# ACE OF PACE

**ADVANCED (CODE: 11)** **ANSWERS KEY** **DATE: 22/04/2018**

| Question | Answer | Question | Answer |
|---|---|---|---|
| 1 | A | 26 | A |
| 2 | A | 27 | A |
| 3 | B | 28 | B |
| 4 | D | 29 | A |
| 5 | A | 30 | A |
| 6 | A | 31 | B |
| 7 | C | 32 | D |
| 8 | B | 33 | A |
| 9 | B | 34 | D |
| 10 | C | 35 | B |
| 11 | A | 36 | A |
| 12 | A | 37 | A |
| 13 | A | 38 | C |
| 14 | C | 39 | B |
| 15 | A | 40 | B |
| 16 | B | 41 | B |
| 17 | B | 42 | B |
| 18 | A | 43 | D |
| 19 | B | 44 | A |
| 20 | A | 45 | A |
| 21 | D | 46 | C |
| 22 | B | 47 | C |
| 23 | D | 48 | B |
| 24 | B | 49 | D |
| 25 | A | 50 | B |

# ACE OF PACE

**ADVANCED (CODE: 11)** **SOLUTION** **DATE: 22/04/2018**

**1.** **(A)**

$y=\sqrt{3}+2$

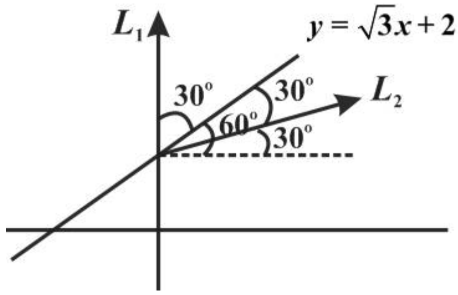
$m_1=\sqrt{3}$ (angle at $60^o$ to the $x$-axis)

$\theta=30^o$ let after slope be $m_2=m$

Then, $\tan\theta=\left|\frac{m_1-m_2}{1+m_1m_2}\right|$

$\tan 30=\left|\frac{\sqrt{3}-m}{1+\sqrt{3}m}\right|$

$\frac{1}{\sqrt{3}}=\left|\frac{\sqrt{3}-m}{1+\sqrt{3}m}\right|$




So, $\frac{1}{\sqrt{3}}=\left|\frac{\sqrt{3}-m}{1+\sqrt{3}m}\right|$

$1+\sqrt{3}m=3-\sqrt{3}m$

$\cancel{2}\sqrt{3}m=\cancel{2}$

$m=\frac{1}{\sqrt{3}}$

So, first slope may be $30^o$ and second line can be parallel to $y$-axis.

**2.** **(A)**

$\log_{10}\left(2^x+x-41\right)=x\left(1-\log_{10}5\right)$

$\log_{10}\left(2^x+x-41\right)=x\log_{10}2=\log_{10}\left(2^x\right)$

$2^x+x-41=2^x$

$\Rightarrow \quad x=41$

**3.** **(B)**

Take log both sides with base 2

$\left(\frac{3}{4}(\log_2 x)^2+\log_2 x-\frac{5}{4}\right)\log_2 x=\frac{1}{2}$

Net $\log_2 x=y$

$\left(\frac{3}{4}y^2+y-\frac{5}{4}\right)y=\frac{1}{2}$

$3y^3+4y^2-5y-2=0$

$3y^2\left(y-1\right)+7y\left(y-1\right)+2\left(y-1\right)=0$

$\Rightarrow \left(y-1\right)\left(3y^2+7y+2\right)=0$

$$\Rightarrow (y-1)(3y+1)(y+2)=0$$

$$\Rightarrow y=1,\ y=-2,\ y=-\frac{1}{3}$$

$$\therefore x=2;\frac{1}{4};\frac{1}{2^{1/3}}$$

$$\Rightarrow x_1x_2x_3=\frac{1}{\sqrt[3]{16}}$$

So, $a=16,\ b=3$

$a+b=19$

**4.** **(D)**

Let $y=\frac{1}{\log_2 N}\cdot\frac{1}{\log_N 8}\cdot\frac{1}{\log_{32} N}\cdot\frac{1}{\log_N 128}$

$$=\frac{1}{\frac{\log N}{\log 2}}\cdot\frac{1}{\frac{\log 8}{\log N}}\cdot\frac{1}{\frac{\log N}{\log 32}}\cdot\frac{1}{\frac{\log 128}{\log N}}$$

$$=\frac{1}{\frac{1}{\log 2}}\cdot\frac{1}{3\log 2}\cdot\frac{1}{\frac{1}{6\log 2}}\cdot\frac{1}{7\log 2}$$

$$y=\frac{5}{21}$$

**5.** **(A)**

Let $y=\sqrt{\frac{5}{4}+\sqrt{\frac{3}{2}}}+\sqrt{\frac{5}{4}-\sqrt{\frac{3}{2}}}$

$$y^2=\frac{5}{4}+\sqrt{\frac{3}{2}}+\frac{5}{4}-\sqrt{\frac{3}{2}}+2\left(\sqrt{\frac{5}{4}+\sqrt{\frac{3}{2}}}\right)\left(\sqrt{\frac{5}{4}-\sqrt{\frac{3}{2}}}\right)$$

$$y^2=\frac{5}{2}+2\left(\sqrt{\frac{25}{16}-\frac{3}{2}}\right)$$

$$y^2=\frac{5}{2}+2\left(\sqrt{\frac{25-24}{16}}\right)$$

$$y^2=\frac{5}{2}+2\cdot\frac{1}{4}$$

$$y^2=\frac{5}{2}+\frac{1}{2}$$

$$y^2=\frac{6}{2}=3$$

$$y=\pm\sqrt{3}$$

So, $\sqrt{3}=\tan 60=\tan\frac{\pi}{3}$

**6.** **(A)**

$7x\le -59$ and $7x\ge -80$

$$x\in\left[\frac{-80}{7},\frac{-59}{7}\right]$$

**7.** **(C)**

$y = a^{\frac{1}{1-\log_a x}}$

$\log_a y = \frac{1}{(1-\log_a x)}\log_a a$

$\log_a y = \frac{1}{1-\log_a x}$

$z = a^{\frac{1}{1-\log_a y}}$

$\log_a z = \left(\frac{1}{1-\log_a y}\right)\log_a a$

$\log_a z = \frac{1}{1-\log_a y}$

$\Rightarrow \quad \log_a z = \frac{1}{\frac{1}{1} - \frac{1}{1-\log_a x}}$

$\log_a x = \frac{1-\log_a x}{1-\log_a x - 1}$

$\log_a z = \frac{1-\log_a x}{-\log_a x}$

$-\log_a z \log_a x = 1 - \log_a x$

$\log_a x - \log_a z \log_a x = 1$

$\log_a x(1-\log_a z) = 1$

$\log_a x = \frac{1}{1-\log_a z} \qquad \Rightarrow \qquad x = a^{\frac{1}{1-\log_a z}}$

**8.** **(B)**

$4^{-x+0.5} - 7.2^{-x} - 4 < 0$

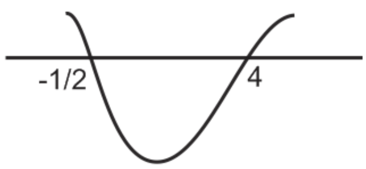
$(4^{-x})(4^{0.5}) - 7.2^{-x} - 4 < 0$

$(2^{-x})^2 \cdot 2 - 7.2^{-x} - 4 < 0$

Let $2^{-x} = y$

$2y^2 - 7y - 4 < 0$

$(2y+1)(y-4) < 0$




$y < 4$

$2^{-x} < 2^2$

$x > -2$

$x \in (-2, \infty)$

**9.** **(B)**

Let $x$ & $y$ be 2 numbers

$a = \frac{x+y}{2} \qquad ...(1)$

Also $p$ and $q$ are 2g.m.s between $x$ & $y$

Then common ratio $r = \left(\frac{y}{x}\right)^{\frac{1}{3}}$

$\therefore \qquad p = xr = x^{2/3}y^{1/3}$

And $\qquad q = xr^2 = x^{1/3}y^{2/3}$

$\Rightarrow \qquad p^3 + q^3 = x^2y + xy^2$

$= xy(x+y)$

$= 2axy$
$= 2apq$

**10.** **(C)**

6, *a* and *b* are in H.P.

$\Rightarrow \frac{1}{6}, \frac{1}{a}, \frac{1}{b}$ are in A.P.

$\Rightarrow \frac{2}{a} = \frac{1}{6} + \frac{1}{b} \Rightarrow b = \frac{6a}{12-a}$

$a \in \{3, 4, 6, 8, 9, 10, 11\}$

**11.** **(A)**

$e^{\left\{\left(\sin^2 x + \sin^4 x + ........\infty\right)\log_e 2\right\}}$

$= e^{\left\{\left(\frac{\sin^2 x}{1-\sin^2 x}\right)\log_e 2\right\}}$

$= e^{\frac{\sin^2 x}{\cos^2 x}} + \log_e 2$

$= e^{\tan^2 x \log_e 2}$

$= e^{\log_e 2 \tan^2 x}$

$= 2^{\tan^2 x}$

And $x^2 - 17x + 16 = 0$

$x^2 - 16x - x + 16 = 0$

$x(x-16) - 1(x-16) = 0$

$(x-1)(x-16) = 0$

$x = 1$ or $x = 16$

| | |
|---|---|
| $2^{\tan^2 x} = 2^o$ | $2^{\tan^2 x} = 2^4$ |
| $\tan^2 x = 0$ | $\tan^2 x = 4$ |
| $\tan x = 0$ | $\tan x = 2$ |

So, $y = \frac{2\cos x}{\sin x + 2\cos x} = \frac{1}{\frac{1}{2}\frac{\sin x}{\cos x} + 1}$

$= \frac{1}{\frac{1}{2}\tan x + 1}$

$= \frac{1}{\frac{1}{\cancel{2}} \cdot \cancel{2} + 1} = \frac{1}{2}$

**12.** **(A)**

$2 + 3 + 6 + 11 + 18 + ......... = 2 + (2 + 1^2) + (2 + 2^2) + (2 + 3^2) + ....$

$\therefore t_n = 2 + (n-1)^2$

$\Rightarrow t_{50} = 2 + (50-1)^2$

$= 2 + 49^2$

**13. (A)**

Let $ax^2+bx+c=0$ has roots $\alpha$ & $\beta$

Given, $\alpha+\beta=\frac{1}{\alpha^2}+\frac{1}{\beta^2}$

$$-\frac{b}{a}=\frac{\alpha^2+\beta^2}{\alpha^2\beta^2}$$

$$-\frac{b}{a}=\frac{(\alpha+\beta)^2-2\alpha\beta}{(\alpha\beta)^2}$$

$$-\frac{b}{a}=\frac{b^2/a^2-2c/a}{c^2/a^2}$$

$$\left(-\frac{b}{a}\right)=\frac{\frac{b^2-2ac}{a^2}}{\frac{c^2}{a^2}}$$

$$-\frac{b}{a}=\frac{b^2-2ac}{c^2}$$

$$-bc^2=ab^2-2a^2c$$

$$2a^2c=ab^2+bc^2$$

$\therefore$ A.P.

**14. (C)**

$$x^2-3Kx+2e^{2\log K}-1=0$$

Product $=\frac{c}{a}=\left(2e^{2\log K}-1\right)=7$

$$2e^{2\log K}=8$$

$$e^{\log_e K^2}=4$$

$$K^2=4$$

$$K=2$$

Sum of roots $=3K$

$=3\cdot 2=6$

**15. (A)**

$$(x-a)(x-b)+(x-b)(x-c)+(x-c)(x-a)=0$$

$$x^2-(a+b)x+ab+x^2-(b+c)x+bc+x^2-(a+c)x+ac=0$$

$$3x^2-(a+b+b+c+c+a)x+(ab+bc+ca)=0$$

$$3x^2-2(a+b+c)x+(ab+bc+ca)=0$$

$\therefore$ Roots equal, $D=0$

$$b^2=4ac$$

$$4(a+b+c)^2=4\cdot 3\cdot(ab+bc+ca)$$

$$a^2+b^2+c^2+2ab+2bc+2ac=3ab+3bc+3ca$$

$$\left(a^2+b^2+c^2\right)=(ab+bc+ca)$$

$\Rightarrow$ $a+b+c\neq 0$

**16.** **(B)**

For $x^2 + 2ax + 10 - 3a > 0 \quad \forall x \in R$

$D < 0$

$4a^2 - 40 + 12a < 0$

$a^2 - 10 + 3a < 0$

$a^2 + 3a - 10 < 0$

$a^2 + 5a - 2a - 10 < 0$

$(a+5)(a-2) < 0$

$\therefore \; -5 < a < 2$

**17.** **(B)**

$3x + 2y + 4 = 0$

$2x + 5y - 1 = 0$

On solving we get $x = -2$, $\quad y = 1$

So, equation will be of the form $(y-1) = m(x+2)$

$(y = mx + 2m + 1)$ or $mx - y + (2m+1) = 0$

Distance from $(2,-1)$ is 2

$$\left|\frac{2m+1+2m+1}{\sqrt{1+m^2}}\right| = 2$$

$$\cancel{2}^{\cancel{2}}(2m+1)^2 = \cancel{4}(1+m^2)$$

$4m^2 + \cancel{1} + 4m = \cancel{1} + m^2$

$3m^2 + 4m = 0$

$m(3m+4) = 0$

$m = 0, \quad m = -\frac{4}{3}$

So either $y = 1$ or $(y-1) = -\frac{4}{3}(x+2)$

$3y - 3 = -4x - 8$

$4x + 3y + 5 = 0$

**18.** **(A)**

$ax + by + c = 0$

$by = -ax - c$

$$y = \left(-\frac{a}{b}\right)x - \left(\frac{c}{b}\right)$$

On comparing with $y = mx + c$

$$m = -\frac{a}{b}$$

Now parallel line have same slopes

So $\quad m = -\frac{a}{b}$

**19.** **(B)**

$$\left(x-\frac{1}{x}\right)=3$$

$$x^4+\frac{1}{x^4}=\left(x^2\right)^2+\left(\frac{1}{x^2}\right)^2+2\left(x^2\right)\left(\frac{1}{x^2}\right)-2$$

$$=\left(x^2+\frac{1}{x^2}\right)^2-2$$

$$=\left[(x)^2+\left(\frac{1}{x}\right)^2-2\left(x^2\right)\left(\frac{1}{x^2}\right)+2\right]^2-2$$

$$=\left[\left(x-\frac{1}{x}\right)^2+2\right]^2-2$$

$$=[9+2]^2-2$$

$$=(11)^2-2$$

$$=121-2$$

$$=119$$

**20.** **(A)**

$$x=2+\sqrt{3}$$

$$x^4=\left(2+\sqrt{3}\right)^4$$

$$x^{-4}=\frac{1}{\left(2+\sqrt{3}\right)^4}$$

Now $2+\sqrt{3}=\dfrac{\left(2+\sqrt{3}\right)\left(2-\sqrt{3}\right)}{\left(2-\sqrt{3}\right)}=\dfrac{4-3}{\left(2-\sqrt{3}\right)}=\dfrac{1}{\left(2-\sqrt{3}\right)}$

So $\dfrac{1}{\left(2+\sqrt{3}\right)^4}=\left(2-\sqrt{3}\right)^4$

So $x^4+x^{-4}=\left(2+\sqrt{3}\right)^4+\left(2-\sqrt{3}\right)^4$

$$=\left({}^4C_0(2)^4+{}^4C_1\cancel{(2)^3}\left(\sqrt{3}\right)+{}^4C_2(2)^2\left(\sqrt{3}\right)^2+{}^4C_3\cancel{(2)^1}\left(\sqrt{3}\right)^3+{}^4C_4\left(\sqrt{3}\right)^4\right)$$
$$+\left({}^4C_0 2^4-{}^4C_1\cancel{(2)^3}\left(\sqrt{3}\right)+{}^4C_2(2)^2\left(\sqrt{3}\right)^2-{}^4C_3\cancel{(2)^1}\left(\sqrt{3}\right)^3+{}^4C_4\left(\sqrt{3}\right)^4\right)$$

$$=2\left(1.2^4+\frac{4.3}{\cancel{2}}.\cancel{4}^2.3+9\right)$$

$$=2(16+72+9)$$

$$=2(97)$$

$$=194$$

**21.** **(D)**

Let $\dfrac{x}{2}=\dfrac{4}{5}=\dfrac{z}{7}=K$, then $x=2K,\ y=5K,\ z=7K$

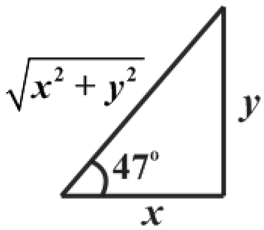
Then $\dfrac{x+y+z}{x}=\dfrac{2K+5K+7K}{2K}=\dfrac{14K}{2K}=7$

**22.** **(B)**

$$\sin 43=\frac{x}{\sqrt{x^2+y^2}}$$

$$\Rightarrow \cos(90-43)=\frac{x}{\sqrt{x^2+y^2}}$$

$$\cos 47=\frac{x}{\sqrt{x^2+y^2}}$$




And $\sin 47=\dfrac{\frac{y}{\sqrt{x^2+y^2}}}{\frac{x}{\sqrt{x^2+y^2}}}=\dfrac{y}{x}$

**23.** **(D)**

$$\alpha^2+\alpha\beta+\beta^2=(\alpha+\beta)^2-2\alpha\beta+\alpha\beta$$

$$=(\alpha+\beta)^2-\alpha\beta$$

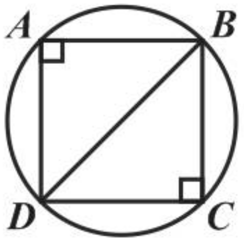
$$\frac{21}{4}=\left(\frac{-5}{2}\right)^2-\frac{K}{2}$$

$$\frac{21}{4}+\left(\frac{5}{2}\right)^2=\frac{-K}{2}$$

$$\frac{21}{4}-\frac{25}{4}=\frac{-K}{2}$$

$$\frac{+K}{2}=\frac{+\cancel{4}}{\cancel{4}}=1$$

**24.** **(B)**




$BD$ is diameter ($b/c$ angle in semicircle is night angle)

$2r=BD=a\sqrt{2}$ ($a$ = side of squre)

$\sqrt{2}2\cdot 16=a\sqrt{2}$

$a=16\sqrt{2}$

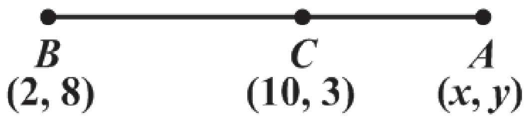
Area of square $=a^2$

$=(16\sqrt{2})^2$

$=16\cdot 16\cdot 2$

$=512$

**25.** **(A)**




$$x=\frac{3(10)-2(2)}{3-2}=\frac{30-4}{1}=26$$

$$y=\frac{3(3)-2(5)}{3-2}=\frac{9-10}{1}=-1$$

$$A(x, y)=(25, -1)$$

**26.** **(A)**

$$S=\{1,2,3,4,5,6\}$$

$$E=\{2,3,5\}$$

$$P(E)=\frac{n(E)}{n(S)}=\frac{3}{6}=\frac{1}{2}$$

**27.** **(A)**

Slope remain same for collinear points

$$\frac{3+K}{3-K}=\frac{6-3}{6-3}$$

$$\frac{3+K}{3-K}=1$$

$$3+K=3-K$$

$$2K=0$$

$$K=0$$

**28.** **(B)**

Face card $=\{K, Q, J, A\}$

No. of Black face cards $n(E)=8$

$$n(S)=52$$

$$P(E)=\frac{8}{52}=\frac{4}{26}=\frac{2}{13}$$

**29.** **(A)**

$$n(S)=\left\{\begin{matrix} (1,1) & (1,2) & (\mathbf{1,3}) & (1,4) & (1,5) & (1,6) \\ (2,1) & (\mathbf{2,2}) & (2,3) & (2,4) & (2,5) & (\mathbf{2,6}) \\ (\mathbf{3,1}) & (3,2) & (3,3) & (3,4) & (\mathbf{3,5}) & (3,6) \\ (4,1) & (4,2) & (4,3) & (\mathbf{4,4}) & (4,5) & (4,6) \\ (5,1) & (5,2) & (\mathbf{5,3}) & (5,4) & (5,5) & (5,6) \\ (6,1) & (\mathbf{6,2}) & (6,3) & (6,4) & (6,5) & (\mathbf{6,6}) \end{matrix}\right\}$$

$$n(E)=\{(1,3)(2,2)(3,1),(2,6)(3,5)(4,4)(5,3)(6,2),(6,6)\}$$

$$P(E)=\frac{9}{36}=\frac{1}{4}$$

**30.** **(A)**

$$x=\cos^4\left(\frac{\pi}{6}\right)-\sin^4\left(\frac{\pi}{6}\right)$$
$$=\left(\cos^2\frac{\pi}{6}-\sin^2\frac{\pi}{6}\right)\left(\cos^2\frac{\pi}{6}+\sin^2\frac{\pi}{6}\right)$$
$$=\cos\left(\frac{2\pi}{6}\right)-1$$
$$=\cos\frac{\pi}{3}$$
$$=\frac{1}{2}$$

**31.** **(B)**

$$4=\frac{1-3+\alpha}{3}, \qquad -6=\frac{-2+3+\beta}{3}$$
$$\alpha+\beta=-5$$

**32.** **(D)**

$$f(x)=2x^2+4x-5$$

Equation $2x^2+4x-5=0$

$a>0$

$$D=b^2-4ac$$
$$=16+4\cdot5\cdot2$$
$$=16+40=50>0$$

Then minimum value occurs at $x=\frac{-6}{2a}$ and $y=\frac{-D}{4a}$

$$\frac{-D}{4a}=\frac{-56}{4\cdot2}=-7$$

**33.** **(A)**

$$\sin x+\sin^2 x=1 \qquad \ldots(1)$$
$$\sin x=1-\sin^2 x$$
$$\sin x=\cos^2 x \qquad \ldots(2)$$

By (2)

$$\cos^6 x+3\cos^8 x+3\cos^{10} x+\cos^{12} x$$
$$=\sin^3 x+3\sin^4 x+3\sin^5 x+\sin^6 x$$
$$=(\sin x)^3+(\sin^2 x)^3+3\sin x(\sin^2 x)^2+3(\sin x)^2\sin^2 x$$
$$=(\sin x+\sin^2 x)^3$$
$$=(1)^3$$
$$=1$$

**34.** **(D)**

Area of parallelogram = $B\times$ H

72 = 8 × H

$H$ = 9 cm

**35.** **(B)**

Area $=\frac{1}{2}b\times h$

$30=\frac{1}{2}x\times b$

$x=\frac{60}{b}$

So point $C=\left(\frac{60}{b},0\right)$

**36.** **(A)**

Series is $a, a+d, a+2d, .........$

$a=16,\ d=3$

So, $a+4d=16+4\times3$

$=28$

**37.** **(A)**

First letter of the first, second, third ……… terms are move one, two, three, four, five, …… steps forward respectively and the second letter are moved two, three, four, five, …….. steps forward respectively. So missing term would be *FL*

**38.** **(C)**

Series is *aaabb*/*aaabb*/*aaabb*.
So, *aabab* is correct.

**39.** **(B)**

In 1st figure, $5\times4+6=26$
In 3rd figure, $8\times3+5=29$
So, $6\times3+4=18+4=22$

**40.** **(B)**

$(9-3)+(7-5)=8$

$(6-4)+(5-0)=7$

So, $(10-5)+(7-5)=9$

**41.** **(B)**

K U M A R
↓ ↓ ↓ ↓ ↓ (+1) letter
L V N B S

E M O T I O N A L
⇒ (+1) letter ↓ ↓ ↓ ↓ ↓ ↓ ↓ ↓ ↓
F N P U J P O B M

**42.** **(B)**

In given word, Z = 1, Y = 2, X = 3, ……. C= 24 , B = 25, A = 26.
So, RAT = 9 + 26 + 7 = 42
CAT = 24 + 26 + 7 = 57
Similarly, LATE = 15 + 26 + 7 + 22 = 70

**43.** **(D)**
Lizard is can animal which crawls & hence is called flying.

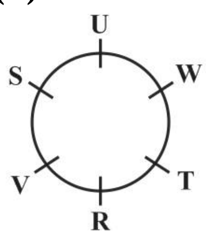
**44.** **(A)**
Second, section, seldom, select, selfish, seller, send
↓
Middle in alphabetical order

**45.** **(A)**
Required numbers in descending order are
57, 54, 51, 48, 45, 42, 39, 36, 33, 30, 27, 24, 21, 18, 15, 12, 9
$10^{th}$ number from bottom is 36.

**46.** **(C)**
Using proper signs
$(3 \times 15 + 19) \div 8 - 6 = 64 \div 8 - 6 = 8 - 6 = 2$

**47.** **(C)**

**48.** **(B)**




**49.** **(D)**
Sex of Q is not given hence the exact relationship between N & Q cannot be established.

**50.** **(B)**