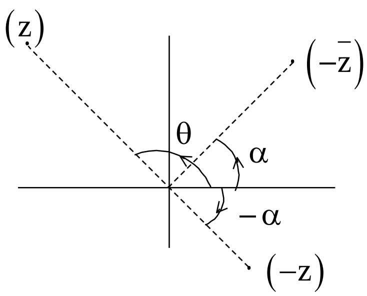
WINDOW TO JEE MAINS

Q.1 (B)

$|z| = |w|$



$\Rightarrow \arg(z) + \arg(w) = \pi$

$\Rightarrow \theta + |-\alpha| = \pi$

$\Rightarrow \theta + \alpha = \pi$

$\Rightarrow \therefore -\bar{z} = w$

$\Rightarrow z = -\overline{w}$

Q.2 (C)

$|z-4| < |z-2|$

$\Rightarrow \sqrt{(x-4)^2 + y^2} < \sqrt{(x-2)^2 + y^2}$

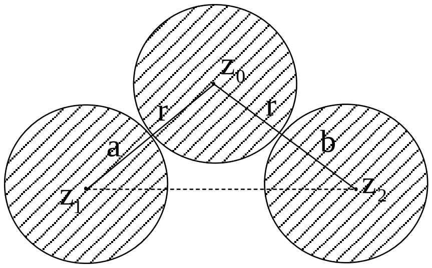
Take square – 8x + 16 < – 4x + 4

$\Rightarrow$ 4x > 12

$\Rightarrow$ x > 3

$\Rightarrow$ Re (z) > 3

Q.3



$|z - z_1| = a$

$\Rightarrow |z - z_2| = b$

$\Rightarrow |z_0 - z_1| = a + r$

$\Rightarrow |z_0 - z_2| = b + r$

$\Rightarrow |z_0 - z_1| - |z_0 - z_2| = (a - b)$

As z_0 is a point more such that subtraction of its distances from 2 fixed point is constant the locus of point z_0 is hyperbola.

Q.4 (D)

$\left(\frac{1+i}{1-i}\right)^x = 1$

$\Rightarrow \left(\frac{\sqrt{2}e^{\frac{i\pi}{4}}}{\sqrt{2}e^{-\frac{i\pi}{4}}}\right)^x = 1$

$\Rightarrow \left(e^{\frac{i\pi}{2}}\right)^x = 1$

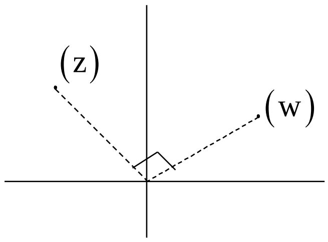
$\Rightarrow (i)^x = 1$

$\Rightarrow$ x = 4n; n is any integer.

Q.5

$|zw| = 1$

$\arg(z) - \arg(w) = \frac{\pi}{2}$



$\Rightarrow z = iwk$

$\Rightarrow \therefore$ where k is any positive constant.

$\Rightarrow \bar{z}w = (-i\bar{w}k)(w)$

$\Rightarrow \bar{z}w = -i|w|^2 k$

$\Rightarrow \because |\bar{z}w| = 1$

So, $\bar{z}w = -i$

Q.6 (B)

$z^2 + az + b = 0$ .... z_1 & z_2

$\Rightarrow z_1 + z_2 = -a$

$\Rightarrow z_1 z_2 = b$

As $0, z_1 \& z_2$ are vertices of equilateral triangle.

$\Rightarrow \therefore$ So, $z_1^2 + z_2^2 + 0^2 = z_1 z_2 + z_2(0) + (0)(z_1)$

$\Rightarrow (z_1 + z_2)^2 = 3z_1 z_2$

$\Rightarrow a^2 = 3b$

Q.8 (D)

$z = x - iy, z^{\frac{1}{3}} = p + iq$

$\Rightarrow z = (p + iq)^3$

$\Rightarrow z - iy = p^3 - iq^3 + 3p(iq)(p + iq)$

$\Rightarrow x = p^3 - 3pq^2, y = q^3 - 3p^2q$

$\Rightarrow \frac{x}{p} + \frac{y}{q} = (p^2 - 3q^2) + (q^2 - 3p^2)$

$\Rightarrow -2p^2 - 2q^2$

$\Rightarrow \therefore \frac{\frac{x}{p} + \frac{y}{q}}{p^2 + q^2} = -2$

Q.9

$|z^2 - 1| = |z|^2 + 1$

By triangle in-quality $|z_1 - z_2| = |z_1| + |-z_2|$

Only if z_1 & z_2 are in opposite direction

$\Rightarrow \therefore z^2 = -k$ where $k > 0$

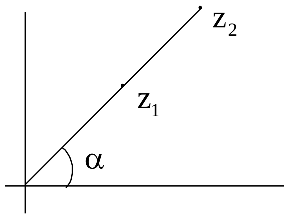
$\Rightarrow \therefore z = i\sqrt{k}$

$\Rightarrow$ z lies on imaginary axis.

Q.10

$|z_1 + z_2| = |z_1| + |z_2|$

Only if z_1 & z_2 are in same direction.



$\Rightarrow \therefore \arg(z_1) - \arg(z_2) = 0$

Q.11 (D)

$w = \dfrac{3z}{3z - i} |w| = 1$

$\Rightarrow w\overline{w} = \left(\dfrac{3z}{3z - i}\right)\left(\dfrac{3\overline{z}}{3\overline{z} + i}\right)$

$\Rightarrow 1\left(9|z|^2 + 3zi - 3i\overline{z} + 1\right) = 9|z|^2$

$\Rightarrow 3i\left(z - \overline{z}\right) = -1$

If z = x + iy

$\Rightarrow \therefore 3i(2iy) = -1$

$\Rightarrow y = \dfrac{1}{6}$

$\Rightarrow$ A straight line

Q.12

$S = \sum_{k=1}^{10}\left(\sin\dfrac{2k\pi}{11} + i\cos\dfrac{2k\pi}{11}\right) = i\sum_{k=1}^{10}\left(\cos\dfrac{2k\pi}{11} - i\sin\dfrac{2k\pi}{11}\right)$

if $\alpha = \cos\left(\dfrac{-2\pi}{11}\right) + i\sin\left(\dfrac{-2\pi}{11}\right)$

$\Rightarrow S = i\left(\alpha + \alpha^2 + \alpha^3 + \ldots\ldots\ldots\ldots + \alpha^{10}\right)$

$\Rightarrow z^{-11} = 1$sum of roots

$\Rightarrow z = (1)^{\frac{-1}{11}}$

$\Rightarrow z = \cos\left(\frac{-2k\pi}{11}\right) + i\sin\left(-\frac{2k\pi}{11}\right)$

$\Rightarrow \alpha^0 + \alpha + \alpha^2 + ... + \alpha^{10} = 0$

$\Rightarrow \alpha + \alpha^2 + \alpha^3 + ... + \alpha^{10} = -1$

$\Rightarrow \therefore S = -i$

Q.13 (D)

$z^2 + z + 1 = 0$ ……….. $w \;\&\; w^2$

Roots are $w \;\&\; w^2$

$\Rightarrow \left(z + \frac{1}{z}\right)^2 + \left(z^2 + \frac{1}{z^2}\right)^2 + ... + \left(z^6 + \frac{1}{z^6}\right)^2$

$\Rightarrow \left(w + \frac{1}{w}\right)^2 + \left(w^2 + \frac{1}{w^2}\right)^2 + \left(w^3 + \frac{1}{w^3}\right)^2 + ... + \left(w^6 + \frac{1}{w^6}\right)^2$

$\Rightarrow (-1)^2 + (-1)^2 + (2)^2 + (-1)^2 + (-1)^2 + (2)^2$

$\Rightarrow 12$

Q.14 (A)

$|z+4| \le 3$

$\Rightarrow ||z|-4| \le |z+4|$

$\Rightarrow ||z|-4| \le 3$

$\Rightarrow 1 \le |z| \le 7$

$\Rightarrow |z+1| \le ||z|-1|$

$\Rightarrow |z+1| \le |7-1|$

$\Rightarrow |z+1| \le 6$

Q.15 (D)

$\bar{z} = \dfrac{1}{i-1}$

$\Rightarrow z = \dfrac{1}{-i-1} = \dfrac{-1}{1+i} \times \dfrac{1-i}{1-i}$

$\Rightarrow z = \dfrac{-1+i}{2}$

Q.16 (C)

$x^2 - x + 1 = 0$

$\Rightarrow x = \dfrac{1 \pm \sqrt{1-4}}{2} = \dfrac{1 \pm i\sqrt{3}}{2}$

$\Rightarrow x = -w, -w^2$

$\Rightarrow \alpha^{2009} + \beta^{2009} = (-w)^{2009} + (-w^2)^{2009}$

$\Rightarrow (-1)\left[w^2 + w^4\right] = 1$

Q.17 (A)

$\alpha, \beta \in R$

$\Rightarrow z^2 + \alpha z + \beta = 0$

For roots Re (z) = 1

$\therefore \text{root } z_0 + 1 + iy$

$\Rightarrow \therefore (1+iy)^2 + \alpha(1+iy) + \beta = 0$

$\Rightarrow (1 - y^2 + \alpha + \beta) = 0,\ 2iy + \alpha iy = 0$

$\Rightarrow \alpha = -2$

$\Rightarrow \therefore 1 - y^2 - 2 + \beta = 0$

$\Rightarrow y^2 - 1 + \beta > 0$

$\Rightarrow \beta > 1$

Q.18 (C)

$w \neq 1$Cube root of unity.

$\Rightarrow (1+w)^7 = (-w^2)^7 = -(w^{14}) = -w^2$

$\Rightarrow$1 + w

$\Rightarrow \therefore$ 1 + w = A + Bw

$\Rightarrow$A = 1, B = 1

Q.19 (A)

$z \neq 1, \frac{z^2}{z-1} = k$; k is constant k + R

$\Rightarrow \frac{z^2}{z-1} = \frac{\bar{z}^2}{\bar{z}-1}$

$\Rightarrow z^2\bar{z} - z^2 = \bar{z}^2 z - \bar{z}^2$

$\Rightarrow z\bar{z}(z - \bar{z}) + (\bar{z} - z)(\bar{z} + z) = 0$

$\Rightarrow (z - \bar{z})(z\bar{z} - \bar{z} - z) = 0$

$\Rightarrow z = \bar{z}$ or $z\bar{z} - \bar{z} - \bar{z} = 0$

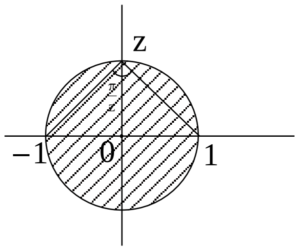
So real axis or circle.

Q.20

$|z| = 1$

$\Rightarrow \arg(z) = \theta$

$\Rightarrow$z lies on unit circle whose centre is “0”.



$$\Rightarrow \arg\left(\frac{z-1}{z+1}\right)=\frac{\pi}{2}$$

$$\Rightarrow \arg\left(\frac{z+1}{z-1}\right)=\frac{-\pi}{2}$$

$$\Rightarrow \arg\left(\frac{z+1}{1-z}\right)=\frac{\pi}{2}$$