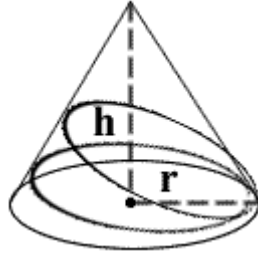


**Congratulations!!! To Arpit Shrivastava for the only fully correct solution.**

### Challenging Cone Problem



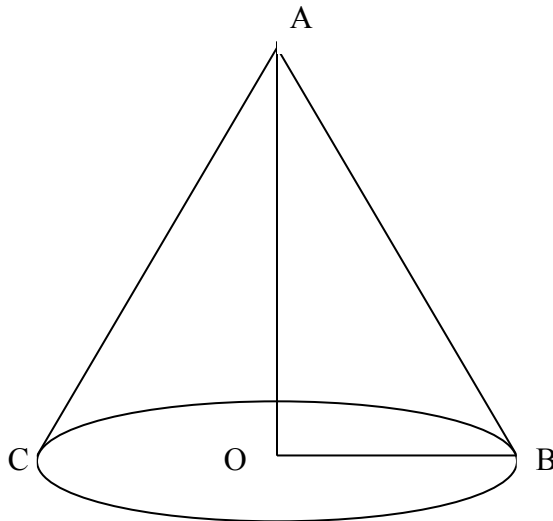
There is a cone with height  $h$  & radius  $r$ . An insect at the bottom edge of the cone needs to complete the journey around the axis of the cone. A few suggested paths are as highlighted. Find the shortest distance the insect has to cover.

### Solution to the Challenging Cone Problem

Let  $ABC$  be the cone with  $AO$ =height of the cone= $H$ ,  $BO$ =radius of the base= $R$  and  $AB$  is the slant height of the cone.

By Pythagoras theorem,  $AB=L=\sqrt{H^2 + R^2}$

Let the insect be situated at  $B$



Now let us suppose that the cone is cut along  $AB$ , then we get a sector of a circle with radius= $AB=L$  and length of the arc as the circumference of the base of the cone= $2\pi R$ .

Let the angle of the sector of the circle be  $\theta$ .

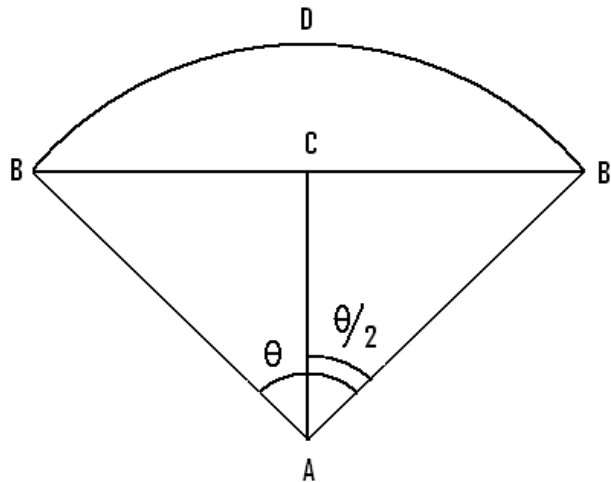
Now following three cases arise:

(i)  $\theta < \pi$

or (ii)  $\theta = \pi$

or (iii)  $\theta > \pi$

Case (i),  $\theta < \pi$



Here the insect is at B which has to reach B' (that is B only). Since the shortest path between two points is the straight line joining them. Therefore the shortest distance which the insect has to travel is BB'.

Here length of the arc = circumference of the cone

$$\Rightarrow (\theta/2\pi) 2\pi\sqrt{(H^2 + R^2)} = 2\pi R$$

$$\Rightarrow (\theta/2\pi) \sqrt{(H^2 + R^2)} = R$$

$$\Rightarrow \theta = 2\pi R / [\sqrt{(H^2 + R^2)}]$$

Now in  $\triangle ODB$ ,

$$\sin(\theta/2) = BD/OB$$

$$\Rightarrow \sin(\pi R / [\sqrt{(H^2 + R^2)}]) = BD / \sqrt{(H^2 + R^2)}$$

$$\Rightarrow BD = \sqrt{(H^2 + R^2)} \cdot \sin(\pi R / [\sqrt{(H^2 + R^2)}])$$

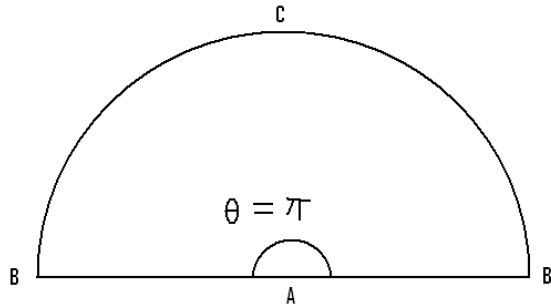
$$\Rightarrow BB' = 2BD$$

$$= 2 \cdot \sqrt{(H^2 + R^2)} \cdot \sin(\pi R / [\sqrt{(H^2 + R^2)}]) \quad \text{----(i)}$$

(This is the shortest distance)

## Case (ii), $\theta < \pi$

Here in this case  $BB' = 2AB = L = 2\sqrt{H^2 + R^2}$  is the shortest path.



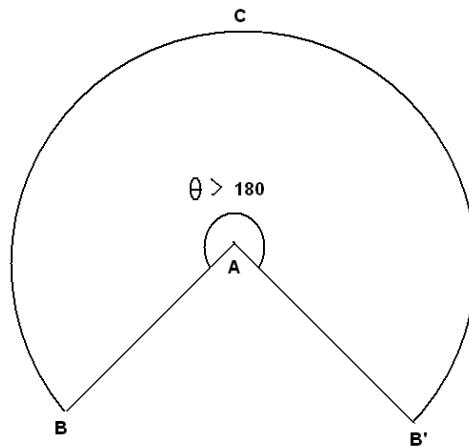
This is in the agreement with (i) as here  $BCB' = 2\pi R$

$$\Rightarrow \pi\sqrt{H^2 + R^2} = 2\pi R$$

$$\Rightarrow \sqrt{H^2 + R^2} = 2R$$

Substituting  $\sqrt{H^2 + R^2} = 2R$  in (i) we get  $BB' = 2\sqrt{H^2 + R^2}$

## Case (ii), $\theta < \pi$



Here in this case also the shortest path is along  $BAB' = BA + AB' = 2L = 2\sqrt{H^2 + R^2}$   
If the insect travels by any other path on the cone it will be greater than  $2L$ .

Therefore the shortest path insect has to travel in this case is  $2\sqrt{H^2 + R^2}$