{2\pi}{3}}, \quad \frac{z_3 - z_0}{z_1 - z_0} = e^{-i\frac{2\pi}{3}}$$

$\Rightarrow z_2 = -1 + i(2 + \sqrt{3})$ and $z_3 = -1 + i(2 - \sqrt{3})$

24. (C)

$|3z - 2| + |3z + 2| = 4$

$$\Rightarrow \left|z - \frac{2}{3}\right| + \left|z + \frac{2}{3}\right| = \frac{4}{3}$$

Sum of distances of $P(z)$ from $A(2/3, 0)$ and $B(-2/3, 0)$ is $4/3$

i.e. $PA + PB = AB$

Hence, locus of P is the line-segment AB.

25. (A)

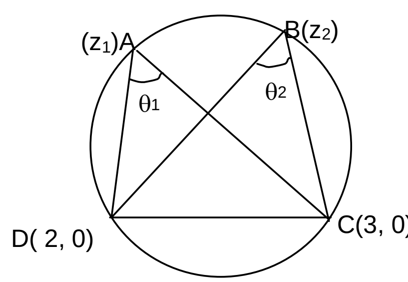
$$\bar{z}_1 = \frac{z_1\bar{z}_1}{z_1} = |z_1|^2 \, z_1^{-1}$$

$\Rightarrow \arg(z_1^{-1}) = \arg(\bar{z}_1) = \arg(z_2) \Rightarrow z_2 = kz_1^{-1}$ $(k > 0)$

26. (A)

$$\text{Arg}\left(\frac{3-z_1}{2-z_1}\right) + \arg\left(\frac{2-z_2}{3-z_2}\right)$$

$$= \arg\left(\frac{3-z_1}{2-z_1}\right)\left(\frac{2-z_2}{3-z_2}\right)$$



Now if $\left(\frac{3-z_1}{2-z_1}\right)\left(\frac{2-z_2}{3-z_2}\right)$ is a +ve real number, then its argument will be zero

So, angles θ_1 and θ_2 are equal in magnitude but opposite in sign.

So chord DC subtends equal angles at A and B. So points are concylic for $k > 0$

27. (C)

$t_n = (n+1)\left(n+\frac{1}{\omega}\right)\left(n+\frac{1}{\omega^2}\right)$

$= n^3 + n^2\left(\frac{1}{\omega^2}+\frac{1}{\omega}+1\right)+n\left(1+\frac{1}{\omega^2}+\frac{1}{\omega}\right)+1$

$= n^3 + n^2(\omega + \omega^2 + 1) + n(\omega + \omega^2 + 1) + 1$

$= n^3 + 1$

$\therefore\ S_n = \sum_{r=1}^{n} t_r = \sum_{r=1}^{n}(r^3+1) = \frac{n^2(n+1)^2}{4} + n$.

28. (B)

We are finding out sum of distances of a complex number z from origin and $(\cos\alpha, \sin\alpha)$. This sum will be minimum if z lies on the line joining the two points and minimum value of sum will be the distance between two points i.e. 1 .

29. (A), (C), (D)

$z^2 + az + a^2 = 0$

$\Rightarrow$ $z = a\omega,\ a\omega^2$ (where 'ω' is non real root of cube unity)

$\Rightarrow$ locus of z is a pair of straight lines

and $\arg(z) = \arg(a) + \arg(\omega)$ or $\arg(a) + \arg(\omega^2)$

$\Rightarrow \arg(z) = \pm\frac{2\pi}{3}$

also, $|z| = |a||\omega|$ or $|a|\,|\omega^2| \Rightarrow |z| = |a|$.

30. (C)

Given $z^{n-1} + z^{n-2} + z^{n-3} + \ . \ . \ . \ + z + 1 = 0$

$\Rightarrow (z-1)(z^{n-1} + z^{n-2} + \ldots + z + 1) = 0, \quad z \neq 1$

$\Rightarrow z^n = 1 = e^{i2r\pi} \qquad (r \in N)$

$\Rightarrow z_r = e^{\frac{i2r\pi}{n}} \qquad r = 1, 2, 3, \ldots, n-1$

$\Rightarrow$ The roots are $e^{\frac{i2\pi}{n}}, e^{\frac{i4\pi}{n}}, e^{\frac{i6\pi}{n}}, \cdots, e^{\frac{i(2n-2)\pi}{n}}$.

Which is a G.P., with common ratio $e^{\frac{i2\pi}{n}}$.

DET

31. **A**

Putting x = 0, we get $e = \begin{vmatrix} 3 & 1 & 4 \\ 1 & 2 & 3 \\ 0 & 1 & 1 \end{vmatrix}$.

$C_3 \to C_3 - C_1 = \begin{vmatrix} 3 & 1 & 1 \\ 2 & 2 & 2 \\ 0 & 1 & 1 \end{vmatrix} = 0.$

32. **D**

$C_3 - C_1 \cos D + C_2 \sin D = 0$

so $\Delta = 0$, hence Δ is independent of A, B, C and D all.

33. **A**

We know that $\left(a^3+b^3+c^3-3abc\right)^2 = \begin{vmatrix} a & c & b \\ b & a & c \\ c & b & a \end{vmatrix}\begin{vmatrix} a & c & b \\ b & a & c \\ c & b & a \end{vmatrix} = \begin{vmatrix} A & B & B \\ B & A & B \\ B & B & A \end{vmatrix} = \begin{vmatrix} B & A & B \\ B & B & A \\ A & B & B \end{vmatrix}$.

34. **C**

Applying $C_1 \to C_1 + C_2$, we get

$$\begin{vmatrix} 2 & \cos^2 x & \sin 2x \\ 2 & 1+\cos^2 x & \sin 2x \\ 1 & \cos^2 x & 1+\sin 2x \end{vmatrix}$$

Applying $R_2 \to R_2 - R_1$ and $R_3 \to R_3 - R_1$, we get

$$\begin{vmatrix} 2 & \cos^2 x & \sin 2x \\ 0 & 1 & 0 \\ -1 & 0 & 1 \end{vmatrix} = 2 + \sin 2x.$$

Since the maximum value of sin2x is 1, and min value of sin2x is (-1).

Therefore $\alpha = 3$, $\beta = 1$.

35. **A**

Since $\begin{vmatrix} x+3 & x+4 & x+\lambda \\ x+4 & x+5 & x+\mu \\ x+5 & x+6 & x+\nu \end{vmatrix} = 0$

$R_2 \to R_2 - \left(\dfrac{R_1+R_3}{2}\right)$.

$$\Rightarrow \begin{vmatrix} x+3 & x+4 & x+\lambda \\ 0 & 0 & \mu-\left(\dfrac{\lambda+\nu}{2}\right) \\ x+5 & x+6 & x+\nu \end{vmatrix} = 0$$

But λ, μ, ν are in A.P. .

$\therefore \mu = \dfrac{\lambda+\nu}{2}$

$$\therefore \begin{vmatrix} x+3 & x+4 & x+\lambda \\ 0 & 0 & 0 \\ x+5 & x+6 & x+\nu \end{vmatrix} = 0 \quad \forall x \in R$$

$\therefore$ An identity in x.

36. **B**

We have $y = \sqrt{x\,z}$, so that the given determinant is equal to

$$\begin{vmatrix} \sqrt{x}\left(\sqrt{x}\,p+\sqrt{z}\right) & x & \sqrt{x\,z} \\ \sqrt{z}\left(\sqrt{x}\,p+\sqrt{z}\right) & \sqrt{xz} & z \\ 0 & \sqrt{x}\left(\sqrt{x}\,p+\sqrt{z}\right) & \sqrt{z}\left(\sqrt{x}\,p+\sqrt{z}\right) \end{vmatrix} = \sqrt{x}\,\sqrt{z}\left(\sqrt{x}\,p+\sqrt{z}\right)^2 \begin{vmatrix} 1 & \sqrt{x} & \sqrt{z} \\ 1 & \sqrt{x} & \sqrt{z} \\ 0 & \sqrt{x} & \sqrt{z} \end{vmatrix} = 0$$

37. **A** Operate $C_1 - bC_2$ and then $C_3 - \frac{c}{a}C_1$ and get

$$\text{LHS} = ab\begin{vmatrix} h & g & a \\ b & f & h \\ f & c & g \end{vmatrix} = ab\begin{vmatrix} g & a & h \\ f & h & b \\ c & g & f \end{vmatrix} = a\begin{vmatrix} bg & a & h \\ bf & h & b \\ bc & g & f \end{vmatrix} = a\begin{vmatrix} ah+bg & a & h \\ ab+bf & h & b \\ af+bc & g & f \end{vmatrix}$$ (Operating $C_1 + aC_3$)

38. **B** The determinant $= \begin{vmatrix} \alpha^2 & 0 & 1 \\ 0 & \beta^2 & 1 \\ -\gamma^2 & -\gamma^2 & 1+\gamma^2 \end{vmatrix} = \alpha^2\beta^2\gamma^2 + \alpha^2\beta^2 + \beta^2\gamma^2 + \alpha^2\gamma^2$

$$\alpha^2\beta^2\gamma^2 + \left(\alpha\beta+\beta\gamma+\gamma\alpha\right)^2 - 2\alpha\beta\gamma\left(\alpha+\beta+\gamma\right) = 1+1-2\times1\times1 = 0.$$

39. **C**

The determinant $= \begin{vmatrix} n-1 & n^2-n & n^2 \\ n & n^2-1 & n^2+n \\ n-1 & n^2+1 & n^2 \end{vmatrix} = \begin{vmatrix} n-1 & n^2-n & n^2 \\ n & n^2-1 & n^2+n \\ 0 & n+1 & 0 \end{vmatrix}$

$= -(n + 1)[n(n^2 - 1) - n^3] = n(n + 1) = 72 = 8 \times 9.$

40. **A**

Area $= \frac{1}{2}\begin{vmatrix} a & x & 1 \\ a+d & xr & 1 \\ a+2d & xr^2 & 1 \end{vmatrix}$

$= \frac{1}{2}\begin{vmatrix} a & x & 1 \\ a & xr & 1 \\ a & xr^2 & 1 \end{vmatrix} + \frac{1}{2}\begin{vmatrix} 0 & x & 1 \\ d & xr & 1 \\ 2d & xr^2 & 1 \end{vmatrix}$

$= 0 + \frac{1}{2}\begin{vmatrix} 0 & x & 1 \\ d & xr & 1 \\ 2d & xr^2 & 1 \end{vmatrix}$

which clearly shows that area is independent of a

41. D

$[\sin^2\theta] = 0 \quad \sin^2\theta \neq 1$

$= 1, \sin^2\theta = 1$

if $\sin^2\theta \neq 1 \Rightarrow D = 2\sin\theta\cos\theta - 2i - 1$

$\text{Re}(D) = 2\sin\theta\cos\theta - 1$

$-2 \le \text{Re}(D) \le 0$

$-\frac{3\pi}{4} \le \arg D \le -\frac{\pi}{2}$

If $\sin^2\theta = 1$, $\sin\theta = \pm 1$, $\cos\theta = 0$

$\text{Arg}(D) = \arg(1 - 2i)$ or $\arg(-1 - 2i)$

42. **B**

Clearly $\alpha = -i$, where $i^2 = -1$

so $\Delta(\alpha) = \alpha^n \cdot \frac{1}{\alpha^n}\begin{vmatrix} 1 & 1 & 1 \\ 1 & \alpha & \alpha^3 \\ \frac{1}{\alpha} & 1 & 0 \end{vmatrix} = \begin{vmatrix} 1 & 1 & 1 \\ 1 & -i & i \\ i & 1 & 0 \end{vmatrix}$

$= 1(-i) + 1(i^2) + (1 + i^2) = -1 - i$, arg is $-\frac{3\pi}{4}$.

43. **A**

$$\Delta'(x) = \begin{vmatrix} e^x & 2\cos 2x & 2x\sec^2 x^2 \\ \ln(1+x) & \cos x & \sin x \\ \cos x^2 & e^x - 1 & \sin x^2 \end{vmatrix} + \begin{vmatrix} e^x & \sin 2x & \tan x^2 \\ \frac{1}{(1+x)} & -\sin x & \cos x \\ \cos x^2 & e^x - 1 & \sin x^2 \end{vmatrix}$$

$$+ \begin{vmatrix} e^x & \sin 2x & \tan x^2 \\ \ln(1+x) & \cos x & \sin x \\ -2x\sin x^2 & e^x & 2x\cos x^2 \end{vmatrix} = B + 2Cx + \ldots$$

Put $x = 0$,

$$B = \begin{vmatrix} 1 & 2 & 0 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \end{vmatrix} + \begin{vmatrix} 1 & 0 & 0 \\ 1 & 0 & -1 \\ 1 & 0 & 0 \end{vmatrix} + \begin{vmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 1 & 0 \end{vmatrix} = 0$$

44. **A**

$\Delta = \begin{vmatrix} 100x_1 + 10y_1 + z_1 & y_1 & z_1 \\ 100x_2 + 10y_2 + z_2 & y_2 & z_2 \\ 100x_3 + 10y_3 + z_3 & y_3 & z_3 \end{vmatrix} = \begin{vmatrix} 2A & y_1 & z_1 \\ 2B & y_2 & z_2 \\ 2C & y_3 & z_3 \end{vmatrix}$, where $A, B, C \in I$

$= 2\begin{vmatrix} A & y_1 & z_1 \\ B & y_2 & z_2 \\ C & y_3 & z_3 \end{vmatrix}$, which is divisible by 2 but not necessarily by 4 or 8.

45. B

$C_1 \to C_1 - \sin\theta\, C_3$ and $C_2 \to C_2 + \sin\theta\, C_3$

$f(\theta) = \begin{vmatrix} 1 & 0 & -\sin\theta \\ 0 & 1 & \cos\theta \\ \sin\theta & -\cos\theta & 0 \end{vmatrix}$

Again $R_3 \; -\sin\theta\, R_1 + \cos\theta\, R_2$, we get

$= \begin{vmatrix} 1 & 0 & -\sin\theta \\ 0 & 1 & \cos\theta \\ 0 & 0 & 1 \end{vmatrix} = 1 \Rightarrow f\left(\frac{\pi}{6}\right) = 1.$

46. **(C)**

The first digit can be chosen in 9 ways(other than zero), the second can be chosen in 9 ways (any digit other then the first digit), the third digit can be chosen in 9 ways (any digit other then the second digit) and so on.

Hence required number of numbers is $9 \times 9 \times .... \times 9$ (n times) $= 9^n$.

47. **(B)**

$3630 = 2 \times 3 \times 5 \times 11^2$

Now a divisor will be of the form (4n + 1) if divisor is formed with the help of (4n + 1) type number or by (4n + 3) types number taken even times.

Hence divisors are $1, 5, 3 \times 11, 11^2, 5 \times 11^2, 5 \times 3 \times 11$, i.e., 6.

48. **(C)**

$^{10}C_{x-1} > 3 \cdot {}^{10}C_x \Rightarrow \frac{1}{11-x} > \frac{3}{x} \Rightarrow 4x > 33 \Rightarrow x \geq 9$, but $x \leq 10$.

So x = 9, 10. Hence there are two solutions

49. **(D)**

Any three numbers x, y, z from {1, 2, 3, …} can be chosen in $^nC_3$ ways and we get unique triplet (x, y, z), x < y < z. Again any two numbers x, z can be chosen from {1, 2, 3,... , n} in $^nC_2$ ways and we get the triplet (x, x, z), x < z. Hence total number of required triplets is $^nC_2 + {}^nC_3$.

50. **(A), (B), (C), (D)**

$\frac{(mn)!}{(m!)^n}$ is the number of ways of distributing mn distinct objects in n persons equally. Hence $\frac{(mn)!}{(m!)^n}$ is an integer $\Rightarrow (m!)^n \mid (mn)!$. Similarly $(n!)^m \mid (mn)!$

Further $m + n < 2m \leq mn \Rightarrow (m + n)! \mid (mn)!$ and $m - n < m < mn$

$\Rightarrow (m - n)! \mid (mn)!$

51. **(B)**

First, 6 distinct digits can be selected in $^{10}C_6$ ways. Now the position of smallest digit in them is fixed i.e. position 4. Of the remaining 5 digits, two digits can be selected in $^5C_2$ ways. These two digits can be placed to the right of 4^{th} position in one way only. The remaining three digits to the left of 4^{th} position are in the required order automatically.

So $n(S) = {}^{10}C_6 \times {}^5C_2 = 210 \times 10 = 2100$.

52. **(A)**

Here $x_1 x_2 x_3 = 2^2 \times 3 \times 5$

Let number of two's given to each of x_1 , x_2 , x_3 be a, b, c .

Then $a + b + c = 2$, $a, b, c \geq 0$

The number of integral solutions of this equations is equal to coefficient of x^2 in $(1 - x)^{-3}$ i.e. $^4C_2$

i.e. the available 2 two's can be distributed among x_1, x_2 and x_3 in $^4C_2 = 6$ ways.

Similarly, the available 1 three can be distributed among x_1, x_2, x_3 in $^3C_2 = 3$ ways. (= coefficient of x in $(1 - x)^{-3}$)

$\therefore$ Total number of ways $= {}^4C_2 \times {}^3C_2 \times {}^3C_2 = 6 \times 3 \times 3 = 54$ ways.

53. **(C)**

21, 22, 23, …, k –1, k

A.M. = $\frac{21+k}{2}$, G.M = $\sqrt{21.k}$

$\Rightarrow$ k = 21. λ^2, $\lambda \in$ I also 100 $\le$ k $\le$ 999 and k should be odd

$\Rightarrow \frac{100}{21} \le \lambda^2 \le \frac{999}{21} \Rightarrow 4.76 \le \lambda^2 \le 47.57$

$\Rightarrow$ λ = 3, 4, 5, 6 but λ should be odd $\Rightarrow$ odd λ = 3, 5

$\Rightarrow$ 'k' can assume 2 different values.

54. **(A)**

Perfect square = $\left[\sqrt{100}\right]$ – 1 = 9 (excluding one)

Perfect cubes = $\left[100^{1/3}\right] - 1 = 3$

Perfect 4^{th} powers = $\left[100^{1/4}\right] - 1 = 3$

Perfect 5^{th} powers = $\left[100^{1/5}\right] - 1 = 1$

Perfect 6^{th} powers = $\left[100^{1/6}\right] - 1 = 1$

Now, perfect 4^{th} powers have already been counted in perfect squares and perfect 6^{th} powers have been counted with perfect squares as well as with perfect cubes.

Hence the total ways = 9 + 3 + 1 = 13.

55. **(A)**

Total number of numbers will be equal to the sum of numbers of all possible 1-digit, 2-digit, 3-digit, 4-diigit and 5-digit numbers.

$\Rightarrow$ Total number of numbers = $3 + 3^2 + 3^3 + 3^4 + 3^4$

$= \frac{3(3^4 - 1)}{2} + 3^4 = \frac{3^5 + 2.3^4 - 3}{2}$.

56. **(B)**

$7! = 2^4 \times 3^2 \times 5 \times 7$

Since the factor should be odd as well as of the form 3t + 1, the factor cannot be a multiple of either 2 or 3. So the factors may be 1, 5, 7and 35 of which only 1 and 7 are of the from 3t +1, whose sum is 8.

57. **(A)**

$n! + (n+1)! + (n+2)! = n! \{1 + n + 1 + (n+2)(n+1)\} = n!(n+2)^2$

$\Rightarrow$ Either 7 divides n + 2 or 49 divides n!

$\Rightarrow$ n = 5, 12, 14.

58. **(A), (B)**

f(n) = 1! + 2! + 3! + ... + n!

f(n+1) = 1! + 2! + 3! + ... + (n+1)!

f(n+2) = 1! + 2! + 3! + ... + (n+2)!

f(n+2) – f(n+1) = (n +2)! = (n+2)(n + 1)!

= (n +2)[f(n+1) – f(n)]

$\Rightarrow$ f(n+2) = (n+3)f(n+1) – (n+2)f(n)

$\Rightarrow$ P(x) = x +3, Q(x) = – x –2

59. **C**

If we put minimum number of balls required in each box. Balls left are $\frac{n(n-1)}{2}$ which can be put in ${}^{\frac{n^2+n-2}{2}}C_{n-1}$ ways without restriction.

60. **(A)**

I	II	III	IV

Two distinct odd digits for the second and fourth places can be selected in ${}^4C_2 = 6$ ways (since we cannot take 1, as first digit will be at least 2). Now these can be arranged in increasing order in one way only. Similarly two distinct even digits for the first and third places can be selected in ${}^4C_2 = 6$ ways (since we cannot take 0). Now these can be arranged in increasing order in one way only.
Now total number of ways of filling the four places is $6 \times 6 = 36$.
But this contains the numbers of the type 6385 which are not needed. So number of such numbers will be less than 36.

QEE

61. (A)
$e^{\cos x} = t \Rightarrow t^2 - 4t - 1 = 0$
$\Rightarrow t = e^{\cos x} = 2 \pm \sqrt{5}$
Since $e^{\cos x} \in [1/e, e]$, so number of real roots is 0.

62. (D) Each term of $\sum_{k=1}^{n}(x-k)^2$ is non-negative, so no real root.

63. (B) Let $f(x) = x^3 - 3x + a$
$f'(x) = 3x^2 - 3$.
For three distinct real roots (i) $f'(x) = 0$ should have two distinct real roots α and β and (ii) $f(\alpha)\, f(\beta) < 0$
Here $\alpha = 1, \beta = -1$.
Now $f(\alpha)\, f(\beta) < 0$
$\Rightarrow (1 - 3 + a)(-1 + 3 + a) < 0 \Rightarrow (a - 2)(a + 2) < 0$
$\Rightarrow -2 < a < 2$.

64. (B)
Since a_1, a_2, a_3 $(a_1 > 0)$ are in G.P.
So, $a_2 = a_1 r$; $a_3 = a_1 r^2$
Given inequality
$9a_1 + 5a_3 > 14a_2$
$9a_1 + 5a_1 r^2 > 14 a_1 r$
$5r^2 - 14r + 9 > 0$
$(r - 1)(r - 9/5) > 0$
$r > 9/5$ and $r < 1$
$r \notin [1, 9/5]$.

65. (A), (B), (C)

Clearly figure represents a downward parabola having its vertex $\left(-\frac{b}{2a}, -\frac{D}{4a}\right)$ in the second quadrant.

$\Rightarrow a < 0$, $-b/2a < 0 \Rightarrow b < 0$

also, $ax^2 + bx + c = 0$ has roots of opposite signs

$\Rightarrow c/a < 0 \Rightarrow c > 0$.

66. (D)

We have, $a > 0$, $c > 0$, $b^2 - 4ac < 0$

Similarly, $a_1 > 0$, $c_1 > 0$, $b_1^2 - 4a_1c_1 < 0$

From the given information sign of $(bb_1)^2 - 4aa_1\,cc_1$ cannot be checked

67. (C)

$x^2 + x - n = 0$, discriminent = $1 + 4n$ = odd number = D(say)

Now given equation would have a integral soluton if D is a perfect square .

Let $D = (2\lambda + 1)^2 \Rightarrow n = \lambda + \lambda^2 = \lambda(\lambda + 1)$ = even number

$\Rightarrow$ n can be 2, 6, 12, 20, 30, 42, 56, 72, 90.

68. (B)

$x + y = 2 \Rightarrow x = 2 - y$

Also, $xy - z^2 = 1 \Rightarrow 2y - y^2 - z^2 = 1 \Rightarrow z^2 + (y - 1)^2 = 0$

$\Rightarrow z = 0, y = 1, x = 1$

69. (C)

$x = \sqrt[3]{7} + \sqrt[3]{49}$

$\Rightarrow x^3 = 7 + 49 + 3\sqrt[3]{7}\,.\sqrt[3]{49}\left(\sqrt[3]{7} + \sqrt[3]{49}\right)$

$\Rightarrow x^3 - 21x - 56 = 0$

$\Rightarrow$ Product of root = 56.

70. (A)

Let m be a positive integer for which $n^2 + 96 = m^2$

$\Rightarrow m^2 - n^2 = 96 \Rightarrow (m + n)(m - n) = 96$

$\Rightarrow (m + n)\{(m + n) - 2n\} = 96$

$\Rightarrow$ m + n and m – n must be both even

$96 = 2 \times 48$ or 4×24 or 6×16 or 8×12

Number of solution = 4.

71. (D)

Discriminant of $3x^2 + 8x + 15 = 0$ is negative. So, the roots are imaginary and therefore conjugate of each other .

So, both roots are common.

$\Rightarrow \frac{a}{3} = \frac{2b}{8} = \frac{3c}{15} \Rightarrow a : b : c = 3 : 4 : 5$

$\Rightarrow \Delta ABC$ is right triangle.

$\Rightarrow \sin^2A + \sin^2B + \sin^2C = 2$.

72. (A)

$(y^2 - 5y + 3)(x^2 + x + 1) < 2x \ \forall x \in R$

$\Rightarrow y^2 - 5y + 3 < \frac{2x}{x^2 + x + 1}$

Let $\frac{2x}{x^2 + x + 1} = p \Rightarrow px^2 + (p - 2)x + p = 0$

Since x is real, $(p - 2)^2 - 4p^2 \geq 0$

$\Rightarrow -2 \leq p \leq \frac{2}{3}$

Minimum value of $\frac{2x}{x^2 + x + 1}$ is -2

So, $y^2 - 5y + 3 < -2 \Rightarrow y^2 - 5y + 5 < 0$

$\Rightarrow y \in \left(\frac{5 - \sqrt{5}}{2}, \frac{5 + \sqrt{5}}{2}\right)$.

73. (C)

74. (A), (B)

x can not be odd integer for if x is odd, x^2 is odd but $2px + 2q$ is even;

so $x^2 + 2px + 2q \neq 0$

x can not be even integer for if x is even, $x^2 + 2px$ is a multiple of 4 but 2q is not.

So $x^2 + 2px + 2q \neq 0$

Also $(x + p)^2 = p^2 - 2q$

$\Rightarrow$ If x is fraction then $(x + p)^2$ is also a fraction but $p^2 - 2q$ is an integer. So, roots cannot be integer or rational numbers.

75. (A)

Let $a^{\cos x} = t$

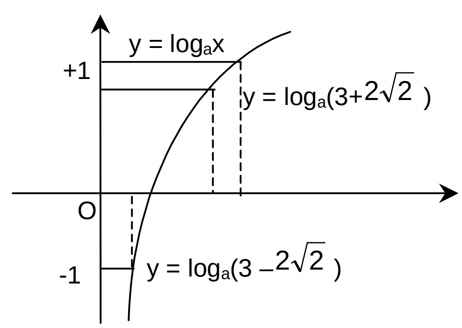
$\Rightarrow t + \frac{1}{t} = 6$

$\Rightarrow t^2 - 6t + 1 = 0$

$\Rightarrow t = \frac{6 \pm \sqrt{36 - 4}}{2} = 3 \pm 2\sqrt{2}$

$\Rightarrow a^{\cos x} = 3 \pm 2\sqrt{2}$

$\Rightarrow \cos x = \log_a(3 \pm 2\sqrt{2})$



since $a > 1$, for all the roots to be real,

we must have $\log_a(3 + 2\sqrt{2}) \leq 1$ and $\log_a(3 - 2\sqrt{2}) \geq -1$,

Both are true for $a \geq 3 + 2\sqrt{2}$.

PS

76. **A**

$t_r = \frac{2r + 5}{(r + 1)(r + 2)}\left(\frac{1}{3}\right)^r = \frac{3(r + 2) - (r + 1)}{(r + 1)(r + 2)}\left(\frac{1}{3}\right)^r = \frac{1}{r + 1}\left(\frac{1}{3}\right)^{r-1} - \frac{1}{r + 2}\left(\frac{1}{3}\right)^r$

$\sum_{r=1}^{10} t_r = \frac{1}{2} - \frac{1}{12} \quad \frac{1}{3^{10}}$.

77. **B**

$a_1 + a_2 + ... + 2a_n \ge n2^{1/n}$

equality holds when

$a_1 = a_2 = ... = 2a_n = k$ (say)

Then (k.k ... (n – 1) times) $. \frac{k}{2} = 1$

$\Rightarrow k = 2^{\frac{1}{n}}$

$\Rightarrow a_n = 2^{\frac{1-n}{n}}$.

78. **B**

S = (0 + 1) + (0 + 2) + ... + (0 + n)

+ (1 + 2) + (1 + 3) + ... + (1 + n)

.....

....

+ ((n – 1) + n)

$= n(1 + 2 + 3 + ... + n) = \frac{n^2(n+1)}{2}$

putting n = 10, we get 50 × 11 = 550.

79. C

$\frac{1}{n} = a + (m-1)d, \quad \frac{1}{m} = a + (n-1)d$

$\Rightarrow d = \frac{1}{mn}, a = \frac{1}{mn} \Rightarrow T_{mn} = 1.$

80. (C)

Clearly $x_1 = ar$, $x_2 = ar^2$, also $y_1 = bs$ and $y_2 = bs^2$

$\therefore$ Area of triangle $= \frac{1}{2}\begin{vmatrix} a & b & 1 \\ ar & bs & 1 \\ ar^2 & bs^2 & 1 \end{vmatrix} = \frac{1}{2}\begin{vmatrix} a(1-r) & b(1-s) & 0 \\ ar(1-r) & bs(1-s) & 0 \\ ar^2 & bs^2 & 1 \end{vmatrix}$

Operating, $R_1 \to R_1 - R_2$ and $R_2 \to R_2 - R_3$.

$= \frac{1}{2}ab(r-1)(s-1)(s-r)$

81. B

82. **C**

Apply A.M ≥ G.M.

83. **C**

Let $\frac{1}{H_{i+1}} - \frac{1}{H_i} = k$

$\sum_{i=1}^{2n}(-1)^i\left(\frac{H_i + H_{i+1}}{H_i - H_{i+1}}\right) = \sum_{i=1}^{2n}\frac{(-1)^i}{k}\left(\frac{1}{H_{i+1}} + \frac{1}{H_i}\right) = 2n.$

84. **D**

$$\frac{1}{a}+\frac{1}{a-2b}+\frac{1}{c}+\frac{1}{c-2b}=(a+c-2b)\left(\frac{1}{a(c-2b)}+\frac{1}{c(a-2b)}\right)=0$$

as $a + c - 2b \neq 0 \Rightarrow \frac{1}{b}=\frac{1}{a}+\frac{1}{c}$.

85. **B**

$$\sum_{r=1}^{n}\frac{r^2-r-1}{r+1!}=\sum_{r=1}^{n}\left(\frac{r-1}{r!}-\frac{r}{(r+1)!}\right)=-\frac{n}{(n+1)!}.$$

86. **D**

$T_n = S_n - S_{n-1} = n(n + 1)(n + 2)(n + 3) - (n - 1)n(n + 1)(n + 2) = 4n(n + 1)(n + 2)$

$$\frac{1}{T_r}=\frac{1}{4r(r+1)(r+2)}=\frac{r+2-r}{8r(r+1)(r+2)}=\frac{1}{8}\left[\frac{1}{r(r+1)}-\frac{1}{(r+1)(r+2)}\right]$$

$$\frac{1}{T_1}=\frac{1}{8}\left[\frac{1}{1\cdot 2}-\frac{1}{2\cdot 3}\right]$$

$$\frac{1}{T_2}=\frac{1}{8}\left[\frac{1}{2\cdot 3}-\frac{1}{3\cdot 4}\right]$$

:   :   :

:   :   :

$$\frac{1}{T_{10}}=\frac{1}{8}\left[\frac{1}{10\cdot 11}-\frac{1}{11\cdot 12}\right]$$

$$\sum_{r=1}^{10}\frac{1}{T_r}=\frac{1}{8}\left[\frac{1}{2}-\frac{1}{132}\right]=\frac{65}{1056}.$$

87. **B**

Since y, x, z are in A.P. $\Rightarrow 2x = y + z$

$\Rightarrow (x + z) + (x + y) = 2(y + z)$

$\Rightarrow x + y, y + z$ and $x + z$ are in A.P.

$\Rightarrow 2^{x+y}, 2^{y+z}$ and 2^{x+z} are in G.P.

88. **C**

Given that $T_m = AR^{m-1} = \frac{1}{n^2}$ and $T_n = AR^{n-1} = \frac{1}{m^2}$

$\Rightarrow A^2R^{m+n-2} = \frac{1}{m^2n^2}$

$\Rightarrow AR^{\frac{m+n}{2}-1}=\frac{1}{mn}$

$\Rightarrow T_{\frac{m+n}{2}}=\frac{1}{mn}$.