IIT – JEE: 2024 **TW TEST** **DATE: 10/07/22**

TOPIC: VECTORS

Answers & Solutions

1. (B)

$\left|\vec{a}-\vec{b}\right| = \sqrt{a^2+b^2-2ab\cos\theta}$ ($\theta = \angle$ between $\vec{a}$ & $\vec{b}$)

$30 = \sqrt{11^2+23^2-2\times11\times23\cos\theta} \Rightarrow \cos\theta = -\frac{250}{506}$

$\therefore \left|\vec{a}+\vec{b}\right| = \sqrt{a^2+b^2+2ab\cos\theta}$

$= \sqrt{11^2+23^2-250} = 20$

2. (B)

Let forces are F and 2F and angle between them is θ and resultant makes an angle α with the force F.

$\tan\alpha = \frac{2F\sin\theta}{F+2F\cos\theta} = \tan 90 = \infty$

$\Rightarrow F+2F\cos\theta = 0 \quad \therefore \cos\theta = -1/2 \text{ or } \theta = 120^0$

3. (B)

$A\times B$ is a vector $\perp$ to both $\bar{A}$ and $\bar{B}$

Now $\bar{A}\times\bar{B} = (\bar{i}-2\bar{j}+\bar{k})\times(3\bar{i}+\bar{j}-2\bar{k}) = 3\bar{i}+5\bar{j}+7\bar{k}$

Now, $B = \frac{\bar{A}\times\bar{B}}{|\bar{A}\times\bar{B}|}$

$= \frac{3\bar{i}+5\bar{j}+7\bar{k}}{\sqrt{3^2+5^2+7^2}} = \frac{3\bar{i}+5\bar{j}+7\bar{k}}{\sqrt{83}}$

Hence, (B) is correct.

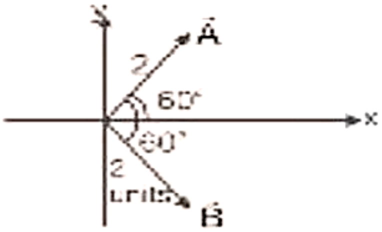
4. (A)

$\vec{r}_F - \vec{r}_1 =$ Displacment $\vec{r}_F =$ Final position vector

$\bar{r} =$ Initial position vector

$\therefore \vec{r}_F = \vec{r}_1 +$ Displacement

5. (B)



$\overline{A} = 2\cos 60^0 \hat{i} + 2\sin 60^0 \hat{j} = \hat{i} + \sqrt{3}\hat{j}$

$\vec{B} = 2\cos 60^0 \hat{i} - 2\sin 60^0 \hat{j} = i - \sqrt{3}j$

$\therefore \vec{A} + \vec{B} = 2\hat{i}$

6. (D)

$|\vec{A} + \vec{B}| = |\vec{A}| = |\vec{B}| = K$ Let

$|\vec{A} + \vec{B}| = \sqrt{A^2 + B^2 + 2AB\cos\theta}$

$K = \sqrt{K^2 + K^2 + 2KK\cos\theta}$

7. (C)

If a vector $\vec{A} = a\hat{i} + b\hat{j} + c\hat{k}$ makes angles α, β and γ with x,y and z axes respectively then

$\cos\alpha = \frac{a}{\sqrt{a^2 + b^2 + c^2}}, \cos\beta = \frac{b}{\sqrt{a^2 + b^2 + c^2}}$ and $\cos\gamma = \frac{c}{\sqrt{a^2 + b^2 + c^2}} = \frac{1}{\sqrt{2}} \Rightarrow \gamma = 45^o$

$\therefore \cos\alpha = \frac{1}{\sqrt{1^2 + 1^2\left(\sqrt{2}\right)^2}} = \frac{1}{2} \Rightarrow \alpha = 60^0$

$\cos\beta = \frac{1}{\sqrt{1^2 + 1^2 + \left(\sqrt{2}\right)^2}} = \frac{1}{2} \Rightarrow \beta = 60^0$

8. (A)

$\vec{F}_{net} = \vec{F}_1 + \vec{F}_2 + \vec{F}_3 = (P+1)\hat{i} + 4\hat{j}$

$\vec{F}_{net} = 5 \Rightarrow P + 1 = 3$ or $P + 1 = -3$

$\Rightarrow P = 2$ or $P = -4$

$\therefore$ Product of possible values of P is -8.

9. (D)

$\frac{\pi}{3}$ radians

$\vec{a}+\vec{b}+\vec{c}=0$

$-\vec{c}=\vec{a}+\vec{b}$

$|-\vec{c}|-=|\vec{a}+\vec{b}|$

$|-\vec{c}|=\sqrt{a^2+b^2+2ab\cos\theta}$

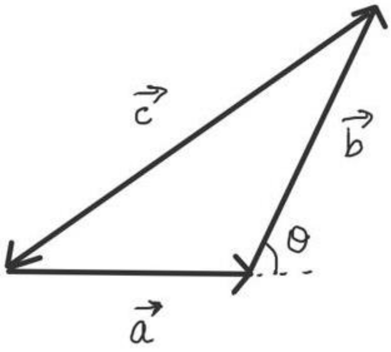
$7=\sqrt{3^2+5^2+2(3)(5)\cos\theta}$

$49=34+30\cos\theta$

$15=30\cos\theta$

$\cos\theta=\frac{1}{2}$

$\theta=\frac{\pi}{3}$



10. (D)

Vector joining A:(4, -4, 0) and B: (-2, -2, 0) is given by

$\overline{AB}=(-2-4)\hat{i}+(-2-(-4))\hat{j}+(0-0)\hat{k}=-6\hat{i}+2\hat{j}$

$|\overline{AB}|=\sqrt{(-6)^2+(2)^2}=\sqrt{40}=2\sqrt{10}$

11. (B)

Let the required vector be $\vec{A}$

$\vec{A}+(\hat{i}-2\hat{j}+2\hat{k})+(2\hat{i}+\hat{j}-\hat{k})=\hat{i}$

$\Rightarrow \vec{A}=-2\hat{i}+\hat{j}-\hat{k}$

12. (D)

$\vec{a}.\hat{i}=3$

13. (A)

$\vec{B}=\vec{C}+\vec{D}$

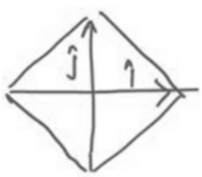
$=5\hat{i}+5\hat{j}-5\hat{k}$

$\vec{B}=5(\hat{i}+\hat{j}-\hat{k})$

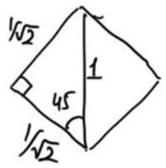
$\vec{B}=5\vec{A}$

Hence $\vec{B}$ and $\vec{A}$ are like vectors

14. (A)



Diagonals are $\perp^r$ and equal in length hence the 11gm is a square.



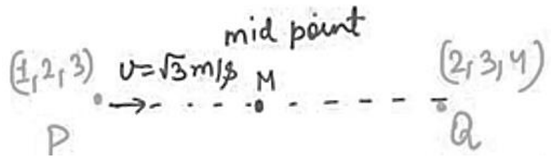
Area of square = $(\text{side})^2$

$$= \left(\frac{1}{\sqrt{2}}\right)^2 = \frac{1}{2}$$

15. (A)

16. (A)

$\mu = \sqrt{3}\,\text{m/s}$



$|\overrightarrow{PQ}| = \sqrt{3}\,\text{m}$

$|\overrightarrow{PM}| = \frac{\sqrt{3}}{2}\,\text{m}$

$t = \frac{|\overrightarrow{PM}|}{\text{speed}} = \frac{\sqrt{3}}{2\sqrt{3}}\,\text{sec} = \frac{1}{2}\,\text{seconds}$

17. (C)

$(\vec{a} - \vec{b}) \times (\vec{a} + \vec{b})$

$= (\vec{a} \times \vec{a}) + \vec{a} \times \vec{b} - \vec{b} \times \vec{a} - (\vec{b} \times \vec{b})$

$= 2(\vec{a} \times \vec{b}) \qquad \left[\because \vec{a} \times \vec{a} = \vec{b} \times \vec{b} = \vec{0} \text{ and } \vec{a} \times \vec{b} = -(\vec{b} \times \vec{a})\right]$

18. (D)

$\because \cos^2\alpha + \cos^2\beta + \cos^2\gamma = 1$

$\sin^2\alpha + \sin^2\beta + \sin^2\gamma = 3 - (\cos^2\alpha + \cos^2\beta + \cos^2\gamma)$

$=2$

19. (A)

$|\hat{p} - \hat{q}| = \sqrt{|\hat{p}|^2 + |\hat{q}|^2 + 2|\hat{p}||\hat{q}|\cos(180 - \theta)}$

$= \sqrt{1 + 1 - 2\cos\theta} = \sqrt{2(1 - \cos\theta)}$

$= \sqrt{4\sin^2(\theta/2)} = 2\sin(\theta/2)$

20. (B)

$\hat{a}.\hat{b} = \cos 60 \cos 45 + \sin 60 \sin 45$

$= \cos(60 - 45)$

$= \cos 15^0$

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TOPIC: MOLE CONCEPT

Answers & Solutions

21. (A)

$\frac{4}{9} = \frac{40/A_X}{60/16} \Rightarrow$ Atomic mass of X, $A_X = 24$

22. (B)

Average mass number $= 0.6 \times 35 + 0.4 \times 37 = 35.8$

23. (B)

Number of electrons $= (2 \times 6 + 4 \times 8 + 2) \times \frac{4.4}{88} \times N_A = 2.3\ N_A$

24. (C)

Number of g-molecules of oxygen $= \frac{6.022 \times 10^{24}}{6.022 \times 10^{23}} \times \frac{1}{2} = 5$

25. (C)

$12 = \frac{3 \times 80}{304 + 106n} \times 100 \Rightarrow n = 16$

26. (C)

$C_8H_{18} + \frac{25}{2} O_2 \rightarrow 8CO_2 + 9H_2O$

8×44 g　　9×18 g

$\therefore$ 7.04 g　　$\frac{9 \times 18}{8 \times 44} \times 7.04 = 3.24$ g

27. (A)

$3SrCl_2 + 2K_3PO_4 \rightarrow Sr_3(PO_4)_2 + 6KCl$

0.3 mole　0.2 mole　　0.6 mole

28. (B)

$Fe + S \rightarrow FeS$

56 g　32 g

29. (B)

$I_2 + Cl_2 \rightarrow 2ICl$ and $I_2 + 3Cl_2 \rightarrow 2ICl_3$

30. (D)

$2NaI \rightarrow Na_2SO_4$

2×150 g   142 g

$\therefore x$ g $\frac{142}{300} \times x$ g

$2NaCl \rightarrow Na_2SO_4$

2×58.5 g   142 g

$\therefore (100 - x)$ g $\frac{142}{1170} \times (100 - x)$ g

From the question, $\frac{142}{300} \times x + \frac{142}{117} \times (100 - x) = 100$

Hence, the percentage of NaI in the mixture = 28.86%

31. (C)

$2a \times 12 + a \times 32 = 14$

$\Rightarrow$ Moles of carbon = $2a$ = 0.5

32. (D)

Volume of solution $= \frac{100}{10} \times 180 = 1800\,\text{ml}$

33. (C)

$d = \frac{25}{20} = 1.25\,\text{g/mL}$

34. (A)

1 mole of $F_2 \equiv$ 2 mole of CO $F_3 \equiv \frac{1}{2n}$ mole $(CF_2)_n$

= 38 g $= \frac{1}{2n} \times 50n = 25\,\text{g}$

$\therefore$ Mass of F_2 needed $= \frac{38}{25} \times 1 = 1.52$ kg

35. (C)

Overall reaction: A + B + C $\rightarrow$ D

A	B	C	D
5 moles	6 moles	excess	5 moles
L.R.			

36. (A)

$\frac{1}{2} \times 1 + \frac{1}{2} \times 3 = n \times 2$ (POAC on Fe)

37. (C)

No. of moles of $P_4 = \frac{12.4}{124} = 0.1$

No. of moles of $O_2 = \frac{12.8}{32} = 0.4$

Let moles of P_4O_6 and P_4O_{10} be n_1 and n_2 respectively.

POAC on 'P' $4 \times 0.1 = n_1 \times 4 + n_2 \times 4$ ...(i)

POAC on 'O' $2 \times 0.4 = n_1 \times 6 + n_2 \times 10$ ...(ii)

$\Rightarrow \quad n_1 = n_2 = 0.05\,\text{mol}$

38. (D)

$M \times 20 = 18.46 \times 0.042 \times 2$

39. (B)

$\frac{100 \times 0.002 \times 3}{1000}$

40. (D)

$\frac{\left[\frac{20 \times 0.1}{1000}\right] \times \left[\frac{1}{2}\right] \times [74.5]}{1} \times 100$

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TOPIC: QUDRTIC EQUATIONS

Answers & Solutions

41. (D)

Since a, b, c $\in$ integers & one root is $3+\sqrt{5}$

other root has to be $3-\sqrt{5} \Rightarrow \boxed{\text{sum}=6}$

42. (A)

Product of roots $=\dfrac{\alpha^2+1}{\alpha}=-2$

$\Rightarrow \alpha^2+2\alpha+1=0 \qquad \Rightarrow (\alpha+1)^2=0$

$\Rightarrow \alpha=-1$

43. (C)

Given $\alpha+\beta=-b$

$\alpha\beta=-c$

$\therefore$ quadratic whose roots b, c is $x^2-(b+c)x+bc=0$

Or $x^2+(\alpha+\beta+\alpha\beta)x+\alpha\beta(\alpha+\beta)=0$

44. (D)

$x^2+ax+10=0$

$x^2+bx-10=0$

Common root condition

$400=(b-a)(-10)(a+b)$

$\therefore a^2-b^2=40$

45. (D)

For roots imaginary $D<0 \Rightarrow 25-4k<0 \Rightarrow \boxed{k>\dfrac{25}{4}}$

Hence least integral k is 7

46. (A)

If p, q roots of $x^2+px+q=0 \qquad \Rightarrow p+q=-p$ &

$pq=q \Rightarrow q=0$ or $p=1$

If $q=0 \Rightarrow p=0$ or if $p=1, q=-2$

47. (B)

Clearly $x^3+2x^2+2x+1=0$

$\Rightarrow \alpha+\beta+\gamma=-2$ we know

$\alpha\beta+\beta\gamma+\gamma\alpha=2$ $\alpha^2+\beta^2+\gamma^2=(\alpha+\beta+\gamma)^2-2(\alpha\beta+\beta\gamma+\gamma\alpha)$

$=4-4=0$

48. (C)

For equal roots $D=0$

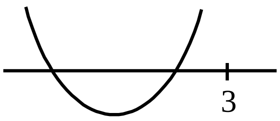
$\Rightarrow (m+2)^2-4(m^2-4m+4)=0$

$\Rightarrow m^2+4m+4-4m^2+16m-16=0$

$\Rightarrow 3m^2-20m+12=0$ $\Rightarrow (3m-2)(m-6)=0$

$\Rightarrow m=2/3, m=6$

49. (B)



For both roots to be less than 3

(i) $D \geq 0$ $\Rightarrow$ $a \in (-\infty, 3]$

(ii) $\frac{-b}{2a} < 3$ $\Rightarrow$ $a \in (-\infty, 3)$

(iii) $f(3) > 0$ $\Rightarrow$ $a \in (-\infty, 2) \cup (3, \infty)$

Upon taking intersection $a \in (-\infty, 2)$

50. (D)

Let $y=\frac{x^2-2x+5}{x^2-2x-8}$

$\Rightarrow$ $(y-1)x^2-2(y-1)x-(8y+5)=0$

For x real $D \geq 0$ $4(y-1)^2+4(y-1)(8y+5) \geq 0$

$\Rightarrow (y-1)[y-1+8y+5] \geq 0$

$\Rightarrow (y-1)(9y+4) \geq 0$

$\Rightarrow y \in \left(-\infty, \frac{-4}{9}\right] \cup [1, \infty)$

But for no x, y =1 $\Rightarrow$ Range $\left(-\infty, -\frac{4}{9}\right] \cup (1, \infty)$

51. (D)

Let quadratic be $ax^2+bx+c=0$ ..(i)

$\therefore$ 1st student $ax^2+bx+p=0$ roots 3, 2 $\Rightarrow 5=-b/a$

2nd student $ax^2+qx+c=0$ roots $-6,1 \Rightarrow -6=c/a$

Hence equation (i) becomes $x^2-5x-6=0$

$$\Rightarrow (x-6)(x+1)=0 \quad \text{roots } -1, 6$$

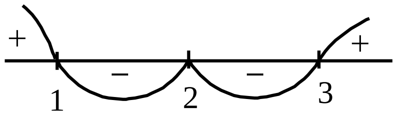
52. (C)

Given $(x-\alpha)(x-\beta)=(x-a)(x-b)-c$

$\Rightarrow \quad (x-a)(x-b)=(x-\alpha)(x-\beta)+c$

The roots of $(x-\alpha)(x-\beta)+c=0$ are a, b

53. (B)



Given $(x-1)^7(3-x)^5(x-2)^4>0$

$\Rightarrow \quad (x-1)^7(x-3)^5(x-2)^4<0$

Hence $x\in(1,2)\cup(2,3)$

54. (A)

$$x-\frac{1}{x^2-4}=2-\frac{1}{x^2-4}$$

$\Rightarrow x=2$ but 2 is not indomain $\Rightarrow$ No solution

55. (A)

$$x=\sqrt{1+\sqrt{1+\sqrt{1+.....}}}$$

$\Rightarrow x=\sqrt{1+x} \quad \Rightarrow x^2-x-1=0$

$$\Rightarrow x=\frac{1+\sqrt{5}}{2}$$

Negative solution rejected

56. (C)

Given $C=2,\ \dfrac{-b}{2a}=1$ & $\dfrac{-d}{4a}=3$

$\Rightarrow b=-2a$ & $2-\dfrac{b^2}{4a}=3$

$\Rightarrow 2-a=3\Rightarrow a=-1, b=2$

So quadratic is $-x^2+2x+2=0$

Or $x^2-2x-2=0$

$\Rightarrow$ both roots irrational

57. (D)

Let roots be $\alpha, \alpha/3$

$\Rightarrow \frac{4\alpha}{3} = \frac{1+4\lambda}{3}$ & $\frac{\alpha^2}{3} = \frac{\lambda^2+2}{3}$

$\Rightarrow \alpha = \frac{1+4\lambda}{4}$ $\Rightarrow$ $\frac{(4\lambda+1)^2}{16} = \lambda^2+2$

$\Rightarrow$ $\cancel{16\lambda^2} + 32\lambda + 1 = \cancel{16\lambda^2} + 32$

$\lambda = \frac{31}{32}$

58. (B)

If $a+b+c=0$ $\Rightarrow$ one root is 1

Clearly (B) option isn't feasible

As if $\sec\theta = 1$ $\Rightarrow \text{cosec}\,\theta \to \infty$ & vice-versa

59. (A)

$\sqrt{x+1} - \sqrt{x-1} = \sqrt{4x+1}$

Square $x+1+x-1-2\sqrt{x^2-1} = 4x+1$

$-2\sqrt{x^2-1} = 2x+1$

Square $4(\cancel{x^2}-1) = \cancel{4x^2} + 4x + 1$

$\Rightarrow$ $x = -\frac{5}{4} \rightarrow$ Which doesn't satisfy original equation

60. (C)

If $-3+5i$ is a root

$\Rightarrow -3-5i$ is another root

Sum = $-6 = -P$ $\therefore p+q = 40$

Product = $34 = q$